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978-1-107-12377-9 — Introduction to Chemical Engineering Fluid Mechanics
William M. Deen
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Part I

Use of experimental data

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Properties, dimensions, and scales

1.1 INTRODUCTION

Fluids, including both gases and liquids, are materials that deform continuously when subjected to shearing forces. If a flowing fluid contains a dye or other tracer, the labeled region tends to change shape from instant to instant. The viscosity of a fluid is a reflection of its resistance to such deformations. This chapter begins with definitions of viscosity and three other material properties that are important in fluid mechanics, namely density, kinematic viscosity, and surface tension. Representative values of each are presented, and some of the differences between Newtonian and non-Newtonian fluids are described.

Physical quantities have *dimensions*, such as mass (M), length (L), and time (T). The concepts of mass, length, and time are more fundamental than the *units* in a particular system of measurement, such as the kilogram (kg), meter (m), and second (s) that underlie the SI system. To have general validity, a physical relationship must be independent of the observer, including the observer's choice of units. That can be achieved by making each variable or parameter in an equation *dimensionless*, which means that the physical quantities are grouped in such a way that their dimensions cancel. Dimensionless parameters are ratios, and often the numerator and denominator are each the *scale* of a variable. A scale is the maximum value of something, such as a velocity or force. Thus, the numerical value of such a group reveals how two things compare, and thereby offers insight into what is important in the process or phenomenon under consideration and what might be negligible. Several dimensionless groups that arise in fluid mechanics are discussed.

Dimensional analysis, the last major topic of the chapter, provides a systematic way to identify the dimensionless groups that are involved in something of interest. In addition to yielding relationships that are more universal, grouping quantities in this manner minimizes the number of independent variables. This greatly facilitates the design of experiments and interpretation of data, as discussed here and explored further in Chapters 2 and 3.

1.2 FLUID PROPERTIES

Viscosity

A definition of viscosity is provided by the idealized experiment in Fig. 1.1. Imagine that a fluid fills a space of thickness H between parallel plates, each of which has an area A. Suppose that the upper plate is pulled to the right (in the x direction) at a constant velocity U, while the lower one is held in place. Because fluids tend to adhere to surfaces in a way that prevents relative motion or "slip," the fluid contacting the top plate also moves at velocity U, while that next to the bottom remains stationary. Once enough time

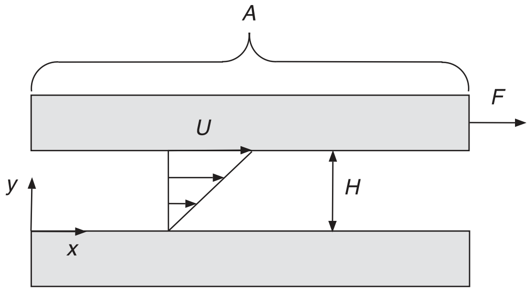

Figure 1.1 Shear flow between parallel plates. A force F applied to the top plate moves it to the right at velocity U, while the bottom plate is kept stationary. The resulting fluid velocity varies linearly from zero at the bottom to U at the top.

has elapsed, the fluid velocity varies linearly with height (the y coordinate), as shown by the arrows. As elements of the fluid at different heights move to the right at different speeds, the distance between any two of them increases. Such elongation of imaginary lines connecting pairs of points indicates that the fluid is being deformed.

A constant force F, to the right, must be applied to the top plate to keep it moving, and the absence of slip transfers that force to the adjacent fluid. The fluid itself may be thought of as having innumerable layers, each of which resists moving at a velocity different from the adjacent ones. This internal friction, which is proportional to the viscosity, transmits the horizontal force through successive fluid layers and to the bottom plate. (Not shown in Fig. 1.1 is the restraining force of magnitude F, to the left, which would be needed to keep the bottom plate stationary.)

