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Partially Ionized Plasmas Here and Everywhere

1.1 Plasma, a Matter of State

A plasma is a large collection of electrically charged particles along with some neutral atoms and molecules. It is generated in a laboratory by heating a solid which sequentially transforms into a liquid and a gaseous state before becoming partially or completely ionized. This is the reason for christening a plasma the fourth state of matter.

About fifteen and odd billion years ago, the embryonic fluid consisting of radiation, electrons, positrons and other exotic particles is believed to have formed in the aftermath of the big bang explosion. It is from this cosmic soup that the three states of matter viz gas, liquid and solid, in that order, came into being as the universe expanded and cooled to its present state, which is embedded in a relic radiation, the cosmic microwave background radiation. This radiation has a black body spectrum corresponding to a temperature of 2.73 degrees Kelvin. So, should not a plasma be known as the first state of matter? Yes, it should be!

1.2 Partial Ionization in the Early Universe

The thermal history of the expanding universe from the big bang onwards is marked by two major epochs. As the temperature T of the universe approaches the rest mass energy of an electron i.e.,

$$K_B T \approx m_e c^2 = 500\ \mathrm{KeV} \tag{1.1}$$

where $K_B = 1.38 \times 10^{-23}$ JK^{-1} is the Boltzmann constant, the electron mass $m_e = 9.1 \times 10^{-31}$ kg and the speed of light $c = 3 \times 10^8$ m/s. At this temperature, $T \approx 5 \times 10^9$ K, the electrons and positrons begin to combine and contribute to the energy density of the photons. The compton scattering of the photons and the electrons (and other charged species) keeps the matter coupled to the radiation so that it behaves as a single-fluid with a common temperature.

The cooling of the universe continues due to its expansion. When the temperature falls to a value near $\approx$ 4000 K[1], the photons cannot maintain the ionization level and the process of neutralization sets in. The electrons and the protons combine to form neutral hydrogen atoms. This happens when the age of the universe is nearly a hundred thousand years[2] Fig. 1.1.

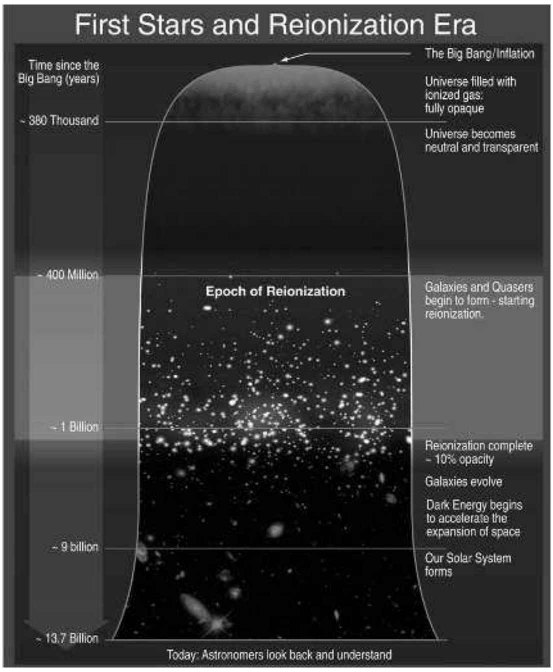

Fig. 1.1 Time line of the universe. Credit: Wikipedia

The equilibrium between photoionization wherein an hydrogen atom absorbs a photon and ionizes to a pair of proton and electron described as

$$\mathrm{H} + \mathrm{h}\nu = p + e \tag{1.2}$$

and neutralization (recombination) wherein the reverse process

$$p + e = \mathrm{H} + \mathrm{h}\nu \tag{1.3}$$

[1]Note that this temperature is much lower than the temperature derived from the ionization potential 13.6 eV of an hydrogen atom. This becomes possible as there are enough photons in the Planck distribution to keep the hydrogen atoms ionized. Besides the number density of photons is much larger than that of matter; this is the radiation dominated era.

