

INTRODUCTION TO CHEMICAL ENGINEERING FLUID MECHANICS

Designed for undergraduate courses in fluid mechanics for chemical engineers, this textbook illustrates the fundamental concepts and analytical strategies in a rigorous and systematic, yet mathematically accessible, manner. Dimensional analysis and order-of-magnitude estimation are presented as tools to help students identify which forces are important in different settings. The friction factors for pipes and other conduits, the terminal velocities of particles, drops, and bubbles, and flow in porous media, packed beds, and fluidized beds are explained from an experimental viewpoint. The physical and mathematical distinctions among major flow regimes, including unidirectional flow, lubrication flow, creeping flow, pseudosteady flow, irrotational flow, laminar boundary layers, turbulent shear flow, and compressible flow are described. Including a full solutions manual for instructors available at www.cambridge.org/deen, this is the ideal text for a one-semester course.

William M. Deen is the Carbon P. Dubbs Professor in the Department of Chemical Engineering at MIT. He is an author of some 200 research publications in bioengineering, colloid science, membrane science, quantitative physiology, and toxicology, most involving aspects of diffusion or fluid flow. During his 40 years of teaching at MIT, he has focused on undergraduate and graduate fluid mechanics, heat transfer, and mass transfer. He is the author of *Analysis of Transport Phenomena* (2012), which is used internationally in graduate-level transport courses. Among his awards are the 2012 Bose Award for Excellence in Teaching from the MIT School of Engineering and the 2012 Warren K. Lewis Award for Contributions to Chemical Engineering Education from the AIChE.

CAMBRIDGE SERIES IN CHEMICAL ENGINEERING

Series Editor

Arvind Varma, *Purdue University*

Editorial Board

Christopher Bowman, *University of Colorado*

Edward Cussler, *University of Minnesota*

Chaitan Khosla, *Stanford University*

Athanassios Z. Panagiotopoulos, *Princeton University*

Gregory Stephanopoulos, *Massachusetts Institute of Technology*

Jackie Ying, *Institute of Bioengineering and Nanotechnology, Singapore*

Books in Series

Baldea and Daoutidis, *Dynamics and Nonlinear Control of Integrated Process Systems*

Chau, *Process Control: A First Course with MATLAB*

Cussler, *Diffusion: Mass Transfer in Fluid Systems, Third Edition*

Cussler and Moggridge, *Chemical Product Design, Second Edition*

De Pablo and Schieber, *Molecular Engineering Thermodynamics*

Deen, *Introduction to Chemical Engineering Fluid Mechanics*

Denn, *Chemical Engineering: An Introduction*

Denn, *Polymer Melt Processing: Foundations in Fluid Mechanics and Heat Transfer*

Duncan and Reimer, *Chemical Engineering Design and Analysis: An Introduction*

Fan and Zhu, *Principles of Gas–Solid Flows*

Fox, *Computational Models for Turbulent Reacting Flows*

Franses, *Thermodynamics with Chemical Engineering Applications*

Leal, *Advanced Transport Phenomena: Fluid Mechanics and Convective Transport Processes*

Lim and Shin, *Fed-Batch Cultures: Principles and Applications of Semi-Batch Bioreactors*

Marchisio and Fox, *Computational Models for Polydisperse Particulate and Multiphase Systems*

Mewis and Wagner, *Colloidal Suspension Rheology*

Morbidelli, Gavrilidis, and Varma, *Catalyst Design: Optimal Distribution of Catalyst in Pellets, Reactors, and Membranes*

Nicoud, *Chromatographic Processes*

Noble and Terry, *Principles of Chemical Separations with Environmental Applications*

Orbey and Sandler, *Modeling Vapor–Liquid Equilibria: Cubic Equations of State and their Mixing Rules*

Petyluk, *Distillation Theory and its Applications to Optimal Design of Separation Units*

Rao and Nott, *An Introduction to Granular Flow*

Russell, Robinson, and Wagner, *Mass and Heat Transfer: Analysis of Mass Contactors and Heat Exchangers*

Schobert, *Chemistry of Fossil Fuels and Biofuels*

Shell, *Thermodynamics and Statistical Mechanics*

Sirkar, *Separation of Molecules, Macromolecules and Particles: Principles, Phenomena and Processes*

Slattery, *Advanced Transport Phenomena*

Varma, Morbidelli, and Wu, *Parametric Sensitivity in Chemical Systems*

Introduction to Chemical Engineering Fluid Mechanics

William M. Deen

Massachusetts Institute of Technology



CAMBRIDGE
UNIVERSITY PRESS



CAMBRIDGE
UNIVERSITY PRESS

Shaftesbury Road, Cambridge CB2 8EA, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of Cambridge University Press & Assessment,
a department of the University of Cambridge.