A *stress* is a force per unit area. In principle, a stress may be computed at any point on a surface by finding the differential force ΔF acting on a differential area ΔA, and calculating the ratio $\Delta F/\Delta A$ in the limit $\Delta A \to 0$. Intensive variables such as this (i.e., ones independent of system size) are central to the analysis of fluid mechanics. *Shear* stresses act parallel to (tangent to) surfaces; *normal* stresses act perpendicular to surfaces. The shear stress that a solid exerts on a fluid, called the *wall shear stress*, is denoted as τ_w.[1] In the idealized experiment in Fig. 1.1, τ_w happens to be the same everywhere on the top plate. Accordingly, its value at any point equals the shear stress averaged over the entire surface. That is, $\tau_w = F/A$.

Experiments like that in Fig. 1.1 reveal that τ_w for any fluid depends on U/H, but not on U or H separately. The velocity-to-thickness ratio is the *shear rate*, $s = U/H$.[2] It is found that τ_w always increases with s. The shear stress and shear rate are related as

$$\tau_w = \frac{\mu U}{H} = \mu s \tag{1.2-1}$$

where μ is the *viscosity*. In a *Newtonian* fluid, μ is independent of s and the shear stress is exactly proportional to the shear rate. Numerous fluids of practical interest are Newtonian, including all gases, liquids with molecular weights (relative molecular masses) less than about 10^3 (such as water and common organic solvents), and suspensions of non-aggregating particles. For a given Newtonian fluid, μ depends mainly on temperature. In *non-Newtonian* fluids, the value of μ (i.e., the ratio τ_w/s) depends in some way on s.

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[1] This symbol is used in Part I of this book, but a more systematic way to identify stress components is needed later. In the notation introduced in Part II, τ_w at the top plate in Fig. 1.1 would be written as τ_{yx}. The first subscript indicates that the surface (the plane $y = H$) is perpendicular to the y axis, and the second refers to a resultant force in the x direction.

[2] A symbol for shear rate that appears very widely in the literature is $\dot{\gamma}$. Because of the potential for confusing that with surface tension (to be denoted as γ), s is used here instead.

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Non-Newtonian fluids also have other distinctive features, as discussed later in this section. Among such fluids are polymer melts, concentrated solutions of flexible polymers, and concentrated suspensions of particles that deform, orient, or aggregate.

A flow that closely approximates that in Fig. 1.1 is realized in a *Couette viscometer*, in which a liquid sample fills the annular space between long, concentric cylinders. If the gap between cylinders is small compared with their radii, the surfaces will appear to the liquid to be planar, just as the Earth seems flat from human eye level. One cylinder is rotated while the other is held in place. The linear velocity of the rotating cylinder is its radius times the angular velocity, and s is the linear velocity divided by the gap width. The torque measured on either cylinder is proportional to τ_w. This and other viscometers avoid practical problems that would occur with the system in Fig. 1.1, such as keeping the gap width H and fluid–solid contact area A constant.

Density and kinematic viscosity

The *density* ρ of a fluid is its mass per unit volume. In that density times velocity is momentum per unit volume, ρ is needed to describe inertia. Moreover, ρg is the gravitational force per unit volume, g being the gravitational acceleration. In principle, the density at any point in a fluid could be found by repeatedly determining the mass Δm in a volume ΔV, and finding the limit of $\Delta m/\Delta V$ as $\Delta V \to 0$. In liquids, where ρ is nearly independent of pressure (P), it is spatially uniform unless there are significant variations in temperature (T). In gases, ρ is related to P and T by an equation of state, and pressure gradients will cause it to vary with position, even if the gas is isothermal. The equation of state for an *ideal gas*, which is adequate at low to moderate pressures, is

$$\rho = \frac{MP}{RT} \tag{1.2-2}$$

where M is the average molecular weight, R is the universal gas constant, and T is the absolute temperature. Assuming ρ to be constant, even in gases, usually leads to negligible error. Exceptions include gases spanning large heights, flowing in very long pipes, or moving at velocities that approach the speed of sound. Those situations are unusual enough in chemical engineering that constancy of ρ will be our default assumption. The corresponding idealization is called *incompressible flow*. Although such flows are our main concern, an introduction to *compressible flow*, where variations in ρ must be taken into account, is provided in Chapter 12.

