

Contents

	<i>Preface</i>	<i>page</i> xiii
1	The Erdős–Ko–Rado Theorem	1
1.1	The original proof	2
1.2	Non-canonical intersecting set systems	6
1.3	The complete Erdős–Ko–Rado Theorem	7
1.4	The shifting technique	10
1.5	The Kruskal–Katona Theorem	11
1.6	The Hilton–Milner Theorem	15
1.7	Cross-intersecting sets	17
1.8	Separated sets	19
1.9	Exercises	21
2	Bounds on cocliques	24
2.1	A clique-coclique bound	24
2.2	Equitable partitions	27
2.3	Interlacing	29
2.4	The ratio bound for cocliques	30
2.5	Application: Erdős–Ko–Rado	33
2.6	A ratio bound for cliques	35
2.7	Ratio bound for cross cocliques	36
2.8	Cocliques in neighborhoods	38
2.9	The inertia bound	39
2.10	Inertia bound: Kneser graphs	40
2.11	Inertia bound: folded cubes	40
2.12	Fractional chromatic number	41
2.13	Homomorphisms	43
		vii

2.14	Cyclic intervals	44
2.15	Exercises	45
3	Association schemes	49
3.1	Definitions	50
3.2	Commutants	51
3.3	The conjugacy class scheme	52
3.4	A basis of idempotents	54
3.5	Some fundamental identities	58
3.6	An example	60
3.7	The clique bound	62
3.8	The clique-coclique bound	64
3.9	Exercises	68
4	Distance-regular graphs	70
4.1	Metric schemes	71
4.2	A three-term recurrence	73
4.3	Cometric schemes	74
4.4	Intersection numbers and Krein parameters	76
4.5	Coding theory	77
4.6	Sturm sequences	79
4.7	Classical parameters	83
4.8	Exercises	84
5	Strongly regular graphs	87
5.1	An association scheme	87
5.2	Eigenvalues	88
5.3	Designs	90
5.4	The Witt graph	94
5.5	Orthogonal arrays	96
5.6	Partial geometries	100
5.7	Eigenspaces of point and line graphs	103
5.8	Paley graphs	105
5.9	Paley graphs of square order	107
5.10	Exercises	108
6	The Johnson scheme	112
6.1	Graphs in the Johnson scheme	112
6.2	A third basis	114
6.3	Eigenvalues of the Johnson graph	116
6.4	A fourth basis	118
6.5	Eigenvalues of the Johnson scheme	120

<i>Contents</i>		ix
6.6	Kneser graphs	122
6.7	Exercises	123
7	Polytopes	126
7.1	Convex polytopes	126
7.2	A polytope from the Johnson graph	128
7.3	Perfect matching polytopes	129
7.4	Perfect matchings in complete graphs	131
7.5	Perfect matchings in complete bipartite graphs	133
7.6	Exercises	133
8	The exact bound in the EKR Theorem	135
8.1	A certificate for maximality	135
8.2	Special cases	139
8.3	The EKR bound for 2-intersecting sets	140
8.4	Wilson's matrix	142
8.5	Eigenvalues of Wilson's matrix	144
8.6	Width and dual width	148
8.7	Equality in the width bound	150
8.8	Intersecting families of maximum size	151
8.9	Equality in the dual width bound	153
8.10	Narrow subsets	157
8.11	Exercises	159
9	The Grassmann scheme	161
9.1	q -Binomial coefficients	162
9.2	q -Commuting variables	162
9.3	Counting subspaces	164
9.4	Subspace incidence matrices	167
9.5	Another basis	169
9.6	Eigenvalues of the Grassmann scheme	170
9.7	The EKR bound for q -Kneser graphs	172
9.8	Cocliques in the q -Kneser graphs	173
9.9	t -Intersecting families	175
9.10	The certifying matrix	176
9.11	Bilinear forms graphs	180
9.12	Dual polar graphs	181
9.13	Exercises	182
10	The Hamming scheme	184
10.1	Eigenvalues of the Hamming scheme	185
10.2	Idempotents	187

