Mathematical Methods in the Earth and Environmental Sciences

The Earth and environmental sciences are becoming progressively more quantitative due to the increased use of mathematical models and new data analysis techniques. This accessible introduction presents an overview of the mathematical methods essential for understanding the Earth's processes, providing an invaluable resource for students and early career researchers who may have missed (or forgotten) the mathematics they need to succeed as scientists. Topics build gently from basic methods such as calculus to more advanced techniques including linear algebra and differential equations. The practical applications of the mathematical methods to a variety of topics are discussed, ranging from atmospheric science and oceanography to biogeochemistry and geophysics. Including over 530 exercises and end-of-chapter problems, as well as additional computer codes in Python and MATLAB, this book supports readers in applying appropriate analytical or computational methods to solving real research questions.

Adrian Burd is an associate professor at the Department of Marine Sciences at the University of Georgia. As a marine scientist, he applies mathematical tools to understand marine systems, including the carbon cycle in the oceans, the health of seagrass and salt marshes, and the fate of oil spills. His work has taken him around the globe, from the heat of Laguna Madre and Florida Bay to the cold climes of Antarctica.

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Preface

The Earth and environmental sciences, like all scientific disciplines, are rooted in observational studies. However, recent years have seen an increasing demand for researchers who are also comfortable with the language of mathematics. Mathematical and computer models are now commonplace research tools as the power of desktop computers has increased and the public release of computer codes has made modeling more accessible to those who previously may have hesitated at using a computer model in their research. In addition, the increasing availability of large regional and global multivariate data sets has stimulated the use of new data analysis techniques requiring an understanding of the mathematics that underlies them if they are not to be used as black boxes. Consequently, there is an increasing need for students and researchers who are comfortable manipulating mathematical expressions and who can understand the mathematics underlying the analysis methods that they use.

Many students enter the Earth and environmental sciences with diverse academic backgrounds. As a result, they often find themselves unprepared for the level of mathematics that they need for their coursework or research. This is often because there can be gaps of several years between when students first encounter mathematical techniques such as calculus and when they finally end up using those techniques on a regular basis. Others may never have been exposed to the mathematical tools in the first place. This book is aimed primarily at those students and researchers who find themselves in such situations and need a gentle reminder or introduction to the mathematical methods they need.

Many students, and dare I say even some experienced researchers, are either intimidated by mathematics or find it of little value. I have found an informal approach, providing context for the uses of mathematics, makes things less intimidating. Some may disagree, and I too find myself tempted to veer into the beauty and technical aspects of mathematics from time to time as my background in theoretical physics and astronomy come to the fore. However, in this book I have resisted this and tried to stick to a practical and informal style at the expense of cutting a few corners from time to time. It is my hope that teachers who see the need for more rigor can use this book as a framework to introduce students to the concepts and techniques they need, and then backfill the rigor within the classroom.

To the student who is using this book for self-study, you should work through all the derivations and equations in the text. At the beginning of the book, you will find that most of the steps in derivations are presented. However, as your skills and understanding develop as you work through the text, it is assumed that you will be able to fill in missing steps. The exercises and problems are there to help you refine and develop your understanding and intuition, so you should attempt as many as you can. It is also a good idea to have access to another text, because developing an understanding for a new topic is often a function of point of view. I have provided suggestions for further reading at the end of each chapter, and I hope that this will give ideas.

This book is not meant to be an exhaustive exploration, but rather an introduction to give the reader the tools and techniques they need to successfully do their science. Indeed, large books have been written on the subjects of individual chapters in this book. Some students in certain disciplines (such as geophysics) will require more rigorous detail and coverage of topics that are not included in the book. Some teachers may find their favorite topics missing, and I hope they will understand that given the constraints on the size of the text not everything could be covered. The lists of further reading at the end of each chapter give some advice for the reader on where they can find more information if needed. However, I hope that this book will provide a practical and accessible foundation to the tools most will need. My aim is to get students thinking mathematically.

The material for this book has come primarily from three courses that I teach or co-teach at the University of Georgia. Quantitative Methods in Marine Science is a graduate-level course designed to do precisely what this book is aimed to do, introduce new graduate students to the mathematical techniques they will need for their research and other courses. This is an intense, one-semester course and covers the material in Chapters 1 through 5 and the early parts of Chapter 6. I teach a more advanced graduate-level course, Modeling Marine Systems, that covers material from Chapters 6 to 11, but mostly concentrates on material for solving ordinary and partial differential equations. This course has been taken by students from a wide range of disciplines including marine science, geology, agriculture, and other environmental sciences, and it has been these students who have prompted me to explore the mathematical needs of students in related disciplines. Finally, Mathematics and Climate is a course for both undergraduate and graduate students that I co-teach with Professor Malcolm Adams from the University of Georgia Mathematics Department. It involves the application of dynamical systems to understanding climate. This course is also taken by students from many disciplines including mathematics, geology, geography, and economics.

The exercises and problems are an essential component of the book, especially if you are using it for self-study. As others have said before, mathematics is not a spectator sport, and one needs to practice solving problems using the techniques one learns. Most of the exercises embedded within each chapter are short and designed to practice a technique or to develop understanding through solving a small problem. The problem sets at the end of each chapter are generally more involved, some involve practicing techniques, others use those techniques to solve problems relevant to the Earth and environmental sciences. Supplemental problems are also available.

Computers play an integral part in many of the mathematical techniques introduced here, and throughout the book you will come across the symbol 🖥 in the margin. This indicates that there are supplemental computer codes available that are relevant to the material being discussed. These are not aimed to teach programming, but rather the application of techniques and are available from the Cambridge University Press website (www.cambridge.org/burd) for this book as well as the author's Github site (`https://github.com/BurdLab`).

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