The ratio of viscosity to density occurs frequently enough to have its own name, the *kinematic viscosity*. It is denoted by the Greek "nu,"

$$\nu = \frac{\mu}{\rho}. \tag{1.2-3}$$

Units and values

The SI system, in which the unit of force is the newton ($\mathrm{N = kg\,m\,s^{-2}}$), is used in this book. The units of energy and power are the joule ($\mathrm{J = N\,m = kg\,m^2\,s^{-2}}$) and watt ($\mathrm{W = J\,s^{-1} = kg\,m^2\,s^{-3}}$), respectively. The unit of pressure (or stress) is the pascal ($\mathrm{Pa = N\,m^{-2} = kg\,m^{-1}\,s^{-2}}$). One atmosphere equals 1.013×10^5 Pa = 101.3 kPa. The standard gravitational acceleration is $g = 9.807\ \mathrm{m\ s^{-2}}$ and the gas constant is $R = 8314\ \mathrm{kg\,m^2\,s^{-2}\,kg\text{-}mol^{-1}\,K^{-1}}$.

When making engineering estimates, one should have in mind typical values for the various fluid properties. Table 1.1 lists properties of several gases at near-ambient

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Table 1.1 Viscosity, density, and kinematic viscosity of gases (at 27 °C and 100 kPa)[a]

Gas	M(daltons)	$\mu(10^{-5}\ \mathrm{Pa \cdot s})$	$\rho(\mathrm{kg\ m^{-3}})$	$\nu(10^{-5}\ \mathrm{m^2\ s^{-1}})$
H_2	2	0.90	0.0819	11.0
CH_4	16	1.12	0.652	1.72
N_2	28	1.79	1.14	1.57
Air	29	1.86	1.18	1.58
C_2H_6	30	0.95	1.22	0.78
O_2	32	2.08	1.30	1.60
CO_2	44	1.50	1.79	0.84

[a] The viscosities are from Lide (1990), the densities are from Eq. (1.2-2), and the kinematic viscosities were calculated from those viscosities and densities. “Air” is dry air.

Table 1.2 Viscosity, density, and kinematic viscosity of Newtonian liquids[a]

Liquid	T(°C)	$\mu(10^{-3}\ \mathrm{Pa \cdot s})$	$\rho(10^3\ \mathrm{kg\ m^{-3}})$	$\nu(10^{-6}\ \mathrm{m^2\ s^{-1}})$
Water	20	1.002	0.9982	1.004
Benzene	20	0.652	0.8765	0.744
Ethanol	20	1.200	0.7893	1.520
Mercury	20	1.554	13.55	0.115
Phenol	18	12.7	1.0576	12.0
Olive oil	20	84.0	0.918	91.5
Machine oils	16	114–661	0.87	130–760
Glycerol	20	1490	1.2613	1181
Honey	20	19,000	1.42	13,400

[a] All viscosities and densities are from Lide (1990), except as noted below, and ν was calculated from μ and ρ. The density for the “machine oils” is a value typical of motor oils. The properties of a representative honey are from www.airborne.co.nz.

conditions. The entries are arranged in order of increasing molecular weight. Despite 20-fold variations in M, the viscosities fall within a narrow range, from about 1×10^{-5} to 2×10^{-5} Pa · s. With little difference in μ and with ρ proportional to M, the variations in ν are due largely to the differences in M. Given that most gas densities are roughly $1\ \mathrm{kg\,m^{-3}}$, a typical kinematic viscosity at ambient temperature and pressure is about $1 \times 10^{-5}\ \mathrm{m^2\,s^{-1}}$.

The properties of a number of Newtonian liquids are shown in Table 1.2. The viscosity of water at room temperature is 1×10^{-3} Pa · s, its density is $1 \times 10^3\ \mathrm{kg\,m^{-3}}$, and its kinematic viscosity is therefore $1 \times 10^{-6}\ \mathrm{m^2\,s^{-1}}$. Although the viscosity of water is about 100 times that for gases, its kinematic viscosity is about 10 times smaller because its density is 1000 times larger. The viscosity, density, and kinematic viscosity of benzene and ethanol, which are representative of common organic solvents, are each the same order of magnitude as those for water. The variations in μ and ν among the other liquids are noteworthy; they range over factors of about 10^4 and 10^5, respectively. Differences in viscosity should not be confused with differences in density, as occurs sometimes in casual speech or writing. Despite its high density, mercury has an ordinary viscosity, whereas the liquids in the last five lines of Table 1.2 have increasingly large viscosities, but ordinary densities.