We share the University's mission to contribute to society through the pursuit of
education, learning and research at the highest international levels of excellence.

www.cambridge.org

Information on this title: www.cambridge.org/9781107123779

© W. Deen 2016

This publication is in copyright. Subject to statutory exception and to the provisions
of relevant collective licensing agreements, no reproduction of any part may take
place without the written permission of Cambridge University Press & Assessment.

First published 2016

A catalogue record for this publication is available from the British Library

ISBN 978-1-107-12377-9 Hardback

Additional resources for this publication at www.cambridge.org/9781107123779

Cambridge University Press & Assessment has no responsibility for the persistence
or accuracy of URLs for external or third-party internet websites referred to in this
publication and does not guarantee that any content on such websites is, or will
remain, accurate or appropriate.

Cambridge University Press & Assessment

978-1-107-12377-9 — Introduction to Chemical Engineering Fluid Mechanics

William M. Deen

Frontmatter

[More Information](#)

To Meredith and Michael

Contents

Preface	page xxi
List of symbols	xxiv
Part I Use of experimental data	
I Properties, dimensions, and scales	3
1.1 Introduction	3
1.2 Fluid properties	3
Viscosity	3
Density and kinematic viscosity	5
Units and values	5
Non-Newtonian liquids	7
Surface tension	10
Continuum approximation	11
1.3 Scales and dimensionless groups	12
Scales	12
Dimensions	13
Stress scales	13
Dimensionless groups	14
Example 1.3-1 Deep-water waves	16
Example 1.3-2 Inkjet printing	16
1.4 Dimensional analysis	17
Pi theorem	17
Example 1.4-1 Speed of water waves	18
Example 1.4-2 Shear stress in pipe flow	20
Example 1.4-3 Energy of an atomic blast	21
Dynamic similarity	22
1.5 Conclusion	22
References	23
Problems	24
1.1 Falling body	24
1.2 Pendulum	24
1.3 Salad dressing	24
1.4 Heat transfer coefficient	24
1.5 Oscillating drops	25
1.6 Dip coating	25
	ix

Contents

1.7	Breakup of liquid jets	26
1.8	Valve scale-up	27
1.9	Ship scale-up	27
1.10	Power input in a stirred tank	28
1.11	Underwater swimming	28
2	Pipe flow	30
2.1	Introduction	30
2.2	Shear stress	30
	Fundamental quantities	30
	Friction factor	31
2.3	Pressure drop and dynamic pressure	34
	Friction factor and pressure drop	34
	Circuit analogy	36
	Example 2.3-1 Pressure drop for water in process pipes	36
	Example 2.3-2 Pressure drop in an oil pipeline	37
	Example 2.3-3 Flow rate in an oil pipeline	38
	Example 2.3-4 Capillary viscometer	38
2.4	Noncircular cross-sections	39
	Turbulent flow	39
	Laminar flow	40
	Example 2.4-1 Pressure drop for air in a triangular duct	41
	Example 2.4-2 Material efficiency of square and circular ducts	41
2.5	Wall roughness	42
	Example 2.5-1 Effect of roughness on water flow in a process pipe	45
	Example 2.5-2 Practical smoothness	45
2.6	Conclusion	46
	References	47
	Problems	47
2.1	Cavitation	47
2.2	Bottling honey	48
2.3	Filling a boiler	48
2.4	Syringe pump	48
2.5	Flue gases	49
2.6	Hydraulic fracturing	49
2.7	Drag reduction	49
2.8	Economic pipe diameter	50
2.9	Microfluidic device	51
2.10	Murray's law	51
2.11	Open-channel flow	52
3	Drag, particles, and porous media	54
3.1	Introduction	54
3.2	Drag	54
	Origins	54
	Drag coefficient	55
	Spheres	56

Contents

	Disks	57
	Cylinders	58
	Flat plates	59
	Example 3.2-1 Drag on a cylinder in water	61
	Example 3.2-2 Comparative drag on a cylinder and a flat plate	61
3.3	Terminal velocity	62
	Buoyancy and gravity	62
	Terminal velocities for solid spheres	62
	Example 3.3-1 Sand grain falling in air	64
	Example 3.3-2 Microfluidic cell separation	65
	Terminal velocities for fluid spheres	65
	Approach to terminal velocity	66
	Example 3.3-3 Approach to terminal velocity for large spheres	66
3.4	Porous media	68
	Darcy permeability	68
	Microstructural models	70
	Example 3.4-1 Air flow through a packed bed of spheres	72
	Example 3.4-2 Comparative properties of granular and fibrous media	73
3.5	Packed beds and fluidized beds	73
	Packed beds	73
	Fluidized beds	75
	Example 3.5-1 Fluidization at low Reynolds number	76
3.6	Conclusion	76
	References	77
	Problems	78
3.1	Chain-link fence	78
3.2	Rowing power	78
3.3	Dispersion of pollen	78
3.4	Downhill ski racing	78
3.5	Homogenized milk	79
3.6	Approach to terminal velocity for small fluid spheres	79
3.7	Inhaled particles	79
3.8	Flocculation	81
3.9	Hydrogel disks	81
3.10	Bypassing a packed bed	81
3.11	Fluidization at high Reynolds number	82

