A MODERN COURSE IN TRANSPORT PHENOMENA

This advanced text presents a unique approach to studying transport phenomena. Bringing together concepts from both chemical engineering and physics, it makes extensive use of nonequilibrium thermodynamics, discusses kinetic theory, and sets out the tools needed to describe the physics of interfaces and boundaries. More traditional topics such as diffusive and convective transport of momentum, energy, and mass are also covered. This is an ideal text for advanced courses in transport phenomena, and for researchers looking to expand their knowledge of the subject.

Also included:

- Novel applications such as complex fluids, transport at interfaces, and biological systems
- Approximately 250 exercises with solutions (included separately) designed to enhance understanding and reinforce key concepts
- End-of-chapter summaries

DAVID C. VENERUS is a Professor of Chemical Engineering in the Department of Chemical and Biological Engineering at Illinois Institute of Technology in Chicago. His research interests are in the areas of transport phenomena in soft matter, polymer science, and the rheology of complex fluids. Professor Venerus has received numerous teaching awards both within the Department and at the College of Engineering at Illinois Institute of Technology. He is a member of the American Institute of Chemical Engineers and of the Society of Rheology.

HANS CHRISTIAN ÖTTINGER is Professor of Polymer Physics at the ETH Zürich. His main research interest is in developing a general framework of nonequilibrium thermodynamics as a tool for describing dissipative classical and quantum systems. He is the author of *Stochastic Processes in Polymeric Fluids* (Springer, 1996), *Beyond Equilibrium Thermodynamics* (Wiley, 2005), and *A Philosophical Approach to Quantum Field Theory* (Cambridge, 2017).

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DAVID C. VENERUS
Illinois Institute of Technology, USA

HANS CHRISTIAN ÖTTINGER
ETH Zürich, Switzerland

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi - 110025, India

79 Anson Road, #06-04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781107129207
DOI: 10.1017/9781316416174

First published 2018

Printed in the United Kingdom by Clays, St Ives plc

A catalogue record for this publication is available from the British Library

Library of Congress Cataloging-in-Publication data
Names: Venerus, David, author. | Ottinger, Hans Christian, 1958– author.
Title: A modern course in transport phenomena / David Venerus, Illinois Institute of Technology, Hans Christian Ottinger, ETH Zentrum, Switzerland.
Description: New York, NY, USA : Cambridge University Press, [2017] | Includes bibliographical references and indexes.
Identifiers: LCCN 2017041983 | ISBN 9781107129207 (hardback : alk. paper)
Subjects: LCSH: Transport theory – Textbooks.
Classification: LCC QC175.25 .V46 2017 | DDC 530.4/75 – dc23 LC record available at https://lccn.loc.gov/2017041983

ISBN 978-1-107-12920-7 Hardback

To our families

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Preface

The subject of 'Transport Phenomena' is almost synonymous with the names Bird, Stewart, and Lightfoot (BSL). When these authors published their pioneering textbook in 1960, their goal was to establish this subject as one of the key engineering sciences:[1] "Knowledge of the basic laws of mass, momentum, and energy transport has certainly become important, if not indispensable, in engineering analysis. In addition, material in this text may be of interest to some who are working in physical chemistry, soil physics, meteorology, and biology." BSL certainly reached their declared goal, and *Transport Phenomena* became a scientific bestseller known to every chemical engineer. Based on their enormous success, with the second edition, BSL proceeded to expand into new fields of technological importance:[2] "While momentum, heat, and mass transfer developed independently as branches of classical physics long ago, their unified study has found its place as one of the fundamental engineering sciences. This development, in turn, less than half a century old, continues to grow and to find applications in new fields such as biotechnology, microelectronics, nanotechnology, and polymer science."

In view of the outstanding book by BSL (and a number of additional good books on transport phenomena), why would we write another textbook on the subject of transport phenomena? Most of the existing textbooks have been written by chemical engineers and focus on developing problem-solving skills for engineering purposes. As a team of a chemical engineer (DCV) and a theoretical physicist (HCO), we would like to develop transport phenomena into a subject that appeals to both engineers and natural scientists. Ultimately, the unified approach to "momentum, heat, and mass transfer developed independently as branches of classical physics" should return as a rich, coherent, and attractive core subject to physics, offering many highly

[1] Preface of first edition of Bird, Stewart & Lightfoot, *Transport Phenomena* (Wiley, 1960).
[2] Preface of second edition of Bird, Stewart & Lightfoot, *Transport Phenomena* (Wiley, 2001).

relevant applications in science and engineering, ranging from developing new key technologies to understanding the origin of life. Hence, we focus on basic ideas and principles, conceptual clarity, illuminating interconnections, modern applications, and deep understanding.

There are a number of distinguishing features of *A Modern Course in Transport Phenomena* compared with previous books on this subject.