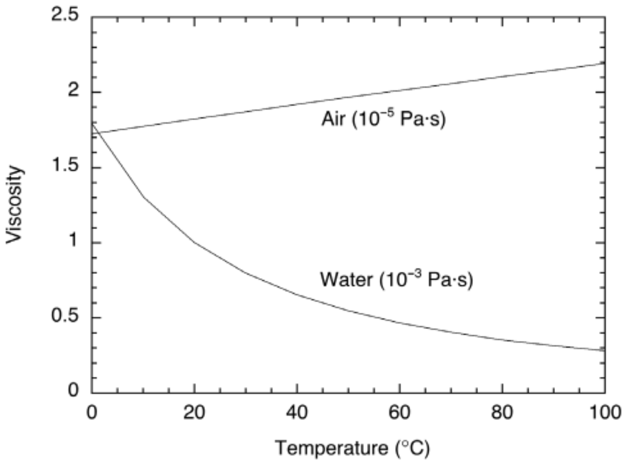

Figure 1.2 Viscosities of air and water as a function of temperature. The values for air are from a correlation in Kadoya *et al.* (1985) for dry air and those for liquid water are from Lide (1990).

Gas viscosities increase with temperature (approximately as $T^{1/2}$), whereas those for liquids decrease, as exemplified by the results for air and water in Fig. 1.2. Liquid viscosities are more temperature-sensitive. As T increases from 0 to 100 °C, μ for liquid water decreases by 84%, whereas μ for air increases by only 27%.

Both for gases and for liquids, the effects of pressure on viscosity are generally negligible (Blevins, 2003). That is true for gases if $P/P_c < 0.2$ and $T/T_c > 1$, where P_c and T_c are the critical temperature and pressure, respectively. For air, $P_c = 36.4$ atm and $T_c = 78.6$ K. For water at 50 atm, the kinematic viscosity is only about 1% less than that at 1 atm.

Estimation methods to use when specific data are unavailable may be found in Reid *et al.* (1987). Gas viscosities are explained well by kinetic theory (Hirschfelder *et al.*, 1954; Kennard, 1938). Although there are no comparably simple and predictive theories for liquids, molecular-dynamics simulations provide a way to relate liquid viscosities and other transport properties to intermolecular forces (Allen and Tildesley, 1987).

Non-Newtonian liquids

As already mentioned, a dependence of viscosity on shear rate is the hallmark of non-Newtonian liquids. The experimental and theoretical investigation of stresses in such fluids constitutes the field of *rheology*. Aside from the dependence of μ on s, the behavior of polymeric liquids, in particular, can differ strikingly from that of low-molecular-weight liquids. For example, when a Newtonian liquid is stirred, the air–liquid interface is depressed in the vicinity of the stir rod; with various polymer solutions the interface is elevated, and the solution may even climb the rod as stirring proceeds. Also, a steady stream of Newtonian liquid leaving a small tube reaches a diameter that does not differ greatly from that of the tube, whereas jets of polymeric liquids can swell to several times the tube diameter. Further, flexible polymer molecules resist elongation and, because of their size, can require several seconds to adjust shape in response to applied forces. This can give the fluid an elastic character and cause viscous stresses to depend not just on instantaneous shear rates, but also the recent history of the sample. Fluids in which μ is time-dependent are *viscoelastic*. These and other special characteristics of polymeric

