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Introduction

We begin with some background material. First, we need to establish the formalism and definitions for the imaginary signals we will be shining on our imaginary detectors. Second, we will describe general detector characteristics so we can judge the merits of the various types as they are discussed. This discussion introduces some common metrics: (1) the quantum efficiency, i.e., the fraction of the incoming photon stream absorbed; (2) noise and the ratio of signal to noise; (3) the fidelity of images produced by a detector array or similar arrangement; and (4) the speed of response of a detector.

1.1 Radiometry

1.1.1 Concepts and Terminology

There are some general aspects of electromagnetic radiation that need to be defined before we discuss how it is detected. Figure 1.1 illustrates schematically a photon of light with terms used to describe it. One should imagine that time has been frozen, but that the photon has been moving at the speed of light in the direction of the arrow. We often discuss the photon in terms of wavefronts, lines marking the surfaces of constant phase and hence separated by one wavelength.

As electromagnetic radiation, a photon has both electric and magnetic components, oscillating in phase perpendicular to each other and perpendicular to the direction of energy propagation. The amplitude of the electric field, its wavelength and phase, and the direction it is moving characterize the photon. The behavior of the electric field can be expressed as

$$E = E_0 cos(\omega t + \phi), \tag{1.1}$$

where E_0 is the amplitude, ω is the angular frequency, and ϕ is the phase. Alternatively, the behavior is conveniently expressed in complex notation as

$$E(t) = E_0 e^{-j\omega t} = E_0 cos(\omega t) - jE_0 sin(\omega t), \tag{1.2}$$

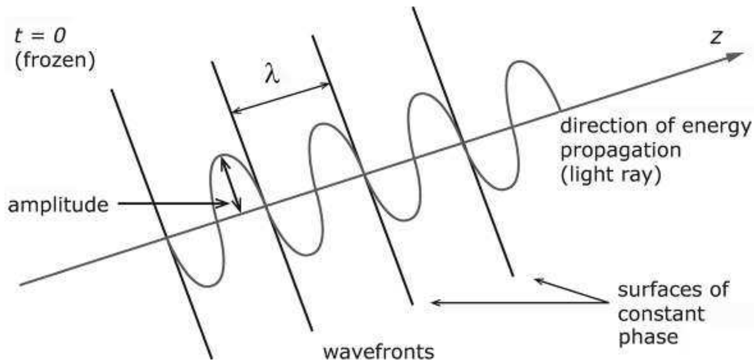

Figure 1.1 Terms describing the propagation of a photon, or a light ray.

where j is the imaginary square root of -1. In this case, the quantity is envisioned as a vector on a two-dimensional diagram with the real component along the usual x-axis and the imaginary one along the usual y-axis. The angle of this vector from the origin and relative to the positive real axis represents the phase.

Most of the time we will treat light as photons of energy; wave aspects will be important only for heterodyne detectors. A photon has an energy of

$$E_{ph} = h\nu = hc/\lambda, \tag{1.3}$$

where h ($= 6.626 \times 10^{-34}$ J s) is Planck's constant, ν and λ are, respectively, the frequency (in hertz = 1/seconds) and wavelength (in meters) of the electromagnetic wave, and c ($= 2.998 \times 10^8$ m s^{-1}) is the speed of light. In the following discussion, we define a number of expressions for the power output of sources of photons; conversion from power to photons per second can be achieved by dividing by the desired form of equation 1.3.

The spectral radiance per frequency interval, L_ν, is the power (in watts) leaving a unit projected area of the surface of the source (in square meters) into a unit solid angle (in steradians) and unit frequency interval (in hertz). The projected area of a surface element dA onto a plane perpendicular to the direction of observation is $dA\ cos\theta$, where θ is the angle between the direction of observation and the outward normal to dA; see Figure 1.2. L_ν has units of W m^{-2} Hz^{-1} ster^{-1}. The spectral radiance per wavelength interval, L_λ, has units of W m^{-3} ster^{-1}. The radiance, L, is the spectral radiance integrated over all frequencies or wavelengths; it has units of W m^{-2} ster^{-1}. The radiant exitance, M, is the integral of the radiance over solid angle, and it is a measure of the total power emitted per unit surface area in units of W m^{-2}.