Part II Fundamentals of fluid dynamics

4	Fluid statics: pressure, gravity, and surface tension	85
4.1	Introduction	85
4.2	Pressure in static fluids	85
	Properties of pressure	85
	Static pressure equation	85
	Pressure distributions	87
	Example 4.2-1 Manometer	87
	Example 4.2-2 Layered fluids	88
	Additional note: Pascal's law	89

Contents

4.3	Pressure forces	89
	Stress and force vectors	89
	Boundaries	90
	Example 4.3-1 Rectangular tank	90
	Example 4.3-2 Inclined planar surface	91
	Projected areas	92
	Immersed objects at constant pressure	93
	Buoyancy	94
	Example 4.3-3 Buoyancy of a sphere	95
4.4	Surface tension	96
	Tensile forces and contact lines	96
	Example 4.4-1 Young–Laplace equation	97
	Example 4.4-2 Capillary rise	98
	Interfaces with variable curvature	99
4.5	Conclusion	101
	References	101
	Problems	101
4.1	Manometry for liquid pipe flow	101
4.2	Hydraulic lift	102
4.3	Static pressure variations in air	103
4.4	Force on Hoover Dam	103
4.5	Floating cup	103
4.6	Sedimentation in a sucrose gradient	104
4.7	Half-submerged cylinder	105
4.8	Buoyancy of a cone	105
4.9	Formation of small bubbles	105
4.10	Capillary adhesion	106
4.11	Capillary flotation	107
4.12	Plateau–Rayleigh instability	107
5	Fluid kinematics	110
5.1	Introduction	110
5.2	Continuity	110
	Example 5.2-1 Unknown velocity component	111
	Example 5.2-2 Expansion of the Universe	112
	Example 5.2-3 Filtration in a hollow fiber	113
5.3	Rates of change for moving observers	115
	Example 5.3-1 Temperature changes sensed by a weather balloon	116
5.4	Rate of strain	116
	Example 5.4-1 Rate of strain in simple shear flow	117
	Example 5.4-2 Rate of strain in pure dilatation	118
5.5	Vorticity	119
	Definition	119
	Irrotational flow	120
5.6	Stream function	121
	Definitions	121
	Streamlines and streaklines	122

Contents

	Example 5.6-1 Streamlines from the stream function	123
	Trajectories	124
	Example 5.6-2 Streamlines from trajectories	125
5.7	Conclusion	125
	References	126
	Problems	126
5.1	Flow past a bubble	126
5.2	Channel with wavy walls	126
5.3	Condensation on a vertical wall	127
5.4	Flow past a solid sphere	128
5.5	Wedge flow	128
5.6	Flow between porous and solid disks	128
5.7	Trajectories of sedimenting particles	129
6	Stress and momentum	130
6.1	Introduction	130
6.2	Stress vector and stress tensor	130
	Stress notation	131
	Stress at an arbitrary surface	132
6.3	Force at a point	132
6.4	Conservation of momentum	134
	Additional note: stress equilibrium	136
6.5	Viscous stress	139
	Rate-of-strain tensor	139
	Example 6.5-1 Rate of strain in simple shear flow	141
	Newtonian fluids	141
	Non-Newtonian fluids	143
	Additional note: stress symmetry	145
6.6	Governing equations	146
	Newtonian fluids with constant properties	146
	Example 6.6-1 Pressure in planar stagnation flow	147
	Fluids with varying viscosity	149
	Velocities at phase boundaries	149
	Stresses at phase boundaries	151
	Example 6.6-2 Shear-stress boundary condition with variable surface tension	151
	Force calculations	152
	Example 6.6-3 General expression for the drag on a sphere	153
6.7	Conclusion	154
	References	155
	Problems	155
6.1	Stress vector and tensor	155
6.2	Effect of surface orientation on the stress vector	156
6.3	Force balance for plane Couette flow	156
6.4	Force balance for plane Poiseuille flow	156
6.5	Normal viscous stress at a solid surface	157
6.6	Drag on a cylinder at high Reynolds number	157