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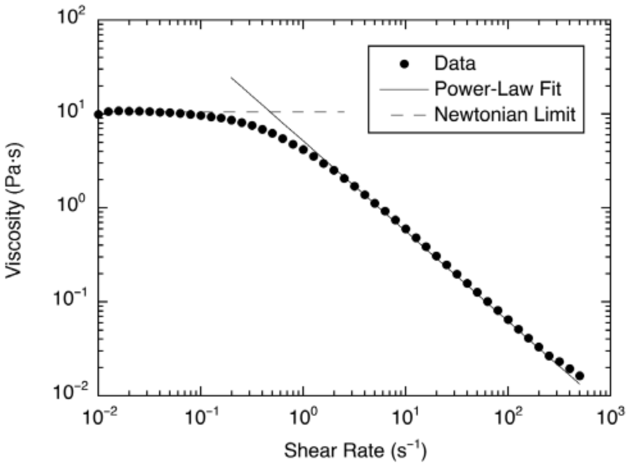

Figure 1.3 Viscosity as a function of shear rate for a solution containing 66 mM cetylpyridinium chloride and 40 mM sodium salicilate (Haward and McKinley, 2012).

liquids are surveyed in Chapter 2 of Bird *et al.* (1987). Colloidal dispersions and other particle suspensions in low-molecular-weight solvents can also be non-Newtonian, especially when very concentrated. For example, whole blood, which is almost half red cells by volume, has a viscosity that depends on shear rate when s is small.

Many features of polymeric liquids are beyond the scope of this book. The dependence of μ on s, and its consequences for predicting the flow rates of non-Newtonian liquids in pipes or other conduits, tends to be of greatest practical interest and will be our focus. A relationship that describes data for many liquids over practical ranges of s is

$$\mu = ms^{n-1} \tag{1.2-4}$$

where m and n are positive constants. The values of m and n are specific to a given material and temperature. In polymer solutions, m and n both depend on concentration. They are obtained by substituting Eq. (1.2-4) into Eq. (1.2-1) and fitting the resulting equation to data for τ_w as a function of s. A material that obeys Eq. (1.2-4) is called a *power-law fluid*, and such liquids can follow this relationship for values of s spanning several orders of magnitude. For polymer solutions, n often ranges from 0.2 to 0.8 (Bird *et al.*, 1987, pp. 172–175; Tanner, 2000, pp. 18–19). A fluid with $n < 1$, such that μ decreases with increasing s, is referred to as *shear-thinning*. In the rarer situations where $n > 1$, such that μ increases with increasing s, it is *shear-thickening*. For $n = 1$, the viscosity is independent of s and the Newtonian case is recovered, with $\mu = m$.

Equation (1.2-4) implies that $\mu \to \infty$ as $s \to 0$ for a shear-thinning fluid. In reality, μ tends to be constant at very low shear rates, as exemplified by the data in Fig. 1.3. What was studied in this case was a surfactant solution that forms highly elongated micelles. However, the overall trends are like those found for numerous polymer solutions and polymer melts. At low shear rates the solution was Newtonian with a viscosity of about $10\,\text{Pa}\cdot\text{s}$, as indicated by the dashed line. At higher shear rates there was a power-law region where

$$\mu = \mu_0 \left(\frac{s}{s_0}\right)^{n-1} \tag{1.2-5}$$

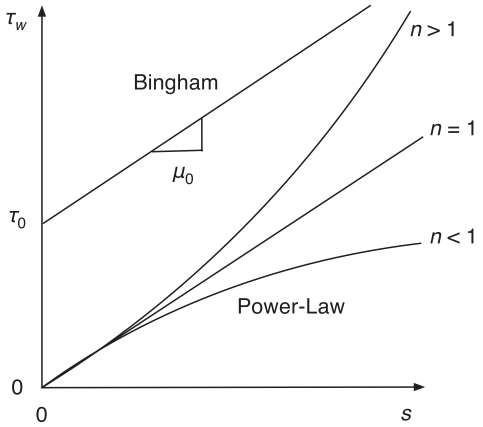

Figure 1.4 Shear stress as a function of shear rate for power-law and Bingham fluids.