- We begin with a discussion of the transport of probability and the general mathematical form of diffusion equations. This approach offers a deeper understanding of the ubiquitous diffusion equations as well as versatile stochastic simulation techniques.
- We make extensive use of nonequilibrium thermodynamics, which we consider to be an integral part of transport phenomena. Analysis of the entropy production in terms of thermodynamic forces and fluxes guides the formulation of constitutive equations for the fluxes to be used in balance equations.
- We develop a new, more intuitive approach to interfacial balance and transport equations. By focusing on the motion of Gibbs dividing surfaces and interpreting the ambiguity behind their precise location as a gauge degree of freedom, we gain conceptual clarity and insight. In particular, we develop the tools for describing the physics of interfaces and boundaries, going far beyond the usual formulation of boundary conditions.
- We develop a number of novel applications and material characterization techniques that include field-flow fractionation, complex fluids or soft matter, relativistic hydrodynamics (which turns out to be intimately related to complex fluids), fiber spinning, Czochralski crystal growth, molecular motors, ion pumps, microbead rheology, and dynamic light scattering.

The structure, contents, and style of *A Modern Course in Transport Phenomena* arose as the product of blending the very different perspectives of an engineer and a physicist. Not surprisingly, this process was dissipative at times. However, more often, it forced the authors to broaden their perspectives, leading to a deeper understanding of the subject. Several chapters are based on lectures given over the past 25 years in advanced undergraduate and graduate courses on transport phenomena for chemical engineering students at the IIT Chicago. This material has been revised and significantly expanded in connection with a new two-semester course for graduate students in the materials program at the ETH Zürich, offered for the first time in 2012–13. There are, of course, several important topics we have chosen to exclude from this book. Our decision to exclude, or touch only briefly, topics such as turbulence, boundary layer theory, stability analysis, etc., was

based on a combination of factors that include space limitations and the preferences and expertise of the authors.

Our approach to the subject of transport phenomena has been inspired by Ravel's *Boléro*.[3] The basic theme appears throughout the book, but more instruments, maybe more adequately referred to as tools in the present context, join in from chapter to chapter. The faint beginning is with one-dimensional transport of probability, for which the basic theme is already there. The dimensionality is then increased and the transport of mass, momentum, and energy join in harmonically. Once the constitutive equations have been found by means of thermodynamics, one can really start to enjoy the piece by solving a number of problems with a variety of methods. Complex fluids or soft matter add a modern sound. Nonlinearity requires powerful new instruments. Interfaces come in as trumpets – our possibilities open up enormously. The orchestration becomes complete when kinetic theories join in with the theme of transport phenomena. Fully solved exercises, études of a kind, invite the audience to not just listen but join in with the theme. A striking difference between Ravel's *Boléro* and our transport phenomena is that we need more than 17 minutes from our audience.

How to Teach from This Book

The most traditional audience for *A Modern Course in Transport Phenomena* is chemical engineers, but interesting courses for several other areas of study are easy to envision. These areas include physics, biology, physical chemistry, and applied mathematics, as well as biomedical, environmental, materials, mechanical, molecular, and nuclear engineering. The material in this book is more than enough to cover two, one-semester, graduate-level courses on transport phenomena. Prior exposure to the subject of transport phenomena is useful, but not essential; the book assumes that the reader has a basic knowledge of classical physics, physical chemistry, vector and tensor analysis, and applied mathematics.

By intention, the book has a non-rigid, non-traditional structure. Each chapter can be covered in roughly 2–3 hours of lecture time. The introductory Chapters 1, 2, and 3 provide motivation and a healthy warm-up. The chapters that follow form the main structure of the book, which is based on blocks of related chapters. Two fundamental blocks contain indispensable core chapters on thermodynamics, balance equations, and constitutive

[3] *Boléro* is a one-movement orchestral piece composed by Maurice Ravel (1875–1937), performed for the first time in 1928. It is based on a single theme repeated over and over again; instead of developing the theme, Ravel gradually increases the orchestra to make the piece interesting in a captivatingly dramatic way.

equations: Chapters 4–6 for bulk phases; Chapters 13–15 for interfaces (chapters inside [...] may be omitted or skimmed by readers with prior knowledge of these topics).

[4] Equilibrium thermodynamics 5 Balance equations 6 Forces and fluxes	13 Thermodynamics of interfaces 14 Interfacial balance equations 15 Interfacial force–flux relations

Each of the two fundamental blocks is supplemented by a block of applications:

7 Measuring transport coefficients 8 Pressure-driven flow 9 Heat exchangers 10 Gas absorption 11 Driven separations 12 Complex fluids	16 Polymer processing 17 Transport around a sphere 18 Bubble growth and dissolution 19 Semi-conductor processing

The application blocks contain chapters involving fluid mechanics, heat transfer, mass diffusion, and chemical reaction. A number of applications to choose from are offered in these non-monolithic blocks. Chapters 7–10 focus on classical problems in transport phenomena, and Chapter 11 covers more modern applications. The modern subject of complex fluids is presented in Chapter 12 and provides essential background for later chapters. The block on the right containing Chapters 16–19 covers important applications where interfacial transport is emphasized.