[2]This epoch is also described in terms of the red shift z. The frequency of a photon decreases as it travels through an expanding medium. It is said to be red shifted. The ratio, (frequency of the emitted photon/frequency of the observed photon) is equal to the value of the red shift z. The neutralization process happens at $z \approx 1500$. The primordial helium nuclei also undergo neutralization with the expansion of the universe. However, since the ionization potential of helium is four times that of the hydrogen, helium becomes neutral at a red shift $z \approx 6000$.

of an electron and a proton combining to form an hydrogen atom fixes the net ionization fraction.

The models of this phase of the universe conclude that the neutralization of the nuclei is not complete. There still remain some electrons, protons and helium nuclei. The rate of recombination which depends on the product of the electron and the ion densities becomes extremely slow as the particle densities decrease. The limiting value of the ionization fraction is achieved when the recombination time and the age of the universe become comparable. The assumption that a plasma is in near thermal equilibrium permits us to use the Saha ionization formula to determine the ionization fraction. The Saha ionization formula gives the number density n_i of the ions at a temperature T to be[3]

$$n_i^2 \approx 2.4 \times 10^{21} n_n T^{3/2} \exp\left(-\frac{I}{K_B T}\right) \text{ m}^{-3} \tag{1.4}$$

where n_n is the number density of the neutral atoms and I is the ionization energy. It is customary to define the ionization fraction X as

$$X = \frac{n_i}{n_B} \tag{1.5}$$

where

$$n_B = n_i + n_n \tag{1.6}$$

is known as the total baryon density. The Saha Eq. (1.4) can be rewritten in terms of X as

$$\frac{1-X}{X^2} \approx 0.4 \times 10^{-21} n_B T^{-3/2} \exp\left(\frac{I}{K_B T}\right) \tag{1.7}$$

It is seen that X can become nearly one if the right hand side becomes nearly zero. This happens even when the temperature is much below the ionization temperature provided n_B is sufficiently small. As an example, let us consider the ionization of hydrogen gas for which $I = 13.6$ eV. At $(I/K_B T) = 3$, the temperature turns out to be $\approx 5 \times 10^4$ K and

$$\frac{1-X}{X^2} \approx 10^{-27} n_B \tag{1.8}$$

Thus $X \approx 1$ for $n_B \ll 10^{27}$ m^{-3} and $X \ll 1$ for $n_B \gg 10^{27}$ m^{-3}. The variation of the baryon density with the red shift is modelled as

$$n_B \approx 1.6(1+z)^3 \text{ m}^{-3} \tag{1.9}$$

[3] For more details, see Plasmas, The First State Of Matter, Vinod Krishan, Cambridge University Press, 2014, page 43.

and the temperature varies as

$$T \approx 2.7(1+z)\mathrm{K} \tag{1.10}$$

Plugging in for n_B and T in Eq. (1.7), the variation of the ionization fraction with the red shift, $X(z)$, can be determined from

$$\frac{1-X}{X^2} \approx 10^{-22}(1+z)^{3/2}\exp\left(\frac{6\times 10^4}{1+z}\right) \tag{1.11}$$

We find, for example, that $T = 4052$ K, $X \approx 0.61$ at $z = 1500$ and $T = 2700$ K, $X \approx 6.6 \times 10^{-5}$ at $z = 1000$. However, as mentioned earlier, the lower bound on the ionization fraction is fixed by the extremely large neutralization time.

The electrons and the ions can combine to form neutral atoms in the presence of a third body in which case it is called the three-body recombination. If the recombination takes place with the emission of photons, the process is called radiative recombination. The rate of change of the electron density n_e, due to recombination, is described as

$$\frac{\partial n_e}{\partial t} = -\alpha_R n_e n_i \tag{1.12}$$

where α_R is called the recombination coefficient. The characteristic recombination time t_R can be found to be

$$t_R \approx \frac{1}{\alpha_R n_i} \tag{1.13}$$

For hydrogen plasma, in the early universe, the radiative recombination is the relevant mechanism. The recombination coefficient[4] is given by

$$\alpha_R \approx < \Sigma_c v_e > \tag{1.17}$$

and has a value $\approx 4 \times 10^{-19}\mathrm{m}^3\mathrm{s}^{-1}$ at a temperature 10^4 K. Here, Σ_c is the electron capture cross-section and v_e is the electron thermal velocity. The recombination time can be expressed as