a relationship that is obtained by setting $m = \mu_0 s_0^{1-n}$ in Eq. (1.2-4). The curve fit shown by the solid line corresponds to $\mu_0 = 10.5$ Pa · s, $n = 0.04$, and $s_0 = 0.48$ s^{-1}. Although beyond the range of these experiments, at extremely high shear rates there would be another Newtonian plateau at a lower viscosity. To describe the upper and lower Newtonian limits, the power-law region, and the transitions in a single expression, an equation with more degrees of freedom is needed. Such an expression is the *Carreau–Yasuda equation*, which involves five fitted constants (Bird *et al.*, 1987, p. 171). Their inaccuracy for extreme values of s notwithstanding, Eqs. (1.2-4) and (1.2-5) are adequate for most pipe-flow calculations.

Another class of materials flows only when the shear stress exceeds a certain threshold, called the *yield stress*. One way to describe their viscosity is

$$\mu = \begin{cases} \infty & \text{for } \tau_w < \tau_0 \\ \mu_0 + \dfrac{\tau_0}{s} & \text{for } \tau_w \geq \tau_0 \end{cases} \tag{1.2-6}$$

where τ_0 is the yield stress and μ_0 is a second material-specific constant [not to be confused with μ_0 in Eq. (1.2-5)]. A material that obeys Eq. (1.2-6) is called a *Bingham fluid* or *Bingham plastic*. The infinite viscosity for $\tau_w < \tau_0$ means simply that the material behaves as a rigid solid when subjected to insufficient shear. However, for $\tau_w \geq \tau_0$ the viscosity becomes finite and decreases with increasing s. If s greatly exceeds τ_0/μ_0, the fluid becomes Newtonian with $\mu = \mu_0$. House paint and various foods behave much like this.

The qualitative dependence of shear stress on shear rate for Newtonian, power-law, and Bingham fluids is shown in Fig. 1.4. For a Newtonian fluid ($n = 1$), the straight line through the origin indicates that μ (which in this case equals the slope) is constant and that there is no yield stress. For a power-law fluid with $n \neq 1$, the curvature of the function $\tau_w(s)$ signifies a variable μ, but the passing of the curve through the origin indicates the absence of a yield stress. For the Bingham model, the nonzero intercept is τ_0 and the constant slope is μ_0.

Equations (1.2-4)–(1.2-6) are examples of rheological *constitutive equations*, which relate viscous stresses to rates of shear or deformation. The constitutive equations for Newtonian, power-law, and Bingham fluids are presented in more general form in Section 6.5. Many other non-Newtonian constitutive equations have been proposed, some

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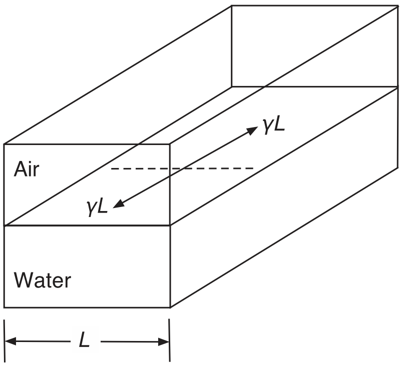

Figure 1.5 Surface tension at a flat air–water interface. A force of magnitude γL pulls away from each side of the imaginary cut shown by the dashed line.

purely empirical and others derived from molecular-level mechanics (Bird *et al.*, 1987; Tanner, 2000).

Surface tension

There is a surface tension (γ) associated with the interface between a gas and a liquid, or between immiscible liquids. Surface tension may be viewed either as an energy per unit area or as a force per unit length (J/m^2 or N/m in SI units). In thermodynamics, it is the energy required to increase the interfacial area; in mechanics, the usual perspective in this book, it is a force that acts on an imaginary line or contour within an interface. The direction of the force is tangent to the interface and away from the line or contour. The surface tension force at a planar interface is illustrated in Fig. 1.5, which shows a container of width L that is partly filled with water. At an imaginary cut through the air–water interface (dashed line), there are opposing forces of magnitude γL acting horizontally on each side of the line of length L. The forces are *tensile*, meaning that each pulls on the imaginary line.