Taken together the four blocks above reflect a natural structure of the *phenomenological approach* to the subject. Horizontal pairs of blocks consist of foundation and application chapters; vertical pairs of blocks contain chapters that emphasize either bulk or interfacial transport phenomena.

The last two blocks of chapters are once more grouped in foundations and applications, where the unifying theme now is the *molecular approach*, that is, the ambitious goal of bridging scales. The first block begins with Chapter 20 and provides background (if necessary) for kinetic theories, which are developed for gases and polymeric liquids in Chapters 21 and 22. The second block containing Chapters 23–26 is offered as a menu to choose from; it includes both theoretical developments and experimental methods.

[20] Statistical mechanics
21 Kinetic theory of gases
22 Kinetic theory of polymers

23 Transport in porous media
24 Transport in biological systems
25 Microbead rheology
26 Dynamic light scattering

Approximately 250 *exercises* can be found throughout this book. Exercises found within the text of a given section serve the purpose of reinforcing concepts, while exercises at the end of a section either allow for the streamlined development of concepts that involve tedious intermediate calculations, or give applications of new concepts. This book has two *appendices*. Appendix A contains several compact and systematically organized tables of thermodynamic relations useful for defining thermodynamic properties and the relationships between them. Appendix B contains tables of the most commonly used differential operators expressed in rectangular, cylindrical, and spherical coordinate systems, which can be used to express balance and evolution equations for particular applications.

Acknowledgments

We have greatly benefited from discussions about this book with a number of colleagues and their feedback has guided us throughout the writing process. In particular, we would like to thank Patrick Anderson, Dick Bedeaux, Manuel Laso, and Jay Schieber for their careful reading of the manuscript at various stages of its development – their feedback had a significant impact on the final version of this book.

The material in this book has been used for several years in graduate courses given at the IIT Chicago and the ETH Zürich. The teaching process has led to important improvements resulting from interactions both with the teaching assistants involved and with the students taking our courses. In particular, we would like to acknowledge the feedback from our teaching assistants Patrick Ilg, David Nieto Simavilla, Maksym Osmanov, Meisam Pourali, Marco Schweizer, Pavlos Stephanou, Laura Stricker, David Taj, and Carl Zinner. Of the many students who have taken our courses, we would like to acknowledge Nora Zimerli and Whitney Fowler for their careful reading of the manuscript, which helped eliminate many typographical errors and improve our grammar. DCV would like to thank Ernest Venerus for his constant support while writing this book, and much more.

Symbols and Notation

Latin Symbols

A	Area of surface
A	Parameter with units of mass per energy
A_1, A_2	Area of entrance, exit surface
A_f	Area of fluid surfaces within A
A_fs	Area of fluid–solid surfaces within V_eff
A_s	Area of solid surfaces within A
A_s	Area of non-entrance and -exit surfaces
$A_\mathrm{si}, A_\mathrm{sp}$	Areas of impermeable, permeable non-entrance and -exit surfaces
$A_\mathrm{ss}, A_\mathrm{sm}$	Areas of stationary, moving non-entrance and -exit surfaces
A_{jk}	Skew-symmetric angular velocity matrix
$A, \boldsymbol{A}$	Drift coefficient, vector
$\boldsymbol{A}$	d' component vector in stochastic differential equation
$\boldsymbol{A}$	Magnetic potential vector
$\boldsymbol{A}$	Matrix describing fluctuating fields in dynamic light scattering
$\mathcal{A}, \mathcal{A}_j$	Affinity of chemical reaction, reaction j
$\mathcal{A}_\mathrm{i}, \mathcal{A}_\mathrm{s}$	Magnitude of incident, scattered wave
a	Unspecified density of an extensive quantity (e, f, g, h, s, u)
a	Distance between binding sites along filament
$\tilde{a}, \hat{a}$	Per mole, mass value of a
$\hat{a}_\alpha$	Partial specific value of a
a_T	Shift factor for time–temperature superposition
a_fs	Pore surface area per unit volume
B	Width
$\boldsymbol{B}$	$d \times d'$ matrix in stochastic differential equation

