

## Physics of Partially Ionized Plasmas

Plasma is considered to be one of the four states of matter. The other three being solid, liquid and gas. It is formed by heating a gas, which ionizes its molecules or atoms. It consists of charged particles, namely positive ions and negative electrons. A partially ionized plasma has a large number of neutral particles too. Molecular clouds, diffuse interstellar gas, solar atmosphere, the earth's ionosphere and laboratory plasmas including fusion plasmas constitute partially ionized plasmas. The topic is an active area of study in plasma physics.

This book provides a comprehensive account of the various aspects of partially ionized plasmas including multifluid description, equilibrium, and waves. It includes a detailed discussion on the reionization phase of the universe along with a brief description of high discharge plasmas, tokamak plasmas and laser plasmas. It also discusses various elastic and inelastic collisions amongst the three particle species.

Using many examples, it demonstrates the novelties of partially ionized plasmas: for instance, in a partially ionized plasma, the magnetic induction is subjected to the ambipolar diffusion and the Hall effect in addition to the usual resistive dissipation. A discussion of kinematic dynamo in partially ionized plasma is also given in the book.

**Vinod Krishan** was Senior Professor at the Indian Institute of Astrophysics, Bangalore. She held visiting positions, at the Indian Institute of Science, the Lawrence Berkeley Laboratory, the National Institute for Space Research (Brazil), the University of Tokyo and the Raman Research Institute. During her long teaching and research career, she taught courses on plasma astrophysics, quantum mechanics and statistical physics at undergraduate and postgraduate levels. She has published more than a hundred research papers. Her major research interests are in the areas of space and astrophysical plasmas.

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Vinod Krishan



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*In loving memory of my father  
Shri Om Prakash Pabbi  
who taught me how to think*

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# Preface

Retirement is a time to indulge oneself, determine your own deadlines and to meet them at your own pace. The cooking breaks are no brakes! This is how I came to write this book, my second, after my superannuation; the first one is called *Plasmas; The First State of Matter*.

I was introduced to the topic of partially ionized plasmas by Professor Kumar Chitre who handed me the A. Brandenburg and E. G. Zweibel paper (1995, Ap. J., 448, 734) during my visit to the Tata Institute of Fundamental Research sometime in the year 2000. This resulted in our paper “Ambipolar diffusion in the solar atmosphere”<sup>1</sup>.

Partially ionized plasmas again came into my line of sight in the year 2005 when I visited the University of Tokyo campus near Edogawadai to work with Professor Yoshida-sansei. Since then, I have been studying plasma-typical problems in partially ionized plasmas. Around the same time I was also collaborating with Professor Swadesh Mahajan on the role of the Hall effect in diverse circumstances. It turned out that in the weakly ionized plasma model of a partially ionized plasma, the ion inertial scale, a hallmark of the Hall effect, gets multiplied by the inverse of the ionization fraction. As a result, the effective ion inertial scale acquires a much larger value than its counterpart in a fully ionized plasma. This was reported in “Equilibrium structures in partially ionized rotating plasmas within Hall magnetohydrodynamics”<sup>2</sup>. During my visit to Professor S. Masuda’s group in Nagoya University, Professor K. Shibata and I discussed the possibility of observing this new inertial scale phenomena on the solar atmosphere with a future solar mission. Additionally, during my visits to the Kyoto University and the National Astronomical Observatory of Japan, Mitaka, Tokyo, I had discussions on the mean field dynamo in partially ionized plasmas with Professor S. Tsuneta’s group. The highly positive response from my peers galvanized me into pursuing the area further, and my own desire to present partially ionized plasmas as a subject in its own right resulted in this book.

The decade and a half’s work could not have been done without the support of colleagues and collaborators, some of whom I have mentioned above. I wish to place on record my appreciation for Professor R. T. Gangadhara (my first student) for his continued collaboration on problems of our common interest. I have also enjoyed working with Professor B. P. Pandey, Dr Baba Varghese, Professor Nikhil Chakraborty, Professor H. Isobe, Professor Y. Hiraki, Dr K. A. P. Singh, and Dr A. Bhowmik.

<sup>1</sup>Chitre, S. M., and V. Krishan. 2001. *Mon. Not. R. Astron.* 323: L23-L25

<sup>2</sup>Krishan, V., and Z. Yoshida. 2006. *Physics of Plasmas*. 13: 092303

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I thank Dr Ebenezer for helping me with the figures.

I am one amongst the many who learnt the hard lesson from B. K. Shivani that one is responsible for all the good and the not-so-good that happens in one's life; a lesson in ultimate freedom.

I thank my housekeeper, Nagaratna for relieving me from some of the daily household chores.

I am blessed with a very supportive extended family.

My husband, Professor Som Krishan, suffice it to say, has tolerated me for forty-seven years. The remaining, I hope, are easy, for him. My daughter, Dr Monika Krishan, a cognitive scientist and a spiritual healer maintains the flow of positive energy in our lives.

Most of all, I am indebted to Professor P. Sreekumar, Director, the Indian Institute of Astrophysics, for generously extending me institute facilities which enabled me to continue my work in a smooth manner.

I hope this book will serve the young and the not so young plasma researchers.