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Second Edition

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To my family

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Preface to the Second Edition

In the twenty plus years since I wrote the first edition of this book there have been many important developments in harmonic and microlocal analysis. For instance, very recently there has been seminal work on multilinear methods through the breakthrough of Bennett, Carbery and Tao [1], the related oscillatory integral estimates of Bourgain and Guth [1] and the decoupling estimates of Bourgain and Demeter [1]. These works and others have led to many applications, including the proof of sharp local smoothing estimates. I have chosen not to include these, partly because they represent a fast moving target, but moreover because they would have added considerably to the book's length. My goal for the first as well as the current edition was to provide a self-contained but by no means exhaustive introduction to harmonic and microlocal analysis.

To this end, I added two new chapters. One, which ties in with the earlier material on Fourier integral operators, presents Hörmander's theory of propagation of singularities and uses this to prove the Duistermaat–Guillemin theorem, which is a triumph of microlocal methods, saying that for generic manifolds one can obtain an improvement in the error term for the Weyl law. This chapter includes material that perhaps should have been included in the first edition and augments the microlocal analysis portion of the book. I also added a new chapter on what could now be considered classical harmonic analysis. It concerns results related to the Kakeya conjecture, including the "bush" method of Bourgain and the "hairbrush" method of Wolff for obtaining Kakeya and Nikodym maximal estimates, as well as the relationship between the Fourier restriction problem and the Kakeya conjecture. Most of the results presented in this chapter were being developed as the first edition of my book was being completed.

I also added a bit of new material to the chapters in the first edition, including Bourgain's counterexample to certain oscillatory integral estimates, which showed that Stein's oscillatory integral theorem was sharp, and I also made a few corrections to the earlier text. This turned out to be a more difficult undertaking than I anticipated due to the fact that the first edition of the book was written in the age of floppy disks and although I had saved the output files on several of them, I did not do the same for the source files for the first edition.

To my delight, Cambridge University Press came to the rescue by re-typesetting all of the old material. They did a wonderful job. I am very grateful to Sam Harrison and Clare Dennison and the typesetters at Cambridge University Press for this. I am also very grateful to Sam Harrison for suggesting that a new edition of my book be written and for his kind encouragement.

As was the case for the first edition, I am also very grateful for the assistance I have received from many of my colleagues. In particular, I would like to thank Changxing Miao and his group for going through the new material and for their suggestions, and I am especially grateful to Yakun Xi for his thorough proofreading of the material prepared by Cambridge University Press as well as the two new chapters. The role that Sam Harrison and Yakun Xi played in this project has been invaluable to me. Finally, I am also grateful to the students at Johns Hopkins University for their feedback and suggestions when the new material was presented in courses over the last couple of years.

Lutherville *C. D. Sogge*

Preface to the First Edition

Except for minor modifications, this monograph represents the lecture notes of a course I gave at UCLA during the winter and spring quarters of 1991. My purpose in the course was to present the necessary background material and to show how ideas from the theory of Fourier integral operators can be useful for studying basic topics in classical analysis, such as oscillatory integrals and maximal functions. The link between the theory of Fourier integral operators and classical analysis is of course not new, since one of the early goals of microlocal analysis was to provide variable coefficient versions of the Fourier transform. However, the primary goal of this subject was to develop tools for the study of partial differential equations and, to some extent, only recently have many classical analysts realized its utility in their subject. In these notes I attempted to stress the unity between these two subjects and only presented the material from microlocal analysis that would be needed for the later applications in Fourier analysis. I did not intend for this course to serve as an introduction to microlocal analysis. For this the reader should be referred to the excellent treatises of Hörmander [5], [7] and Treves [1].

In addition to these sources, I also borrowed heavily from Stein [4]. His work represents lecture notes based on a course that he gave at Princeton while I was his graduate student. As the reader can certainly tell, this course influenced me quite a bit and I am happy to acknowledge my indebtedness. My presentation of the overlapping material is very similar to his, except that I chose to present the material in the chapter on oscillatory integrals more geometrically, using the cotangent bundle. This turns out to be useful in dealing with Fourier analysis on manifolds and it also helps to motivate some results concerning Fourier integral operators, in particular the local smoothing estimates at the end of the monograph.

Roughly speaking, the material is organized as follows. The first two chapters present background material on Fourier analysis and stationary phase that will be used throughout. The next chapter deals with non-homogeneous oscillatory integrals. It contains the L^2 restriction theorem for the Fourier transform, estimates for Riesz means in $\mathbb{R}^n$, and Bourgain's circular maximal theorem. The goal of the rest of the monograph is mainly to develop generalizations of these results. The first step in this direction is to present some basic background material from the theory of pseudo-differential operators, emphasizing the role of stationary phase. After the chapter on pseudo-differential operators comes one dealing with the sharp Weyl formula of Hörmander [4], Avakumovič [1], and Levitan [1]. I followed the exposition in Hörmander's paper, except that the Tauberian condition in the proof of the Weyl formula is stated in terms of L^∞ estimates for eigenfunctions. In the next chapter, this slightly different point of view is used in generalizing some of the earlier results from Fourier analysis in $\mathbb{R}^n$ to the setting of compact manifolds. Finally, the last two chapters are concerned with Fourier integral operators. First, some background material is presented and then the mapping properties of Fourier integral operators are investigated. This is all used to prove some recent local smoothing estimates for Fourier integral operators, which in turn imply variable coefficient versions of Stein's spherical maximal theorem and Bourgain's circular maximal theorem.

It is a pleasure to express my gratitude to the many people who helped me in preparing this monograph. First, I would like to thank everyone who attended the course for their helpful comments and suggestions. I am especially indebted to D. Grieser, A. Iosevich, J. Johnsen, and H. Smith who helped me both mathematically and in proofreading. I am also grateful to M. Cassorla and R. Strichartz for their thorough critical reading of earlier versions of the manuscript. Lastly, I would like to thank all of my collaborators for the important role they have played in the development of many of the central ideas in this course. In this regard, I am particularly indebted to A. Seeger and E. M. Stein.

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Sherman Oaks *C. D. Sogge*