$\boldsymbol{B}$	Magnetic field vector
b	Scattering length
b_1, b_2, b_{11}	Second-order fluid model parameters
c	Molar density
c	Speed of light (2.99792458×10^8 m/s)
$c_{\hat{s}}, c_{\mathrm{T}}$	Speed of sound at constant entropy, temperature
$\hat{c}_{\hat{v}}, \hat{c}_p$	Heat capacity per unit mass at constant volume, pressure
c_α	Molar density of species α
$\boldsymbol{c}$	Relative position vector of non-inertial coordinate system
$\boldsymbol{c}$	Conformation tensor
$D, \boldsymbol{D}$	Diffusion coefficient, tensor
D	Diameter
$D_{\alpha\beta}$	Diffusivity of species α relative to species β
$D_{q\alpha}$	Thermal diffusion coefficient of species α
D_{T}	Thermal diffusion coefficient normalized by temperature
$D_{\mathrm{eff}}, \boldsymbol{D}_{\mathrm{eff}}$	Effective diffusivity, tensor in porous media
$\mathfrak{D}_{\alpha\beta}$	Maxwell–Stefan diffusivity of species α relative to species β
d	Particle diameter
d	Number of dimensions
E	Energy
E_{tot}	Total energy
E_j	Energy of microstate j
E^2	Operator in axisymmetric spherical coordinates
$\tilde{E}_{\mathrm{act}}$	Activation energy
$\boldsymbol{E}$	Electric field vector
$\boldsymbol{E}$	Finite strain tensor
e	Mathematical constant (2.71828)
e	Energy density
e	Elementary charge [$1.6021766208(98) \times 10^{-19}$ C]
$\boldsymbol{e}_i$	Eigenvectors ($i = 1, 2, 3$) for evolution of fluctuating fields in dynamic light scattering
F	Helmholtz free energy
F	Force exerted by molecular motor on filament
$\tilde{F}$	Faraday constant [$\tilde{F} = \tilde{N}_{\mathrm{A}} z_{\mathrm{el}} = 9.648533289(59) \times 10^4$ C/mol]
F_L	Take-up force in fiber spinning
$\boldsymbol{\mathcal{F}}_{\mathrm{s}}$	Force exerted by fluid on solid
f	Helmholtz free energy density
f	Dimensionless force exerted by molecular motor on filament
f_{s}	Friction factor

$f(\boldsymbol{r},\boldsymbol{p})$	Probability density in Boltzmann's equation
$\boldsymbol{f}_\alpha$	External force on species α
f	Number of independent intensive variables in Gibbs phase rule
$\boldsymbol{f}_\mathrm{B}$	Brownian force
$\mathfrak{f}_\alpha$	Fugacity of species α
G	Gibbs free energy
G	Relaxation modulus
G_0	Relaxation modulus at $t=0$
G',G''	Storage, loss modulus
G^*	Complex modulus
$\tilde{G}(\gamma)$	Reaction coordinate-dependent molar Gibbs free energy
$\mathcal{G}_\alpha,\widetilde{\mathcal{G}}_\alpha$	Total mass, molar transfer rate of species α
g	Gibbs free energy density
$\boldsymbol{g}$	Gravitational acceleration
H	Enthalpy
H	Height
H	Hookean spring constant
H_α,H_ω	Positive constants
H_B	Functional in Boltzmann's H-theorem
$\mathcal{H}$	Heaviside step function
h	Enthalpy density
h	Film thickness
h	Heat transfer coefficient
h_g	Heat transfer coefficient in gas
h_m	Heat transfer coefficient in melt
I_A	Spectral density of autocorrelation function of A
$I_\mathrm{i},I_\mathrm{s}$	Intensity of incident, scattered wave
i	$\sqrt{-1}$
i	Integer indicating state $(0,1)$ of molecule conformation
$\boldsymbol{i}$	Electric flux (current)
$J,\boldsymbol{J}$	Probability flux, vector
$\boldsymbol{J}_\alpha$	Diffusive molar flux of species α relative to $\boldsymbol{v}$
$\boldsymbol{J}_\alpha^*$	Diffusive molar flux of species α relative to $\boldsymbol{v}^*$
$\boldsymbol{j}_a$	Diffusive flux of a
$\boldsymbol{j}_\alpha$	Diffusive mass flux of species α relative to $\boldsymbol{v}$
$\boldsymbol{j}_\alpha^\dagger$	Diffusive mass flux of species α relative to $\boldsymbol{v}^\dagger$
$\boldsymbol{j}_\mathrm{el}$	Mass flux of electric charge relative to $\boldsymbol{v}$
$\boldsymbol{j}_q,\boldsymbol{j}_q'$	Diffusive energy flux, modified
$\boldsymbol{j}_s$	Diffusive entropy flux

K	Equilibrium constant for chemical reaction
K	Power-law model viscosity parameter
K_d	Dispersion coefficient
K_i	Dispersion theory functions ($i = 1, 2, ...$)
K_tot	Total kinetic energy
k	Spatial Fourier transform variable
k	Number of components in mixture
k, k'	Rate constants
$k_\mathrm{m}, \tilde{k}_\mathrm{m}$	Mass transfer coefficients
k_B	Boltzmann's constant [$1.3806505(24) \times 10^{-23}$ J/K]
k_H	Henry's law constant
k_w	Scaled inverse of Henry's law constant
k_eff	Effective reaction rate constant in porous media
$\boldsymbol{k}_\mathrm{i}, \boldsymbol{k}_\mathrm{s}$	Wave vector of incident, scattered wave
L	Length (characteristic)
L_b	Length of fixed bed reactor
L_ext	Length of extruder channel
L_i	Phenomenological (transport) coefficient ($i = q, \tau, \Gamma$)
L_g	Length of cubic volume element
L_p	Injection pulse length
L_v	Entrance length for velocity
L_T	Entrance length for temperature
L_{ij}	Matrix of phenomenological (transport) coefficients ($i, j = \alpha, \beta, q, \mathcal{A}$)
l_cap	Capillary length
l_f	Characteristic pore size
l_md	Mean distance between particles
l_mfp	Mean free path of a particle
l_{ij}	Matrix of phenomenological (transport) coefficients ($i, j = \alpha, \beta, q, \mathcal{A}$)
ℓ	Displacement of interface surface
M	Mass
M	Memory function
M_tot	Total mass
$M_{\alpha,\mathrm{tot}}$	Total mass of species α
$\tilde{M}$	Molecular weight (of mixture)
$\tilde{M}_\alpha$	Molecular weight of species α
$\tilde{M}_q, \tilde{M}_{q_j}$	Molecular weight of reactants/products for reaction, reaction j
$\tilde{M}_\mathrm{w}$	Molecular weight (weight-average) of polymer