[4]The process of neutralization reduces the electron (n_e) and the ion number densities $(n_i = n_e = n)$ and the rate of loss is proportional to $n_e n_i = n^2$. Thus, the mass conservation law or the continuity equation becomes

$$\frac{\partial n}{\partial t} = -\alpha_R n^2 \tag{1.14}$$

which has a solution

$$\frac{1}{n(\vec{r},t)} = \frac{1}{n_0(\vec{r})} + \alpha_R t \tag{1.15}$$

where n_0 is the initial density. The time when $n \ll n_0$ is found to be

$$t = \frac{1}{n\alpha_R} \tag{1.16}$$

The time t is typically the electron-ion collision time and is given by $t = 1/(n < \Sigma_c v_e >)$.

$$t_R \approx \frac{1}{\alpha_R n_B X} \tag{1.18}$$

Thus, as both n_B and X decrease with the red shift z, the recombination time increases enormously and a limiting value of X around $10^{-4} - 10^{-5}$ obtains. With the decrease in the number density of the electrons, the photons undergo hardly any Thomson scattering[5]. As a result there is a decoupling of the matter and radiation. The cosmic background radiation travels almost unhindered until the birth of the luminous ultraviolet sources which can reionize the matter. However, with the residual ionization, the cosmic background radiation can get absorbed due to electron-neutral collisions.

In Chapter 5, we have derived a criterion under which a partially ionized plasma can be modelled as a weakly ionized plasma described by the set of Eqs (2.220)–(2.226). The criterion, Eq. (5.2)

$$\frac{n_i}{n_n} \ll 6 \times 10^{-10}\, T^2 \tag{1.19}$$

becomes, in terms of the ionization fraction X, for hydrogen plasma with $n_p = n_i$, at a red shift z,

$$\frac{n_i}{n_n} = \frac{X}{1-X} \ll 4.4 \times 10^{-9}(1+z)^2 \tag{1.20}$$

We can check that the criterion for a weakly ionized model is easily satisfied for $X = 6.6 \times 10^{-5}$ at $z = 1000$.

The cosmic microwave background (CMB) radiation suffered its last scattering with the electrons, the Thomson scattering, at the time of the neutralization of the plasma. The optical depth[6] for the scattering of the CMB photons decreased sharply so that the CMB radiation retained its warts and all since then. There have been several missions to detect the CMB anisotropies and their power spectrum. It becomes therefore necessary to study the neutralization mechanisms in its entirety in order to account for the observed CMB characteristics. The determination of the ionization fraction as a function of the redshift is an industrious process as it necessitates the inclusion of detailed multilevel hydrogen recombination[7]. A schematic variation of the ionization fraction of hydrogen as a function of the redshift is shown in Fig. 1.2. With the emergence of the first luminous sources which is understood to happen at a red shift of $z \approx 10$, the reionization of the universe begins.

The period between the neutralization and the reionization is known as the dark ages as no luminous sources existed then. In addition to the cosmic

[5]The Thomson scattering is the scattering of photons by free electrons; see "Plasmas, The First State Of Matter, Vinod Krishan, Cambridge University Press, 2014, page 213."

[6]Optical depth is the product of the scattering or absorption rate per unit length multiplied by the size of the scattering or absorbing region.

[7]Sara Seager and Dimitar D. Sasselov. The Astrophysical Journal Supplement Series, volume 128, page 407, 2000.

microwave background radiation the only other signal that can be received from this era is the highly red shifted 21 cm. Radiation produced by the spin-flip transition in the neutral hydrogen atoms. This spectral line can be excited by the microwave background radiation as well as by the electron collisions with the neutral hydrogen atoms in the extremely weakly ionized plasma. A lot of effort is being put in to build low frequency radio telescopes to detect the highly red shifted and rather weak 21 cm. radiation to fathom the evolution of the universe.