Contents

6.7	Pressure for creeping flow past a solid sphere	158
6.8	Pressure between porous and solid disks	158

Part III Microscopic analysis

7	Unidirectional flow	161
7.1	Introduction	161
7.2	Fully developed flow	161
	Example 7.2-1 Velocity and pressure for plane Poiseuille flow	161
	Example 7.2-2 Velocity and pressure for Poiseuille flow	163
	Example 7.2-3 Friction factor for laminar tube flow	164
7.3	Moving surfaces	166
	Example 7.3-1 Plane Couette flow	166
	Example 7.3-2 Rotating rod	166
	Example 7.3-3 Plate suddenly set in motion	168
7.4	Free surfaces	172
	Example 7.4-1 Falling film on a vertical wall	172
	Example 7.4-2 Surface of a stirred liquid	174
7.5	Non-Newtonian fluids	175
	Example 7.5-1 Poiseuille flow of a power-law fluid	175
	Example 7.5-2 Plane Couette flow of generalized Newtonian fluids	177
7.6	Symmetry conditions	178
	Cylindrical symmetry	178
	Reflective symmetry	178
7.7	Conclusion	179
	References	179
	Problems	179
	7.1 Couette viscometer	179
	7.2 Annular conduit	180
	7.3 Triangular conduit	180
	7.4 Elliptical conduit	181
	7.5 Slip in tube flow	182
	7.6 Darcy permeability of a fibrous material	182
	7.7 Surface of a liquid in rigid-body rotation	183
	7.8 Layered liquids on an inclined surface	183
	7.9 Liquid film outside a vertical tube	184
	7.10 Film on an upward-moving surface	184
	7.11 Slot coating	185
	7.12 Flow in a cavity	185
	7.13 Falling-cylinder viscometer	186
	7.14 Bubble rising in a tube	187
	7.15 Paint film	188
	7.16 Temperature-dependent viscosity	188
	7.17 Blood rheology	188
8	Approximations for viscous flows	190
8.1	Introduction	190

Contents

8.2	Lubrication approximation	190
	Example 8.2-1 Tapered channel	191
	Example 8.2-2 Permeable tube	194
	Example 8.2-3 Slider bearing	195
8.3	Creeping flow	198
	Stokes' equation	198
	Example 8.3-1 Flow between porous and solid disks	200
	Example 8.3-2 Flow past a sphere	201
	Example 8.3-3 Stokes' law	205
	Porous media	206
8.4	Pseudosteady flow	207
	Example 8.4-1 Parallel-plate channel with a decaying pressure drop	207
	Example 8.4-2 Squeeze flow	208
8.5	Anticipating approximations	209
	Order-of-magnitude estimation	210
	Example 8.5-1 Order-of-magnitude analysis for a tapered channel	211
	Example 8.5-2 Order-of-magnitude analysis for Stokes flow past a sphere	212
	Lubrication approximation	212
	Creeping-flow approximation	213
	Pseudosteady approximation	213
	Example 8.5-3 Order-of-magnitude analysis for squeeze flow	214
	Example 8.5-4 Force on a slider bearing	215
8.6	Conclusion	216
	References	217
	Problems	217
	8.1 Imperfect parallel-plate channel	217
	8.2 Permeable closed-end tube	218
	8.3 Permeation-driven flow in a microchannel	218
	8.4 Candy manufacturing	219
	8.5 Blade coating	219
	8.6 Torque on a rotating sphere	220
	8.7 Velocity and pressure for flow past a bubble	220
	8.8 Terminal velocity of a small bubble	220
	8.9 Rotating and stationary disks	221
	8.10 Cone-and-plate viscometer	221
	8.11 Growing mercury drop	222
	8.12 Drag on a cylinder at low Reynolds number	222
	8.13 Darcy flow in a tumor	223
	8.14 Washburn's law	224
	8.15 Injection molding	225
	8.16 Capillary pump	225
9	Laminar flow with inertia	227
	9.1 Introduction	227
	9.2 Inviscid and irrotational flow	227

Contents

	Inviscid flow	227
	Vorticity transport	228
	Irrotational flow	229
	Example 9.2-1 Velocity for potential flow past a cylinder	230
	Example 9.2-2 Pressure and drag for inviscid and irrotational flow past a cylinder	232
	Example 9.2-3 Water waves	233
9.3	Boundary layers: differential analysis	236
	Boundary-layer approximation	236
	Joining the regions	238
	Example 9.3-1 Blasius solution for a flat plate	239
	Wedge flows	242
	Internal boundary layers	243
	Example 9.3-2 Planar jet	243
9.4	Boundary layers: integral analysis	244
	Integral momentum equation	244
	Example 9.4-1 Integral solution for a flat plate	246
	Boundary-layer separation	247
	Example 9.4-2 Integral solution for a cylinder	248
9.5	Conclusion	252
	References	252
	Problems	253
9.1	Potential flow past a sphere	253
9.2	Lift on a half-cylinder	253
9.3	Axisymmetric stagnation flow	253
9.4	Opposed circular jets	254
9.5	Added mass for a sphere	254
9.6	Spin coating	254
9.7	Bubble growing in a liquid	255
9.8	Entrance length	256
9.9	Axisymmetric jet	256
9.10	Boundary layers in power-law fluids	257
9.11	Normal velocity component for a flat plate	257
9.12	Rotating disk	257
9.13	Flat plate with suction	259
9.14	Terminal velocity of a large bubble	259
9.15	Planar stagnation flow	260
9.16	Flow past a right-angle wedge	260
10	Turbulent flow	261
10.1	Introduction	261
10.2	Characteristics and scales	261
	Basic features	261
	Wall variables	263
	Kolmogorov scales	264
	Example 10.2-1 Turbulence scales for air flow in a pipe	265
10.3	Reynolds averaging	266
	Time-smoothed variables	266