$M_{\text{cycle}}^{(i)}$	ith moment of molecular motor displacement
$\boldsymbol{M}$	Momentum
$\boldsymbol{M}_{\text{tot}}$	Total momentum
$\boldsymbol{\mathcal{M}}_{\text{s}}$	Torque exerted by fluid on solid
m	Mass of a particle, bead
m	Number of fundamental dimensions in n physical quantities
m_{eff}	Effective mass of bead
m_{el}	Mass of electron $[9.10938215(45) \times 10^{-31}$ kg]
$\boldsymbol{m}$	Momentum density
N	Number of moles
N	Number of cycles
N_α	Number of moles of species α
$\tilde{N}_{\text{A}}$	Avagadro's number $[6.022140857(74) \times 10^{23}$ mol$^{-1}]$
N_1, N_2	First, second normal stress differences
$(N_1)_R$	First normal stress difference at capillary wall
N_w	Dimensionless driving force parameter for bubble growth
$N_{\Delta p}$	Dimensionless pressure difference for bubble growth
N_γ	Dimensionless interfacial tension for bubble growth
N_η	Dimensionless viscosity for bubble growth
$\mathcal{N}$	Prefactor integral of $\Phi(\gamma)$
$\boldsymbol{N}_\alpha$	Molar flux of species α relative to stationary frame
n	Number of independent physical quantities
n	Power-law model index parameter
n	Number of particles in system
n, n'	Ratios of phenomenological coefficients
n_{p}	Number density of bead–spring elements
$\boldsymbol{n}$	Unit normal vector
n_α	Elementary charge of species α
$\boldsymbol{n}_\alpha$	Mass flux of species α relative to stationary frame
P_{el}	Electrical power per unit length
P_z	Transition probability
$\mathcal{P}$	Modified pressure
p	Probability density
p	Pressure
p^{L}	Pseudo-pressure
p_n	Eigenfunction of probability density
$\boldsymbol{p}$	Momentum of particle
Q	Heat
$\dot{Q}$	Total rate of heat transfer

Q_{jk}	Orthogonal rotation matrix
$\boldsymbol{Q}$	Structural (connector) vector
q	Coupling coefficient for ion pump
$\boldsymbol{q}$	Momentum of particle
$\boldsymbol{q}$	Scattering vector
R	Radius
$\tilde{R}$	Ideal gas constant [$\tilde{R} = \tilde{N}_{\mathrm{A}} k_{\mathrm{B}} = 8.3144598(48)$ J/(mol K)]
R_{b}	Radius of fixed bed reactor
R_{el}	Electrical resistance per unit length
R_{hyd}	Hydraulic radius
$R_{\mathrm{K}}^{\mathrm{Is}}, R_{\mathrm{K}}^{\mathrm{IIs}}$	Kapitza resistance of phase I, II
R_Q	Autocorrelation function of Q
R_{ij}	Matrix of phenomenological (resistance) coefficients ($i, j = \alpha, \beta, q, \mathcal{A}$)
$\boldsymbol{R}$	Position of detector relative to scattering volume
$\boldsymbol{R}_1, \boldsymbol{R}_2$	Bead position vectors
$\mathcal{R}_\alpha, \widetilde{\mathcal{R}}_\alpha$	Total rate of species α mass, moles produced by chemical reaction
r	Radial coordinate in cylindrical coordinate system
r	Radial coordinate in spherical coordinate system
r	Randomness factor
$\boldsymbol{r}$	Position vector
$\boldsymbol{r}'$	Position vector in non-inertial coordinate system
$\boldsymbol{r}_{\mathrm{b}}$	Bead position vector
S	Entropy
$S(\boldsymbol{q}, \omega)$	Dynamic structure factor
$\dot{S}$	Total rate of entropy transfer
S_{tot}	Total entropy
S_T	Soret coefficient
s	Laplace transform variable
s	Entropy density
s_1	Sedimentation coefficient
T	Temperature
T_{fl}	Temperature of fluid away from surface
T_{g}	Temperature of fluid in gas
T_{m}	Temperature of fluid in melt
$\lvert\boldsymbol{\nabla} T\rvert_\infty$	Temperature gradient (magnitude) around sphere
$T^{\mu\nu}$	Energy-momentum tensor ($\mu, \nu = 0, 1, 2, 3$)
$\boldsymbol{T}$	Maxwell electromagnetic stress tensor
t	Time