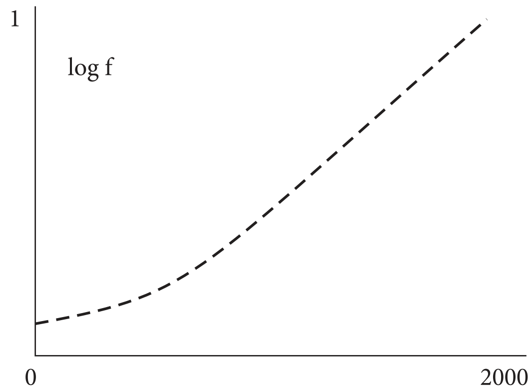

Fig. 1.2 Schematic variation of the ionization fraction, $f = n_e/n_H$, of hydrogen with redshift.

1.3 Reionization Phase of the Universe

It is inferred from the observations that the reionization of the universe happened, with the appearance of the first luminous sources, in a highly inhomogeneous manner. The luminous sources in a cluster produced extended ionized regions which tapered on to a partially ionized state of matter before merging with the background neutral matter. The hot and the cold coexist not often in a state of pressure equilibrium. These clouds and filaments cause anisotropies in the cosmic microwave background (CMB) radiation on the spatial scales of the structures. The large scale anisotropies arise from the variation of the average degree of ionization with the redshift. The free electrons scatter electromagnetic radiation by the Thomson scattering mechanism. The 21 cm radiation of the neutral hydrogen atom is a measure of the presence of the neutral gas. Thus, the 21 cm emission must be anticorrelated with the existence of the Thomson scattered CMB signal. Between the Thomson scattering sources and the 21 cm emission regions lies the partially ionized plasmas.

1.4 Partially Ionized Plasma on Cosmic Scales

The universe harbours an hierarchy of structures from huge superclusters of galaxies to the tiny comets and meteorites. The matter in these structures exhibits a bewildering range of temperatures. This translates into an equally bewildering range of the ionization status of the matter. The hot fully ionized regions of various shapes and sizes are often modelled to be embedded in the cold partially ionized or neutral background matter. Whereas the intracluster medium of a cluster of galaxies, Fig. 1.3, is at a sizzling hundred million degrees Kelvin as concluded from their X-ray emission, the cold regions emitting the 21 cm radiation from the neutral hydrogen atoms abound aplenty around the individual galaxies. The boundary between the fully ionized and the weakly ionized plasma could be pretty sharp to warrant shock-like structures.

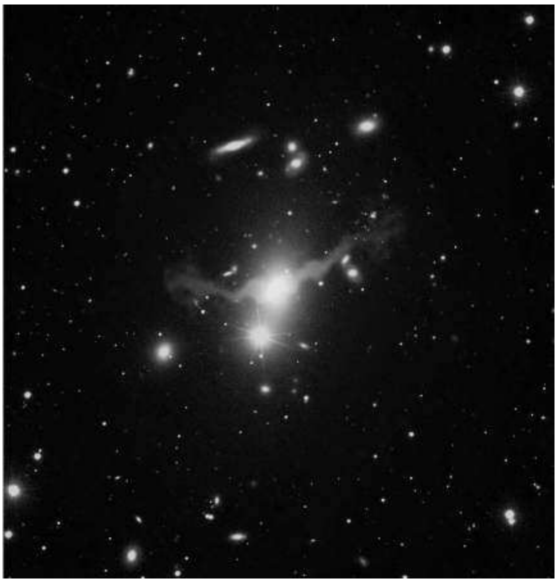

Fig. 1.3 Multi-wavelength image of a cluster of galaxies containing a massive bright elliptical galaxy (large glow) that is ejecting jets of material into the cluster. New results from Chandra find that the ejected material significantly enriches the cluster with iron and other elements. (Credit: NASA, Chandra, SDSS, and GMRT)

1.5 Molecular Clouds

Molecular clouds are the birth places of stars Fig. 1.4. The neutral hydrogen density can be as high as 10^{10} m^{-3}. They have a low degree of ionization produced mostly due to the colliding cosmic rays. The ion to neutral density ratio is of the order of $10^{-7} - 10^{-8}$. Magnetic fields of the order of tens of microGauss have been inferred from the radio observations of the clouds.

