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Ridge Functions

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Preface

This monograph is an attempt to examine and study ridge functions as entities in and of themselves. As such we present what we consider to be various central properties of ridge functions. However, no encyclopedic claims are being made, and the topics chosen are those that we alone considered appropriate. In addition, most chapters contain, either explicitly or implicitly, unresolved questions. It is our hope that this monograph will prove useful and interesting to both researchers and the more casual reader. And, of course, all errors, omissions and other transgressions are totally our responsibility.

No monograph is written in a vacuum, and I would like to especially thank Carl de Boor, Vugar Ismailov and an anonymous referee for various comments and suggestions. Thanks also to Heinz Bauschke, Vitaly Maiorov, Simon Reich and Yuan Xu for their patience, and help with my various inquiries.

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Glossary of Selected Symbols

Symbol	*Meaning*	*Page*
$f(\mathbf{a}\cdot\mathbf{x})$	Ridge function	1
$f(A\mathbf{x})$	Generalized ridge function	1
$\mathcal{M}(\mathbf{a}^1,\ldots,\mathbf{a}^r)$	$\{\sum_{i=1}^r f_i(\mathbf{a}^i\cdot\mathbf{x}) \,:\, f_i \text{ vary}\}$	3
$\mathcal{M}(\Omega)$	$\text{span}\{f(\mathbf{a}\cdot\mathbf{x}) \,:\, f:\mathbb{R}\to\mathbb{R},\, \mathbf{a}\in\Omega\}$	8
$\mathcal{M}(A^1,\ldots,A^r)$	$\{\sum_{i=1}^r f_i(A^i\mathbf{x}) \,:\, f_i:\mathbb{R}^d\to\mathbb{R}\}$	8
$\mathcal{M}(\Omega_d)$	$\text{span}\{f(A\mathbf{x}) \,:\, f:\mathbb{R}^d\to\mathbb{R},\, A\in\Omega_d\}$	9
B^n	$\{\mathbf{x}:\, \|\mathbf{x}\|_2\le 1,\, \mathbf{x}\in\mathbb{R}^n\}$	9
S^{n-1}	$\{\mathbf{x}:\, \|\mathbf{x}\|_2 = 1,\, \mathbf{x}\in\mathbb{R}^n\}$	9
H_m^n	Homogeneous polynomials of degree m in $\mathbb{R}^n$	9
Π_m^n	Algebraic polynomials of degree at most m in $\mathbb{R}^n$	9
$D^{\mathbf{k}}$	$\frac{\partial^{\lvert\mathbf{k}\rvert}}{\partial x_1^{k_1}\cdots\partial x_n^{k_n}}$	10
$D_{\mathbf{c}}$	$\sum_{i=1}^n c_i \frac{\partial}{\partial x_i}$	10
Z_A	$\{\mathbf{x}: A\mathbf{x}=\mathbf{0}\}$	34
H_g	$\left(\frac{\partial^2 g}{\partial x_i \partial x_j}\right)_{i,j=1}^m$	35
$L(\Omega)$	$\{\lambda\mathbf{a}:\, \mathbf{a}\in\Omega,\, \lambda\in\mathbb{R}\}$	36
$\mathcal{P}(\Omega)$	$\{p:\, p\vert_{L(\Omega)}=0,\, p\in\Pi^n\}$	37
$L(A)$	Span of the rows of the matrix A	56
$L(\Omega_d)$	$\bigcup_{A\in\Omega_d} L(A)$	58
$\mathcal{C}(\Omega)$	$\text{span}\{p:\, q(D)p=0 \text{ all } q\in\mathcal{P}(\Omega),\, p\in\Pi^n\}$	70
$\mathcal{N}(\sigma)$	$\text{span}\{\sigma(\mathbf{a}\cdot\mathbf{x}+b) :\, \mathbf{a}\in\mathbb{R}^n,\, b\in\mathbb{R}\}$	73
$\mathcal{N}_1(\sigma)$	$\text{span}\{\sigma(\lambda t+b) :\, \lambda, b\in\mathbb{R}\}$	74
$\mathcal{M}(A;K)$	$\{f(A\mathbf{x}) :\, \mathbf{x}\in K,\, \text{all } f:\mathbb{R}^d\to\mathbb{R}\}$	77
$C(A;K)$	$\mathcal{M}(A;K)\cap C(K)$	87

Symbol	*Meaning*	*Page*
$J(F)$	$\{\mathbf{x} : \lvert F(\mathbf{x})\rvert = \lVert F\rVert\}$	92
$P_A G(\mathbf{y})$	$\frac{1}{2}[\max_{\{\mathbf{x}:A\mathbf{x}=A\mathbf{y}\}\cap K} G(\mathbf{x}) + \min_{\{\mathbf{x}:A\mathbf{x}=A\mathbf{y}\}\cap K} G(\mathbf{x})]$	101
$Z(G)$	$\{x : G(x) = 0\}$	139
$\Delta_{\mathbf{x}}$	$\sum_{i=1}^n \frac{\partial^2}{\partial x_i^2}$	141
ω_n	$\frac{2\pi^{n/2}}{\Gamma(n/2)}$	141
$\mathcal{N}$	$\{\big(G(\mathbf{x}^1), \ldots, G(\mathbf{x}^k)\big) : G \in \mathcal{M}(A^1, \ldots, A^r)\}$	153
Λ_i	$\{A^i\mathbf{x}^j \,:\, j = 1, \ldots, k\}$	154
$\Gamma_A(\mathbf{c})$	$\{\mathbf{x} : A\mathbf{x} = \mathbf{c}\}$	155