If an interface is flat and γ is constant, the surface tension forces will balance and not be observable. However, surface tension can become very evident when an interface is nonplanar, when γ varies with position (due to gradients in temperature or surfactant concentration), or when an interface ends at a three-phase *contact line*. A contact line is created, for example, at the edge of a liquid drop resting on a solid surface, where solid, liquid, and gas all meet. The surroundings exert a force that pulls away from the contour that corresponds to the contact line. That force may cause the drop to either spread or contract, depending on the angle the gas–liquid interface makes with the surface, which is called the *contact angle*. Contact angles and other aspects of surface tension are discussed in much more detail in Chapter 4. Surface tension affects the dynamics of drops, bubbles, and foams, causes wetting of surfaces, and leads to fluid uptake by porous media. These effects are most prominent at small length scales. Many such phenomena are described in Adamson and Gast (1997) and De Gennes *et al.* (2003).

Table 1.3 shows values of γ for several liquids in contact with air. The surface tension for water (about 7×10^{-2} N/m or 70 mN/m) is about three times that for benzene, ethanol, or other common organic solvents (20–30 mN/m). Mercury is unusual in that γ is about 20 times that for the organics (almost 500 mN/m). Surface tensions decrease with increasing temperature, as exemplified by the entries for water and mercury.

The surface tension for water in contact with various immiscible liquids is less than that for water and air. For example, at room temperature $\gamma = 45.7$ mN/m for decane/water and 16.4 mN/m for olive oil/water (Than *et al.*, 1988).

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Table 1.3 Surface tension for liquid–air interfaces[a]

Liquid	T(K)	γ(mN/m)
Water	293	72.88
	303	71.40
	313	69.92
Benzene	293	28.88
Ethanol	293	22.39
Mercury	293	486.5
	303	484.5

[a] From Jasper (1972).

Continuum approximation

Little more is said in this book about molecules, because in continuum mechanics they are ignored. Indeed, continuum descriptions of fluid mechanics were already well advanced by the mid 1700s, long before the existence of molecules was firmly established. Continuum modeling is natural in that, in our everyday experiences with air and water, we are unaware of individual molecules. Of great importance, postulating that velocities, stresses, and material properties are smoothly varying functions of position allows us to use the tools of calculus. In general, this approach hinges on there being enough molecules present in a system to average their effects. That is easiest to visualize with density. When we speak of the local value of ρ as the ratio $\Delta m/\Delta V$ in the limit $\Delta V \to 0$, what is really meant is that there exists a ΔV that is small compared with the system volume, but still large enough to contain a predictable number of molecules. If ΔV were so small that it enclosed only a few molecules, ρ would fluctuate wildly, according to the thermal motion of molecules into or out of the imaginary box. Thus, ρ becomes unpredictable if the system is too small. The same is true for the other properties we have discussed. If the number of molecules present is inadequate, flow models must be stochastic (based on probabilities) rather than deterministic.

To have continuum behavior with property values that are independent of system size, the smallest linear dimension of the system (ℓ) must greatly exceed whichever is the larger of two molecular length scales. One is the intermolecular spacing, which may be expressed as an average center-to-center distance (r). The other is the mean distance traveled in a random "jump." Single molecular displacements in liquids are typically a small fraction of r, whereas the average distance between collisions in a gas (λ, the mean free path) greatly exceeds r. This makes r limiting in liquids and λ limiting in gases. The need to have $\ell \gg r$ is evident already from the discussion of density, and $\ell \gg \lambda$ is required to make collisions among the gas molecules much more frequent than their collisions with boundaries. Collisions among the gas molecules are what transmit momentum in a gas, and their frequency determines μ. If a container is so small that collisions occur mainly with the walls, the usual values of μ will be inapplicable.

A typical value of r for water or organic solvents is 0.4 nm. Allowing for a factor of 10, this suggests $\ell > 4$ nm as a criterion for the continuum modeling of such liquids. From kinetic theory (Kennard, 1938), the mean free path in a gas is

$$\lambda = \frac{k_B T}{\sqrt{2}\,\pi d^2 P} \tag{1.2-7}$$