We will deal only with Lambertian sources; the defining characteristic of such a source is that its radiance is constant regardless of the direction from which it is viewed. A blackbody is one example. The emission of a Lambertian source goes as

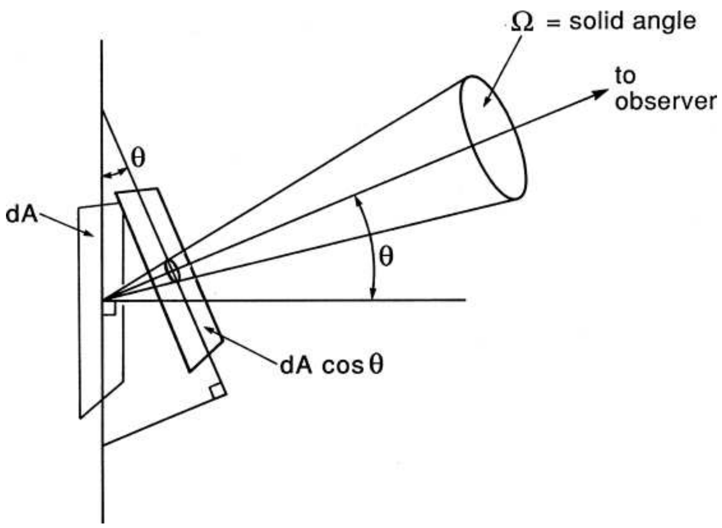

Figure 1.2 Geometry for computing radiance.

the cosine of the angle between the direction of the radiation and the normal to the source surface. From the definition of projected area in the preceding paragraph, it can be seen that this emission pattern exactly compensates for the foreshortening of the surface as it is tilted away from being perpendicular to the line of sight. That is, for the element dA, the projected surface area and the emission decrease by the same cosine factor. Thus, if the entire source has the same temperature and emissivity, every unit area of its projected surface in the plane perpendicular to the observer's line of sight appears to be of the same brightness, independent of its actual angle to the line of sight. Keeping in mind this cosine dependence, and the definition of radiant exitance, the radiance and radiant exitance are related as

$$M = \int L\, cos\,\theta\, d\Omega = 2\pi L \int_0^{\pi/2} sin\,\theta\, cos\,\theta\, d\theta = \pi L. \tag{1.4}$$

The flux emitted by the source, Φ, is the radiant exitance times the total surface area of the source, that is the power emitted by the entire source. For example, for a spherical source of radius R,

$$\Phi = 4\pi R^2 M = 4\pi^2 R^2 L. \tag{1.5}$$

Although there are other types of Lambertian sources, we will consider only sources that have spectra resembling those of blackbodies, for which the spectral radiance in frequency units is

$$L_\nu = \frac{\varepsilon\left[2h\nu^3/(c/\mathrm{n})^2\right]}{e^{h\nu/kT} - 1}, \tag{1.6}$$

where ε is the emissivity of the source, n is the refractive index of the medium into which the source radiates, and k ($= 1.38 \times 10^{-23}$ J K^{-1}) is the Boltzmann constant. The emissivity (ranging from 0 to 1) is the efficiency with which the source radiates compared to that of a perfect blackbody, which by definition has $\varepsilon = 1$. According to Kirchhoff's law, the absorption efficiency, or absorptivity, and the emissivity are equal for any source. In wavelength units, the spectral radiance is

$$L_\lambda = \frac{\varepsilon\left[2h(c/\mathrm{n})^2\right]}{\lambda^5\left(e^{hc/\lambda kT} - 1\right)}. \tag{1.7}$$

It can be easily shown from equations 1.6 and 1.7 that the spectral radiances are related as follows:

$$L_\lambda = \left(\frac{c}{\lambda^2}\right) L_\nu = \left(\frac{\nu}{\lambda}\right) L_\nu. \tag{1.8}$$

According to the Stefan–Boltzmann law, the radiant exitance for a blackbody becomes

$$\begin{aligned} M &= \pi \int_0^\infty L_\nu d\nu = \frac{2\pi k^4 T^4}{c^2 h^3} \int_0^\infty \frac{x^3}{e^x - 1} dx \\ &= \frac{2\pi^5 k^4}{15 c^2 h^3} T^4 = \sigma T^4, \end{aligned} \tag{1.9}$$

where σ ($= 5.67 \times 10^{-8}$ W m^{-2} K^{-4}) is the Stefan–Boltzmann constant.