t'	Lorentz transformed time
$\boldsymbol{t}$	Unit tangent vector
U	Internal energy
u	Internal energy density
$\tilde{u}, \hat{u}$	Internal energy per mole, mass
u^μ	Velocity four-vector ($\mu = 0, 1, 2, 3$)
$\boldsymbol{u}$	Displacement vector
$\boldsymbol{u}$	Diffusion-induced velocity in porous media
V	Volume
V	Velocity (characteristic)
V_{con}	Volume of continuous phase
V_{dis}	Volume of dispersed phase
V_{eff}	Effective volume
V_{f}	Fluid phase volume within V_{eff}
V_{s}	Solid phase volume within V_{eff}
$\hat{v}$	Volume per unit mass
$\hat{v}_\alpha$	Partial specific volume of species α
v_n^{s}	Normal part of interface velocity $\boldsymbol{v}^{\text{s}}$
v_{na}^{s}	Normal part of interface velocity $\boldsymbol{v}^{\text{s}}$ for which $a^{\text{s}} = 0$
$\boldsymbol{v}$	Mass-average (barycentric) velocity
$\boldsymbol{v}^*$	Molar-average velocity
$\boldsymbol{v}^\dagger$	Volume-average velocity
$\boldsymbol{v}_\alpha$	Species α velocity
$\boldsymbol{v}_{\text{b}}$	Sphere or bead velocity
$\boldsymbol{v}^{\text{s}}$	Interface velocity
$\boldsymbol{v}_{\text{tr}}^{\text{s}}$	Translational part of interface velocity $\boldsymbol{v}^{\text{s}}$
$\boldsymbol{v}_{\text{def}}^{\text{s}}$	Deformational part of interface velocity $\boldsymbol{v}^{\text{s}}$
$\boldsymbol{v}_{\parallel}^{\text{s}}$	Tangential part of interface velocity $\boldsymbol{v}^{\text{s}}$
W	Work
$\dot{W}$	Total rate of work
$\mathcal{W}$	Total mass flow rate
$\boldsymbol{W}_j$	d'-dimensional column vector of independent Gaussian random variables
w	Transition rate for momentum exchange
w_α	Mass fraction of species α
$w_{\alpha\text{fl}}, x_{\alpha\text{fl}}$	Mass, mole fraction of species α of fluid away from surface
$\boldsymbol{w}$	Vorticity vector
$\boldsymbol{X}_j$	d-dimensional column vector of random variables
$x, \boldsymbol{x}$	Position, vector
$\boldsymbol{x}$	Vector of fluctuating fields in dynamic light scattering

x_i	Cartesian coordinates ($i = 1, 2, 3$)
x_i'	Cartesian coordinates ($i = 1, 2, 3$) in non-inertial coordinate system
x_i'	Lorentz transformed spatial coordinates ($i = 1, 2, 3$)
x_α	Mole fraction of species α
x^μ, x_μ	Time and space coordinates ($\mu = 0, 1, 2, 3$)
$\boldsymbol{y}$	Position vector within V_{eff}
Z	Thermoelectric figure of merit
Z	Canonical partition sum
Z	Entropy production parameter for ion pump
$\mathcal{Z}$	Prefactor integral of $\Phi(\gamma)$
z	Axial coordinate in cylindrical coordinate system
z	Integer indicating position along filament
z_α	Charge per unit mass of species α
z_{el}	Charge per unit mass of electron

Greek Symbols

α	Angle
$\alpha_t, \boldsymbol{\alpha}_t$	Mean of Gaussian distribution
$\alpha_{\hat{s}}, \alpha_p$	Thermal expansion coefficient at constant entropy, pressure
α_i	Eigenvalues ($i = 1, 2, 3$) for evolution of fluctuating fields in dynamic light scattering
$\alpha^{\mu\nu}$	Structure tensor ($\mu, \nu = 0, 1, 2, 3$)
$\overline{\alpha}_{\mu\nu}, \overset{\circ}{\alpha}_{\mu\nu}$	Trace and trace-free parts of $\alpha^{\mu\nu}$
$\boldsymbol{\alpha}$	Dyadic product of $\boldsymbol{Q}$
$\beta, \bar{\beta}$	Dimensionless parameter
$\Gamma, \tilde{\Gamma}$	Mass, molar rate of chemical reaction per volume
Γ	Attenuation coefficient
$\Gamma_{\text{A}}^{\pm}$	Transition rate between states z, i
Γ_{tot}	Total transition rate
$\boldsymbol{\Gamma}_t$	Phase-space trajectory
γ	Ratio of heat capacities ($= \hat{c}_p/\hat{c}_{\hat{v}}$)
γ	Shear strain
$\dot{\gamma}$	Shear strain rate
γ	Lorentz factor
γ	Interfacial tension
γ	Reaction coordinate
$\bar{\gamma}_\alpha$	Activity coefficient for species α
γ_T	Derivative of interfacial tension γ with respect to T