Fig. 1.4 Image of M16 molecular cloud. Credit: NASA, ESA, STScI, J. Hester and P. Scowen (Arizona State University)

1.6 Accretion Disks

The compact objects such as stars, pulsars and black holes accrete matter from their neighbourhood due to their strong gravitational pull. The accreted matter carrying angular momentum settles around the compact object in the form of a disk, called accretion disk. With a typical radius of 100 AU[8] and a thickness of a few AU, these accretion disks have masses of the order of one tenth of the mass of the sun[9]. The equilibrium and the stability of the accreted matter defines the plasma parameters of a disk such as temperature, density, flow speed and magnetic field. The circumstellar and the protoplanetary disks contain cold partially ionized plasma with ionization fraction anywhere in the range $\approx 10^{-4} - 10^{-11}$ and magnetic field $\approx$ of a few Gauss to milliGauss. The weakly ionized plasma model is quite suitable to study these disks.

The dwarf novae disks Fig. 1.5 form in binary stellar systems where a white dwarf[10] star accretes matter from a main sequence star.[11] The disks often show outbursts with sharp increase in their luminosity. The models show that the disks remain stable predominantly in two phases: (1) of large accretion rate and high ionization and (2) of small accretion rate and low ionization. Typical value of electron density is $\approx 10^{17}$ m^{-3} and of neutral hydrogen density is $\approx 10^{23}$ m^{-3} in the low ionization state.

[8]One AU, the astronomical unit is the distance between the sun and the earth $\approx 200 R_O$ where $R_O \approx 7 \times 10^8$ m is the radius of the sun.
[9]Mass of the sun $\approx 2 \times 10^{30}$ kg.
[10]A white dwarf star has burnt all its fuel, the nuclear reactions have stopped, the star is white hot and just cools its heels!
[11]The main sequence stars are young and burning. Our sun is a middle aged main sequence star.

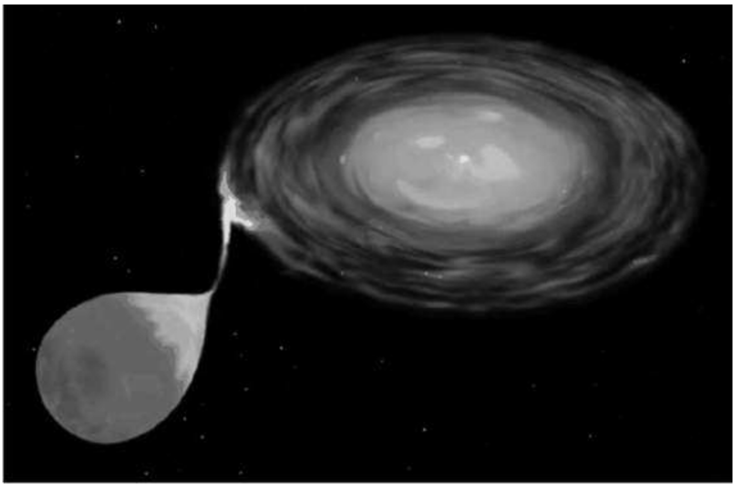

Fig. 1.5 Accretion disk in a binary system, an illustration. Credit: ST ScI, NASA

1.7 Interstellar Partially Ionized Plasma

The spaces amongst the stars show a variety of physical conditions. From a million degree Kelvin fully ionized plasma to a near neutral gas and dust are interspersed with abundance. The extreme ultraviolet and soft X-ray radiation from stars can ionize gas atoms with low ionization potential, for example, Argon, much more easily than hydrogen. This fact is used to determine the degree of ionization in the warm phase of the interstellar medium. The spectroscopic studies of elements such as Argon and oxygen reveal the amount of ionization with typical electron density, n_e, of the order of, 5×10^4 m^{-3} with a neutral hydrogen density n_n of 10^6 m^{-3}. The ratio $n_e/n_n \approx 5 \times 10^{-2}$, though not too small to bring in the characteristics of a weakly ionized plasma, the role of neutrals, nevertheless makes the system highly collisional. This would, to the first order, enhance the dissipation of the magnetohydrodynamic waves which are invoked in several different contexts in the interstellar medium.