Contents

Continuity equation	267
Navier–Stokes equation	268
Closure problem	268
Reynolds stress	268
10.4 Closure schemes	269
Eddy diffusivities	270
Other approaches	272
10.5 Unidirectional flow	272
Example 10.5-1 Velocity profile near a wall	272
Complete velocity profile for tube flow	275
Example 10.5-2 Prandtl–Kármán equation	276
10.6 Boundary layers	277
Example 10.6-1 Flat plate	277
Example 10.6-2 Axisymmetric jet	279
Limitations of mixing-length concept	281
10.7 Conclusion	281
References	282
Problems	283
10.1 Turbulence scales for water flow in a pipe	283
10.2 Cell damage in turbulent flow	283
10.3 Jet velocity from a photograph	283
10.4 Reynolds-stress data	284
10.5 Eddy diffusivity from near-wall velocity data	285
10.6 Mixing length in tube flow	285
10.7 Power-law velocity profile and Blasius friction factor	285
10.8 Improved velocity profile for tube flow	285
10.9 Friction factor and hydraulic diameter	286
10.10 Effects of tube roughness	286
10.11 Planar jet	287
10.12 Eddy diffusivity in a circular jet	287
Part IV Macroscopic analysis	
11 Macroscopic balances for mass, momentum, and energy	291
11.1 Introduction	291
11.2 Conservation of mass	291
General control volume	291
Discrete openings	292
Example 11.2-1 Fluid displacement from a cavity	293
Example 11.2-2 Draining of a tank through a horizontal pipe	293
Integration of the continuity equation	295
11.3 Conservation of momentum	295
General control volume	295
Discrete openings	296
Example 11.3-1 Force on a return bend	298
Example 11.3-2 Acceleration of a force-free rocket	299

Contents

11.4 Mechanical energy balances	300
General control volume	300
Discrete openings	301
Example 11.4-1 Viscous loss in pipe flow	303
Example 11.4-2 Venturi flow meter	304
Example 11.4-3 Hydroelectric power	305
Additional note: mechanical energy derivations	306
11.5 Systems with free surfaces	308
Example 11.5-1 Capillary jet	308
Example 11.5-2 Hydraulic jump	309
Example 11.5-3 Liquid jet striking an inclined plate	311
11.6 Conclusion	313
References	314
Problems	314
11.1 Torricelli's law	314
11.2 Water clock	314
11.3 Forces on nozzles	315
11.4 Drag on a flat plate calculated from the wake velocity	315
11.5 Drag on a cylinder calculated from the wake velocity	316
11.6 Jet ejector	317
11.7 Wave tank	317
11.8 Force in a syringe pump	318
11.9 Plate suspended by a water jet	318
11.10 Viscous losses in laminar pipe flow	319
11.11 Hydroelectric power	319
11.12 Pitot tube	319
11.13 Siphon	320
11.14 Sump pump	320
11.15 Drainage pipe	321
12 Pipe flow: entrance effects, fittings, and compressibility	322
12.1 Introduction	322
12.2 Entrance effects	322
Entrance length	322
Excess pressure drop in entrance regions	323
Example 12.2-1 Entrance correction for a process pipe	325
Example 12.2-2 Entrance correction for a capillary viscometer	325
12.3 Fittings, valves, and pumps	325
Loss coefficients	325
Pump characteristics	327
Example 12.3-1 Force on a return bend (revisited)	328
Example 12.3-2 Borda–Carnot equation	329
Example 12.3-3 Pressure increase at a diverging branch	330
Example 12.3-4 Draining of one tank into another	332
Additional note: pseudosteady approximation for tank filling or emptying	334

Contents

12.4	Compressible flow in long pipes	335
	Engineering Bernoulli equation for variable density	336
	Isothermal pipe flow	337
	Example 12.4-1 Natural-gas pipeline	339
12.5	Compressible flow near the speed of sound	341
	Adiabatic pipe flow	341
	Choked flow	344
	Example 12.5-1 Absence of choking in a natural-gas pipeline	345
	Example 12.5-2 Choked air flow	345
	Varying cross-section: nozzles and diffusers	346
	Example 12.5-3 Converging nozzle	348
12.6	Conclusion	349
	References	350
	Problems	350
12.1	Entrance effects with air flow	350
12.2	Entrance-region model	351
12.3	Nozzle with diffuser	351
12.4	Water siphon	352
12.5	Pumping from a lower to a higher reservoir	352
12.6	Water transfer from a higher to a lower reservoir	352
12.7	Home plumbing	353
12.8	Membrane hydraulic permeability	353
12.9	Design of distribution manifolds	354
12.10	Tubular reactors in parallel	355
12.11	Pumping between tanks	356
12.12	Pumps in series or parallel	356
12.13	Conical diffuser	357
12.14	Balloon inflation	357
12.15	Discharge of a compressed-air tank	358
12.16	Automobile tire inflation	358
12.17	Comparison of isothermal and adiabatic pipe flow	358
12.18	Gas-cylinder hazard	358
12.19	Speed of sound	359
12.20	Transonic flow	360
Appendix. Vectors, tensors, and coordinate systems		362
A.1	Introduction	362
A.2	Notation and fundamentals	362
	Representation of vectors and tensors	362
	Basic operations	363
	Coordinate independence	364
A.3	Vector and tensor products	364
	Vector dot product	364
	Vector cross product	366
	Dyadic product	367
	Tensor products	367
	Identity tensor	368