$\boldsymbol{\gamma}$	Strain tensor
$\dot{\boldsymbol{\gamma}}$	Rate of strain tensor
δ	Dirac's delta function
δ_T	Thermal boundary layer thickness
$\boldsymbol{\delta}, \delta_{ij}$	Identity (unit) tensor, Kronecker delta
$\boldsymbol{\delta}_i$	Base vector
$\boldsymbol{\delta}'_i$	Base vector in non-inertial coordinate system
$\boldsymbol{\delta}_{\parallel}$	Tangential identity tensor
ϵ	Internal energy density (Section 12.6 only)
ϵ	Extensional strain
ϵ	Ratio of spherical particle density to surrounding fluid density
ϵ	Porosity
$\dot{\epsilon}$	Extensional strain rate
$\boldsymbol{\epsilon}, \epsilon_{ijk}$	Alternating identity (unit) tensor, permutation symbol
ε	Dimensionless (perturbation) parameter
$\varepsilon_{\mathrm{el}}$	Seebeck coefficient
ζ	Parameter for streamline
ζ	Friction coefficient
η	Shear viscosity
η	Efficiency of ion pump
η	Effectiveness factor for porous catalyst
η_{d}	Dilatational viscosity
η_0, η_∞	Zero-, infinite-shear rate viscosity
η_j	Viscosity for relaxation time λ_j
$\eta_{\mathrm{B}}, \eta_{\mathrm{E}}$	Equibiaxial, uniaxial elongational viscosities
η_{eff}	Effective viscosity
η', η''	Real, imaginary part of complex viscosity
η^*	Complex viscosity
$\eta^{\mu\nu}, \eta_{\mu\nu}$	Minkowski tensors $(\mu, \nu = 0, 1, 2, 3)$
$\hat{\eta}^{\mu\nu}$	Spatial projection of $\eta^{\mu\nu}$ $(\mu, \nu = 0, 1, 2, 3)$
θ	Azimuthal angle in cylindrical coordinate system
θ	Polar angle in spherical coordinate system
θ	Contact angle
$\boldsymbol{\Theta}$	Second moment of connector vector $\boldsymbol{Q}$
$\Theta_t, \boldsymbol{\Theta}_t$	Variance of Gaussian distribution
$\kappa_{\hat{s}}, \kappa_p$	Compressibility at constant entropy, pressure
$\boldsymbol{\kappa}$	Transpose of gradient of velocity $[= (\boldsymbol{\nabla v})^{\mathrm{T}}]$
$\kappa, \boldsymbol{\kappa}$	Permeability, tensor

Λ	Dimensionless parameter characterizing deviation from equilibrium at interface
Λ_p	Dimensionless parameter for ultracentrifugation
Λ_T	Dimensionless parameter for thermal field flow fractionation
Λ_ξ	Dimensionless slip coefficient
Λ_ϕ	Dimensionless parameter for electric field flow fractionation
λ, λ'	Thermal conductivity, modified
$\boldsymbol{\lambda}$	Thermal conductivity tensor
λ_{eff}	Effective thermal conductivity
$\lambda_0, \lambda_1, \lambda_2$	Relaxation times
λ, λ_i	Relaxation time, spectrum ($i = 1, 2, ...$)
$\lambda_0, \boldsymbol{\lambda}_1, \lambda_2$	Parameters in Maxwellian distribution
λ_n	Eigenvalue of probability density
$\tilde{\mu}, \hat{\mu}$	Chemical potential per mole, mass
$\tilde{\mu}, \hat{\mu}$	Flat-average chemical potential per mole, unit mass
$\tilde{\mu}_\alpha, \hat{\mu}_\alpha$	Chemical potential of species α per mole, unit mass
$\hat{\mu}'_\alpha$	Electrochemical potential of species α per unit mass
ν	Kinematic viscosity
ν_l	Longitudinal kinematic viscosity
$\nu_\alpha, \nu_{\alpha_j}$	Coefficient for species α in chemical reaction, j
$\tilde{\nu}_\alpha, \tilde{\nu}_{\alpha_j}$	Stoichiometric coefficient for species α in chemical reaction, j
ξ, ξ_j	Extent of reaction, reaction j
ξ	Similarity transform variable
ξ_n	Mobility coefficient for normal velocity jump at interface
$\boldsymbol{\xi}_\parallel$	Mobility coefficient for tangential velocity jump at interface
π	Number of phases in Gibbs phase rule
π	Mathematical constant (3.14159)
π_{el}	Peltier coefficient
$\boldsymbol{\pi}$	Pressure (momentum flux) tensor
$\boldsymbol{\pi}_n$	Normal projection of pressure tensor (stress vector)
ρ	Mass density
ρ_α	Mass density of species α
$\dot{\Sigma}$	Total rate of entropy production
σ	Entropy production rate per volume
σ	Yield stress
σ_{el}	Electrical conductivity
τ_Γ	Relaxation time for chemical reaction
τ_{flow}	Characteristic time for flow
τ_R	Shear stress at capillary wall