For Lambertian sources, the optical system feeding a detector will receive a portion of the source power that is determined by a number of geometric factors as illustrated in Figure 1.3. The system will accept radiation from only a limited range of directions determined by the geometry of the optical system as a whole and known as the field of view. The area of the source that is effective in producing a signal is determined by the field of view and the distance from the optical system to the source (or by the size of the source if it all lies within the field of view). This area will emit radiation with some angular dependence. Only the radiation that is emitted in directions where it is intercepted by the optical system can be detected. The range of directions accepted is determined by the solid angle, Ω, that the entrance aperture of the optical system subtends as viewed from the source. In addition, some of the emitted power may be absorbed or scattered by any medium through which it propagates to reach the optical system. For a Lambertian source, the power this system receives is then the radiance in its direction multiplied by the source area within the system field of view, multiplied by the solid angle subtended by the optical system as viewed from the source, and multiplied by the transmittance of the optical path from the source to the system.

Although a general treatment must allow for the field of view to include only a portion of the source, in many cases of interest the entire source lies within the

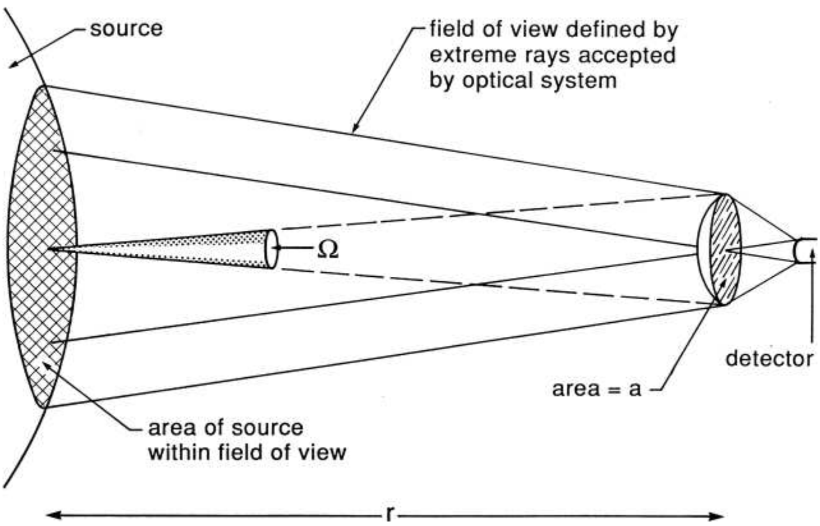

Figure 1.3 Geometry for computing power received by a detector system.

field of view, so the full projected area of the source is used in calculating the signal. For a spherical source of radius R, this area is πR^2. The solid angle subtended by the detector system is

$$\Omega = \frac{\mathrm{a}}{r^2}, \tag{1.10}$$

where a is the area of the entrance aperture of the system (strictly speaking, a is the projected area; we have assumed the system is pointing directly at the source) and r is its distance from the source. For a circular aperture,

$$\Omega = 4\pi \; sin^2(\theta/2), \tag{1.11}$$

where θ is the half-angle of the right circular cone whose base is the detector system entrance aperture, and whose vertex lies on a point on the surface of the source; r is the height of this cone.

It is particularly useful when the angular diameter of the source is small compared with the field of view of the detector system to consider the irradiance, E, which is the power in watts per square meter received at a unit surface element at some distance from the source. For the case described in the preceding paragraph, the irradiance is obtained by first multiplying the radiant exitance (from equation 1.4) by the total surface area of the source, A, to get the flux, $A\pi L$. The flux is then divided by the area of a sphere of radius r centered on the source to give

$$E = \frac{AL}{4r^2}, \tag{1.12}$$

where r is the distance of the source from the irradiated surface element. The spectral irradiance, E_ν or E_λ, is the irradiance per unit frequency or wavelength interval. It is also sometimes called the flux density, and is a very commonly used description of the power received from a source. It can be obtained from equation 1.12 by substituting L_ν or L_λ for L.

The radiometric quantities discussed above are summarized in Table 1.1. Equations are provided for illustration only; in some cases, these examples apply only to specific circumstances. The terminology and symbolism vary substantially from one discipline to another; for example, the last two columns of the table translate some of the commonly used radiometric terms into astronomical nomenclature.

1.1.2 The Detection Process

Only a portion of the power received by the optical system is passed on to the detector. The system will have inefficiencies due to both absorption and scattering of energy in its elements, and because of optical aberrations and diffraction. These effects can be combined into a system transmittance term. In addition, the range of frequencies or wavelengths to which the system is sensitive (that is, the spectral bandwidth of the system in frequency or wavelength units) is usually restricted by a spectral filter plus a combination of characteristics of the detector and other elements of the system as well as by any spectral dependence of the transmittance of the optical path from the source to the entrance aperture. A rigorous accounting of the spectral response requires that the spectral radiance of the source be multiplied point-by-point by the spectral transmittances of all the spectrally active elements in the optical path to the detector, and by the detector spectral response, and the resulting function subsequently integrated over frequency or wavelength to determine the total power effective in generating a signal.