Contents

Example A.3-1 Repeated dot products of a vector with an antisymmetric tensor	369
Example A.3-2 Scalar triple products	369
A.4 Differential and integral identities	370
Gradient	370
Divergence	370
Curl	370
Laplacian	370
Differential identities	371
Example A.4-1 Proof of a differential identity	371
Example A.4-2 Proof of a differential identity	372
Example A.4-3 Proof of a differential identity	372
Example A.4-4 Proof of a differential identity	373
Integral transformations	374
Unit normal and unit tangent vectors	374
Example A.4-5 Integration of a unit normal over a surface	375
A.5 Orthogonal curvilinear coordinates	376
Base vectors	376
Position vectors and scale factors	376
Volumes and surface areas	377
Gradient	378
Scale-factor identities	378
Divergence	379
Curl	379
Laplacian	380
Cartesian coordinates	380
Cylindrical coordinates	380
Spherical coordinates	382
References	384
Author index	385
Subject index	388

Preface

WHAT IS CHEMICAL ENGINEERING FLUID MECHANICS?

Quantitative experimentation with fluids began in antiquity, and the foundations for the mathematical analysis of fluid flow were well established by the mid 1800s. Although a mature subject, fluid mechanics remains a very active area of research in engineering, applied mathematics, and physics. As befits a field that is both fascinating and useful, it has been the subject of innumerable introductory textbooks. However, only a few have focused on the aspects of fluid mechanics that are most vital in chemical engineering.

Certain results that stem from conservation of mass and momentum in fluids cut across all fields. However, the kinds of flow that are of greatest interest differ considerably among the various branches of engineering. One thing that distinguishes fluid mechanics in chemical engineering from that in, say, aeronautical or civil engineering, is the central importance of viscosity. Viscous stresses are at the heart of predicting flow rates in pipes, which has always been the main application of fluid mechanics in process design. Moreover, chemical engineering encompasses many technologies that involve bubbles, drops, particles, porous media, or liquid films, where small length scales amplify the effects of viscosity. Surface tension, usually not a concern in other engineering disciplines, also can be important at such length scales. In addition, in chemical engineering applications even gases usually can be idealized as incompressible. Another feature of chemical engineering fluid mechanics is an emphasis on microscopic analysis to calculate velocity fields. Determining velocities and pressures, and finding the resulting forces or torques, is often not an end in itself. Detailed velocity fields are needed to predict concentration and temperature distributions, which in turn are essential for the analysis and design of reactors and separation devices. Of lesser concern than in some other disciplines are the fluid dynamics of rotating machinery, flow in open channels, and flow at near-sonic velocities (where gas compressibility is important). Thus, chemical engineering fluid mechanics is characterized by a heightened interest in the microscopic analysis of incompressible viscous flows. Biomedical and mechanical engineers share some of the same concerns.

PURPOSE AND ORGANIZATION

This book is designed mainly as a text for chemical engineering undergraduates. The intention is to present fundamental concepts in a rigorous but mathematically accessible manner. A recurring theme is how to identify what is important physically in a novel situation and how to use such insights in modeling. That is illustrated by examples both within and outside the traditional domain of chemical engineering. The end-of-chapter problems tend to be challenging. They are intended not just to provide practice in certain

Preface

kinds of calculations, but to build confidence in analyzing physical systems and to help develop engineering judgment.

The essential prerequisites are introductory mechanics, multivariable calculus, and ordinary differential equations. The information on vectors and tensors that is needed to understand certain derivations is summarized in an appendix, thereby making that part of the mathematics self-contained. Familiarity with a few numerical methods (e.g., solving first-order differential equations) is helpful but not necessary. No prior experience with partial differential equations is assumed; solution methods are explained as the need arises. A basic background in thermodynamics is presumed only in the last chapter.