$\boldsymbol{\tau}$	Extra pressure (stress) tensor
$\Upsilon_{\alpha k}, \tilde{\Upsilon}_{\alpha k}$	Relative mass, molar adsorption for species α
Φ	Velocity potential
Φ	Potential function for bead–spring element
Φ_{tot}	Total potential energy
$\Phi(\gamma)$	Reaction coordinate-dependent potential energy
ϕ	Azimuthal angle in spherical coordinate system
ϕ	Gravitational potential
ϕ	Rotation angle in Q_{jk} matrix
ϕ	Volume fraction
$\phi^{(\text{e})}$	External force potential
ϕ_α	External force potential for species α
$\bar{\phi}_\alpha$	Fugacity coefficient of species α
φ^{s}	Function describing surface
χ	Thermal diffusivity
Ψ_t	Integrating factor for diffusion equation
Ψ_1, Ψ_2	First, second normal stress coefficients
ψ	Stream function
Ω	Angular velocity
Ω	Number of microstates
ω	Oscillation frequency
ω	Time Fourier transform variable
ω^μ	Structure vector ($\mu = 0, 1, 2, 3$)
$\boldsymbol{\omega}$	Angular velocity vector

Other Symbols

δa	Fluctuation of a
$\langle a \rangle$	Average of a
$\tilde{a}$	Spatial Fourier transform of a
$\bar{a}$	Laplace transform of a
(1)	Subscript indicating convected time derivative
a	Subscript indicating surface area average of quantity
eq	Subscript indicating quantity at equilibrium
i	Subscript indicating intrinsic average of quantity
f	Subscript indicating quantity in comoving local reference frame
s	Subscript indicating superficial average of quantity
0	Subscript/superscript indicating initial or reference value
s	Superscript indicating interfacial quantity

$*$	Superscript indicating complex conjugate
I, II	Superscript indicating quantity in phase I, II at interface
$(0), (1), \ldots$	Superscript indicating zero-, first-, ... order perturbation solution
$'$	Superscript indicating quantity in non-inertial coordinate system
T	Superscript indicating transpose of tensor
$\bar{x}$	Rescaled variable x
Δa	Difference in a
$\boldsymbol{\nabla}$	Nabla operator
$\boldsymbol{\nabla}_{\parallel}$	Nabla operator for surface
∇^2	Laplacian operator

Dimensionless Groups

$N_{\mathrm{Bi}} = hL/\lambda$	Biot number
$N_{\mathrm{Br}} = \eta V^2/(\lambda \Delta T)$	Brinkman number
$N_{\mathrm{Da}} = kL^2/D_{12}$	Damköhler number
$N_{\mathrm{De}} = \lambda/\tau_{\mathrm{flow}}$	Deborah number
$N_{\mathrm{Fr}} = V^2/(gL)$	Froude number
$N_{\mathrm{Gr}} = gL^3\alpha_p \Delta T/\nu^2$	Grashof number
$N_{\mathrm{Ka}} = R_{\mathrm{K}}\lambda/L$	Kapitza number
$N_{\mathrm{Kn}} = l_{\mathrm{mfp}}/L$	Knudsen number
$N_{\mathrm{Le}} = \chi/D_{12}$	Lewis number
$N_{\mathrm{Ma}} = V/c_T$	Mach number
$N_{\mathrm{Na}} = V^2\lvert\partial\eta/\partial T\rvert/\lambda$	Nahme–Griffith number
$N_{\mathrm{Nu}} = hL/\lambda_{\mathrm{fl}}$	Nusselt number
$N_{\mathrm{Pe}} = \rho V L/D_{12}$	Péclet number for mass transfer
$N'_{\mathrm{Pe}} = \rho V L/\chi$	Péclet number for energy transfer
$N_{\mathrm{Pr}} = \nu/\chi$	Prandtl number
$N_{\mathrm{Ra}} = gL^3\alpha_p \Delta T/(\nu\chi)$	Rayleigh number
$N_{\mathrm{Re}} = VL/\nu$	Reynolds number
$N_{\mathrm{Sc}} = \nu/D_{12}$	Schmidt number
$N_{\mathrm{Sh}} = k_{\mathrm{m}}L/(\rho D_{12})_{\mathrm{fl}}$	Sherwood number
$N_{\mathrm{St}} = \hat{c}_p \Delta T/\Delta\hat{h}$	Stefan number
$N_{\mathrm{Th}} = L\sqrt{a_{\mathrm{fs}}k^{\mathrm{s}}/D_{\mathrm{AB}}}$	Thiele number (modulus)
$N_{\mathrm{We}} = \rho V^2 L/\gamma$	Weber number
$N_{\mathrm{Wi}} = \lambda\dot{\gamma}$	Weissenberg number