x	<i>Contents</i>	
	10.3	Partial words 189
	10.4	The EKR Theorem for words 191
	10.5	The complete EKR for words 193
	10.6	Cross-intersecting sets of words 194
	10.7	An operation on words 195
	10.8	Bounds on the size of a derived set 196
	10.9	Cocliques in power graphs 200
	10.10	Cocliques in product graphs 203
	10.11	Exercises 208
11	Representation theory	210
	11.1	Representations 210
	11.2	The group algebra 211
	11.3	Operations on representations 212
	11.4	Sums and idempotents 214
	11.5	Irreducible representations 215
	11.6	Schur's Lemma 217
	11.7	Coordinate functions 218
	11.8	Characters 220
	11.9	Orthogonality of characters 221
	11.10	Decomposing the regular representation 223
	11.11	The conjugacy class scheme: idempotents 225
	11.12	The conjugacy class scheme: eigenvalues 226
	11.13	Restriction and induction 228
	11.14	Exercises 230
12	Representation theory of the symmetric group	232
	12.1	Permutation representations 232
	12.2	Examples: orbit counting 233
	12.3	Young subgroups 235
	12.4	Irreducible representations 235
	12.5	Kostka numbers 237
	12.6	Hook length formula 238
	12.7	Branching rule 239
	12.8	Wreath products 242
	12.9	Exercises 243
13	Orbitals	245
	13.1	Arc-transitive directed graphs 245
	13.2	Commutants of permutation groups 247
	13.3	Generously transitive groups 248

<i>Contents</i>		xi
13.4	Multiplicity-free representations	249
13.5	Multiplicity-free representations of the symmetric group	250
13.6	An equitable partition	251
13.7	A homomorphism	253
13.8	Eigenvalues of the orbital scheme	254
13.9	Eigenspaces and Young subgroups	256
13.10	Exercises	258
14	Permutations	260
14.1	The derangement graph	261
14.2	Eigenvalues of the derangement graph	262
14.3	An eigenspace of the derangement graph	264
14.4	Cocliques in the derangement graphs	266
14.5	t -Intersecting permutations	268
14.6	Transitive permutation groups	269
14.7	2-Transitive subgroups	271
14.8	Direct products	275
14.9	Exercises	276
15	Partitions	279
15.1	Intersecting partitions	280
15.2	Perfect matchings	282
15.3	Eigenvalues of the perfect matching graph	284
15.4	The perfect matching association scheme	285
15.5	Modules of the perfect matching graph	286
15.6	EKR Theorem for perfect matchings	289
15.7	Skew partitions	289
15.8	Cliques and cocliques in the skew-partition graph	291
15.9	Eigenvalues of the skew-partition graph	293
15.10	Eigenspaces of the skew-partition graph	295
15.11	3×3 partitions	298
15.12	Inner distributions of cliques	299
15.13	Characterizing cocliques	301
15.14	Meet tables	302
15.15	Exercises	305
16	Open problems	308
16.1	Generalize the ratio bound for cliques	308
16.2	Extend the Katona cycle proof	309
16.3	Prove the EKR Theorem for the block graph of a design	310

16.4	Prove the EKR Theorem for the orthogonal array graph	311
16.5	Find an algebraic proof of the EKR Theorem for the Paley graphs	311
16.6	Determine the chromatic number of the Johnson graphs	312
16.7	Prove the EKR Theorem for 2-transitive groups	313
16.8	Determine cliques in groups that do not have the EKR property	314
16.9	Prove the EKR property holds for other group actions	315
16.10	Calculate the eigenvalues of the perfect matching scheme	315
16.11	Prove the EKR Theorem for partitions	316
16.12	Prove EKR Theorems for other objects	317
16.13	Find maximal cliques in graphs from an orbital scheme	318
16.14	Develop other compression operations	319
	<i>Glossary: Symbols</i>	321
	<i>Glossary: Operations and relations</i>	323
	<i>References</i>	324
	<i>Index</i>	332