The book has four parts. Part I, “Use of Experimental Data” (Chapters 1–3), discusses fluid properties, representative magnitudes of velocities and forces, and certain kinds of design. The information in these chapters is largely empirical. After surveying gas and liquid properties, Chapter 1 introduces dimensional analysis and the several uses of dimensionless groups, with an emphasis on groups that indicate the relative importance of different kinds of forces. Chapter 2 focuses on pressure–flow relationships in long pipes or other conduits. Chapter 3 discusses drag forces, terminal velocities of particles, porous media, packed beds, and fluidized beds. While presenting various experimental results and explaining certain engineering calculations, Chapters 2 and 3 introduce phenomena and relationships that are revisited later from more fundamental viewpoints.

Part II, “Fundamentals of Fluid Dynamics” (Chapters 4–6), lays the groundwork for predictive modeling. Chapter 4, on static fluids, explains the interactions among pressure, gravity, and surface tension and begins to make force calculations more precise. Chapter 5 introduces the continuity equation (the differential equation that embodies conservation of mass), the concept of rate of strain, and other aspects of kinematics, the description of fluid motion. Chapter 6 provides a general description of viscous stresses and combines that with conservation of linear momentum. The main result is the Navier–Stokes equation, the differential equation that ordinarily governs momentum changes within fluids. As aspects of vectors, tensors, and analytical geometry become relevant, the reader is referred to specific sections of the Appendix.

Part III, “Microscopic Analysis” (Chapters 7–10), illustrates how to use the governing equations of Chapter 6 to predict velocity and pressure fields, and how then to calculate fluid forces and torques. Chapter 7 is devoted to unidirectional flow, the simplest set of applications. Chapter 8 discusses how to anticipate and justify simplifications of the Navier–Stokes equation when viscous stresses are much more important than the inertia of the fluid. Introduced there are the lubrication, creeping flow, and pseudosteady approximations. Chapter 9 extends the discussion of approximations to laminar flows where inertia is prominent, as in boundary layers. Concepts unique to turbulent flow (Kolmogorov scales, time-smoothing, Reynolds stress, and mixing lengths) are presented in Chapter 10. Numerous connections are made between results derived in Part III and the experimental observations in Part I.

Part IV, “Macroscopic Analysis” (Chapters 11–12), focuses on flow problems that are too complex for the approaches in Part III, but where a less detailed kind of analysis is useful. Integral forms of the conservation equations are derived in Chapter 11 and simplified to algebraic equations that are practical for applications, such as the engineering Bernoulli equation. Key assumptions are justified by referring to the microscopic results of Part III. The simplified macroscopic balances are applied to a variety of systems in Chapter 11. Chapter 12 revisits pipe flow, including now resistances due to entrance regions and pipe fittings. It concludes with an introduction to compressible flow.

Preface

Although proofs of all key theoretical results are provided, some derivations are put in “additional notes” at the ends of sections. Any subsection so labeled can be skipped without loss of continuity. In contrast, all the examples illustrate core material and merit study. Many of the end-of-chapter problems present additional theoretical or experimental results or describe new kinds of applications. It is recommended that all the problem statements be read as part of the chapter, even if solutions are not to be worked out.

In manuscript form, the book has been used successfully as the text in a one-semester course for chemical engineering undergraduates. In a fast-paced course with four contact hours per week over 14 weeks, approximately 80% of the material was covered. Overall, the content provides a reasonable foundation for practicing chemical engineers and good preparation for graduate-level study of fluid mechanics. If supplemented by a comparable introduction to heat and mass transfer, it would be good preparation for graduate study of transport phenomena.

ACKNOWLEDGMENTS

Some of the examples and problems originated with MIT colleagues or faculty elsewhere. I have identified such unpublished sources with footnotes that state “this problem was suggested by,” a phrase intended to give credit without blame. I have revised what others had written in homework or exam problems, often extensively, or elaborated on what they and I discussed, and therefore take all responsibility for errors or confusion.

I have learned a great deal over the years from MIT faculty with whom I have taught fluid mechanics or otherwise discussed the subject. Among those present or former colleagues are Robert C. Armstrong, Martin Z. Bazant, Robert A. Brown, Fikile R. Brushett, Arup K. Chakraborty, Patrick S. Doyle, Kenneth A. Smith, James W. Swan, and Preetinder S. Virk. As a graduate student at Stanford long ago, I was inspired by a course in viscous flow theory taught by Andreas Acrivos. My education has been advanced no less by interactions with generations of MIT students, who have always impressed me with their curiosity and determination to learn. Responding to their questions has continually sharpened my own understanding of the subject.

Although undertaken after retirement, the writing of this book was supported in part by the Carbon P. Dubbs Chair in the Department of Chemical Engineering at MIT. I am appreciative of that support and the other encouragement I have received from the Department.

Last but not least are those at Cambridge University Press who helped improve the book and make it a reality, including acquisition editor Michelle Carey, copy-editor Steven Holt, and production editor Charles Howell.