In cases where the spectral response is restricted to a range of wavelengths by a bandpass optical filter in the beam, it is generally useful to define the effective wavelength[1] of the system as

$$\lambda_0 = \frac{\int_0^\infty \lambda\, T(\lambda)\, d\lambda}{\int_0^\infty T(\lambda)\, d\lambda}, \tag{1.13}$$

where $T(\lambda)$ is the spectral transmittance of the system. Often the spectral variations of the other transmittance terms can be ignored over the restricted spectral range of the filter. The bandpass of the filter, $\Delta\lambda$, can be taken to be the full width at half maximum (FWHM) of its transmittance function (see Figure 1.4). If the filter cuts

[1] We have characterized the response using the mean wavelength; there are a number of other conventions, but for our purposes the differences are minor and unimportant.

Table 1.1 *Definitions of radiometric quantities*

Symbol	Name	Definition	Units	Equation
L_ν	Spectral radiance (frequency units)	Power leaving unit projected surface area into unit solid angle and unit frequency interval	W m^{-2} Hz^{-1} ster^{-1}	(1.6)
L_λ	Spectral radiance (wavelength units)	Power leaving unit projected surface area into unit solid angle and unit wavelength interval	W m^{-3} ster^{-1}	(1.7)
L	Radiance	Spectral radiance integrated over frequency or wavelength	W m^{-2} ster^{-1}	$L = \int L_\nu d\nu$
M	Radiant exitance	Power emitted per unit surface area	W m^{-2}	$M = \int L(\theta)$
Φ	Flux	Total power emitted by source of area A	W	$\Phi = \int M\, d$
E	Irradiance	Power received at unit surface element; equation applies well removed from the source at distance r	W m^{-2}	$E = \frac{\int M\, dA}{(4\pi r^2)}$
E_ν, E_λ	Spectral irradiance	Power received at unit surface element per unit frequency or wavelength interval	W m^{-2} Hz^{-1}, W m^{-3}	

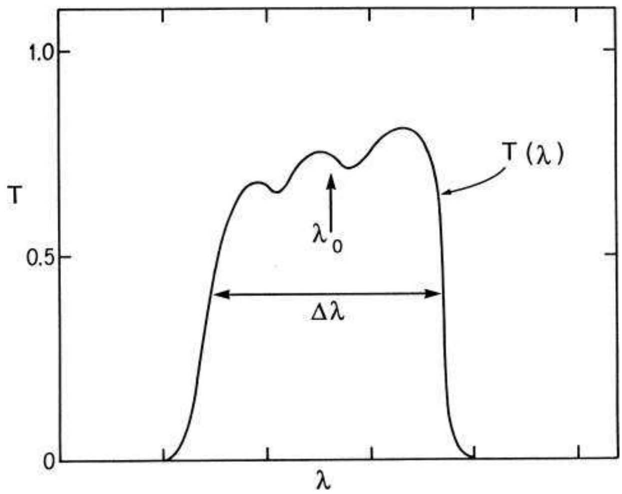

Figure 1.4 Transmittance function $T(\lambda)$ of a filter. The FWHM $\Delta\lambda$ and the effective wavelength λ_0 are indicated.

on and off sharply, its transmittance can be approximated as the average value over the range $\Delta\lambda$:

$$T_F = \frac{\int_{\Delta\lambda} T(\lambda)\, d\lambda}{\Delta\lambda}. \tag{1.14}$$

If $\Delta\lambda/\lambda_0 \leq 0.2$ and the filter cuts on and off sharply, the power effective in generating a signal can usually be estimated in a simplified manner. The signal transmitted by the bandpass filter can be approximated by taking the spectral radiance at λ_0 and multiplying it by $\Delta\lambda$ and the average filter transmittance over the range $\Delta\lambda$. Of course, to obtain the net signal that reaches the detector, this result is multiplied by the various geometric and transmittance terms already discussed for the remainder of the system. However, if λ_0 is substantially shorter than the peak wavelength of the blackbody curve (that is, one is operating in the Wien region of the blackbody) or there is sharp spectral structure within the passband, then this approximation can lead to significant errors, particularly if $\Delta\lambda/\lambda_0$ is relatively large.