1.8 Solar Atmosphere

In order to have a bird's eyeview of the ionization state of the solar atmosphere, it suffices to recall the temperature profile of the entire sun, from the core to the corona. The temperature in the interior of the sun is derived from nucleosynthesis and helioseismology. A rough sketch of the temperature variation with the radius from the core of the sun to its surface is shown in Fig. 1.6. The surface variation of the temperature beginning from the photosphere is shown in Fig. 1.7. The photosphere-chromosphere region of the solar atmosphere is a partially ionized plasma, the photosphere meeting the condition of the weakly ionized plasma. The region houses a variety of identifiable structures with distinct properties.

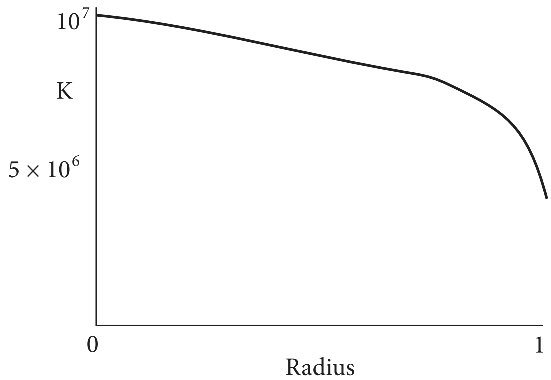

Fig. 1.6 Schematic temperature variation with the radius (scaled to the radius of the sun) from the core to the photosphere

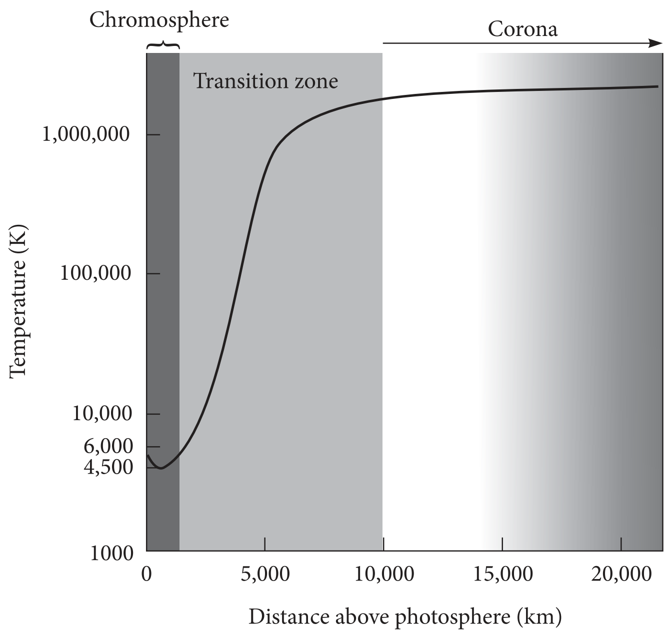

Fig. 1.7 Temperature variation of the sun from the photosphere to the corona. Credit: www.jmm.sgmjournals.org/cgi/content/abstract/60/1/75

The sunspots with dark umbra at their centers and not so dark penumbra at their peripheries have an average temperature < 4000 K Fig. 1.8. They are an excellent example of a strongly magnetized, with a magnetic field strength of $2 - 4$ kiloGauss, partially ionized plasma. The relationship between the sunspot magnetic field and the global magnetic field of the sun is still not so well understood. The need for a surface dynamo action in addition to the sub-surface dynamo has come to the fore. The nature of the Evershed flows[12]

[12] J. Evershed, "Radial Movement in Sunspots", 1909, Monthly Notices of the Royal Astronomical Society, volume, 69, page 454.