W. M. D.

Symbols

Following is a list of the more commonly used symbols. Omitted are coordinates (defined in Section A.5) and many quantities that appear in just one chapter. In general, scalars are italic Roman or Greek (e.g., a or δ), vectors are bold Roman (e.g., \mathbf{v}), and tensors are bold Greek (e.g., $\boldsymbol{\tau}$). Magnitudes of vectors and tensors are denoted usually by the corresponding italic letter (e.g., v or τ), and vector and tensor components are represented using subscripted italics (e.g., v_x or τ_{yx}). Natural and base-10 logarithms are written as “ln” and “log,” respectively.

ROMAN LETTERS

A	Surface area or cross-sectional area.
Ar	Archimedes number.
Bo	Bond number.
C_D	Drag coefficient [Eq. (3.2-1)].
C_f	Friction factor or drag coefficient for a flat plate [Eq. (3.2-13)].
Ca	Capillary number.
D	Diameter (also d).
D_H	Hydraulic diameter [Eq. (2.4-1)].
\mathbf{e}_i	Unit vector associated with coordinate i .
E_c	Rate of mechanical energy loss due to compression [Eq. (11.4-2)].
E_v	Rate of mechanical energy loss due to viscous dissipation [Eq. (11.4-3)].
f	Friction factor for a tube or other conduit [Eq. (2.2-4)].
F_D	Drag force.
\mathbf{F}	Force vector.
\mathbf{F}_0	Force due to static pressure variations.
\mathbf{F}_B	Net buoyancy force, $\mathbf{F}_0 - \mathbf{F}_G$.
\mathbf{F}_G	Gravitational force.
\mathbf{F}_P	Pressure force.
\mathbf{F}_ϕ	Flow-dependent part of pressure force.
\mathbf{F}_τ	Viscous force.
Fr	Froude number.
\mathbf{g}	Gravitational acceleration vector.
\mathbf{G}	Torque vector.
h	Height (usually).
k	Wall roughness parameter (usually) or Darcy permeability (Chapter 3).
K_i	Loss coefficient for event or device i [Eq. (12.3-1)].
L	Length as a dimension.

List of symbols

L	Object length or characteristic length.
L_E	Entrance length for tubes or other conduits.
m	Mass.
M	Mass as a dimension.
Ma	Mach number.
\mathbf{n}	Unit vector normal to a surface, directed outward from a control volume or from phase 1 to phase 2.
P	Absolute or thermodynamic pressure.
\mathcal{P}	Dynamic pressure (also called modified pressure or equivalent pressure).
q	Volume flow rate per unit width in two-dimensional flows.
Q	Volume flow rate.
\mathbf{r}	Position vector.
R	Radius (usually) or universal gas constant (Sections 1.2, 12.4, and 12.5).
Re	Reynolds number.
\mathbf{s}	Stress vector.
S	Surface area.
t	Time.
\mathbf{t}	Unit vector tangent to a surface.
T	Absolute temperature.
T	Time as a dimension.
\mathbf{u}	Interfacial velocity (Chapter 6), turbulent velocity fluctuation (Chapter 10), or control-surface velocity (Chapter 11).
u	In boundary-layer flows, the outer velocity evaluated at the surface (Chapters 9 and 10).
u_τ	Friction velocity [Eq. (10.2-4)].
U	Characteristic velocity, usually a mean fluid velocity or particle velocity.
v_s	Superficial velocity [Eq. (3.4-1)].
\mathbf{v}	Fluid velocity.
V	Volume (usually) or velocity.
w	Mass flow rate (Chapters 11 and 12).
\mathbf{w}	Vorticity vector.
W_m	Rate of work done on a system by moving surfaces (shaft work).

GREEK LETTERS

α	Contact angle at a three-phase contact line.
β	At a contraction or expansion, the smaller diameter divided by the larger one.
δ	Boundary-layer thickness (usually).
δ_{ij}	Kronecker delta [Eq. (A.3-6)].
δ	Identity tensor.
Δ	Difference along the direction of flow (downstream value minus upstream value) or differential change.
ε	Volume fraction of fluid (void fraction) in porous media or packed beds.
ε_{ijk}	Permutation symbol [Eq. (A.3-15)].
ϕ	Volume fraction of solids in porous media (Chapter 3) or velocity potential (Chapters 5 and 9).
$\mathbf{\Gamma}$	Rate-of-strain tensor [Eq. (6.5-1)].
γ	Surface tension (usually) or heat-capacity ratio [Eq. (12.4-1)].
μ	Viscosity.

List of symbols

ν	Kinematic viscosity, μ / ρ .
ρ	Density.
σ	Total stress tensor.
τ	Viscous stress tensor.
τ_w	Shear stress at a wall or other solid surface (also τ_0).
Ω	Vorticity tensor [Eq. (6.5-3)].
ψ	Stream function.

SPECIAL SYMBOLS

D/Dt	Material derivative, $\partial/\partial t + \mathbf{v} \cdot \nabla$.
∇	Gradient operator.
∇^2	Laplacian operator, $\nabla \cdot \nabla$.
\sim	Order-of-magnitude (OM) equality.