Continuing with the approximation just discussed, we can derive a useful expression for estimating the power falling on the detector:

$$P_D \approx \frac{A_{proj}\ \mathrm{a}\ T_P(\lambda_0)\ T_O(\lambda_0)\ T_F\ L_\lambda(\lambda_0)\ \Delta\lambda}{r^2}. \tag{1.15}$$

Here A_{proj} is the area of the source projected onto the plane perpendicular to the line of sight from the source to the optical receiver. T_P, T_O, and T_F are the transmittances, respectively, of the optical path from the source to the receiver, of the receiver optics (excluding the bandpass filter), and of the bandpass filter. The area of the receiver entrance aperture is a, and the distance of the receiver from the source is r. An analogous expression holds in frequency units. The major underlying assumptions for equation 1.15 are (a) the field of view of the receiver includes the entire source; (b) the source is a Lambertian emitter; and (c) the spectral response of the detector is limited by a filter with a narrow or moderate bandpass that is sharply defined.

1.2 Detector Types

Nearly all detectors act as transducers that receive photons and produce an electrical response that can be amplified and converted into a form intelligible to suitably conditioned human beings. There are three basic ways that detectors carry out this function:

(a) *Photodetectors* respond directly to individual photons. An absorbed photon releases one or more bound charge carriers in the detector that may (1) modulate the electric current in the material; (2) move directly to an output amplifier; or (3) lead to a chemical change. The most common photodetectors are based on semiconducting materials and are used throughout the X-ray, ultraviolet, visible, and infrared spectral regions. Examples that we will discuss are photoconductors (Chapters 2 and 3), photodiodes (Chapter 3), charge coupled devices (CCDs) (Chapter 5), photographic materials (Chapter 6), photoemissive detectors (Chapter 6), and quantum well detectors (Chapter 6), plus some less common examples scattered about these chapters. The sheer number of types of semiconductor photodetectors provides an indication of their broad application. The unique properties of superconductors enable additional types of photodetector with applications in the submillimeter/millimeter wavelength or with the potential to provide spectral resolution within the detection process. Chapter 7 discusses two examples, microwave kinetic inductance detectors (MKIDs), and superconducting tunnel junctions (STJs).

(b) *Thermal detectors* absorb photons and thermalize their energy. In most cases, this energy changes the electrical properties of the detector material, resulting in a modulation of the electric current passing through it. Thermal detectors have a very broad and nonspecific spectral response, but they are particularly important at infrared and submillimeter wavelengths, and as X-ray detectors. Bolometers and other thermal detectors will be discussed in Chapter 8.

(c) *Coherent detectors* respond to the electric field strength of the signal and can preserve phase information about the incoming photons. They operate by interference of the electric field of the incident photon with the electric field from a local oscillator. These detectors are primarily used in the radio and submillimeter regions but also have specialized applications in the visible and infrared.

Coherent detectors for the visible and infrared are discussed in Chapter 9, and those for the submillimeter are discussed in Chapter 10.

1.3 Performance Characteristics

Good detectors preserve a large proportion of the information contained in the incoming stream of photons. A variety of parameters are relevant to this goal:

(a) *Spectral response* – the total wavelength or frequency range over which photons can be detected with reasonable efficiency.

(b) *Spectral bandwidth* – the wavelength or frequency range over which photons are detected at any one time; some detectors can operate in one or more bands placed within a broader range of spectral response.

(c) *Linearity* – the degree to which the output signal is proportional to the number of incoming photons that were received to produce the signal.

(d) *Dynamic range* – the maximum variation in signal over which the detector output represents the photon flux without losing significant amounts of information.

(e) *Quantum efficiency* – the fraction of the incoming photon stream that is converted into signal.

(f) *Noise* – the uncertainty in the output signal. Ideally, the noise consists only of statistical fluctuations due to the finite number of photons producing the signal.

(g) *Imaging properties* – e.g., the number of detectors ("pixels") in an array. Because signal may blend from one pixel to adjacent ones, the resolution that can be realized may be less, however, than indicated just by the pixel count.

(h) *Time response* – the minimum interval of time over which the detector can distinguish changes in the photon arrival rate.

The first two items in this listing should be clear from our discussion of radiometry, and the next two are more or less self-explanatory. However, the remaining entries include subtleties that call for more discussion.

1.3.1 Quantum Efficiency

To be detected, photons must be absorbed. The absorption coefficient in the detector material is indicated as $a(\lambda)$ and conventionally has units of cm^{-1}. The absorption length is defined as $1/a(\lambda)$. The absorption of a flux of photons, S, passing through a differential thickness element dl is expressed by

$$\frac{dS}{dl} = -a(\lambda)S, \tag{1.16}$$

with the solution for the remaining flux at depth l being

$$S = S_0 e^{-a(\lambda)l}. \tag{1.17}$$