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| d -adic interval, | 491 | $h_{\mu}(f, \mathcal{P})$ | entropy relative to a partition, 250 |
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| $e(\psi, x)$ | conditional expectation, 150 | $h_{\mu}^{\pm}(f, \varepsilon, x)$ | entropy (local) at a point, 263 |
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