# NONLINEAR SOLID MECHANICS FOR FINITE ELEMENT ANALYSIS: DYNAMICS

Designing engineering components that make optimal use of materials requires consideration of the nonlinear static and dynamic characteristics associated with both manufacturing and working environments. The modeling of these characteristics can only be done through numerical formulation and simulation, which requires an understanding of both the theoretical background and associated computer solution techniques.

By presenting nonlinear solid mechanics, dynamic conservation laws and principles, and the associated finite element techniques together, the authors provide in this second book a unified treatment of the dynamic simulation of nonlinear solids.

Alongside a number of worked examples and exercises are user instructions, program descriptions, and examples for two MATLAB computer implementations for which source codes are available online.

While this book is designed to complement postgraduate courses, it is also relevant to those in industry requiring an appreciation of the way their computer simulation programs work.

JAVIER BONET is a Professor of Engineering, currently Deputy Vice-chancellor for Research and Enterprise at the University of Greenwich and formerly Head of the College of Engineering at Swansea University. He has extensive experience of teaching topics in structural mechanics and dynamics, including large strain nonlinear solid mechanics, to undergraduate and graduate engineering students.

ANTONIO J. GIL is a Professor in the Zienkiewicz Centre for Computational Engineering, Swansea University. He has numerous publications in various areas of computational mechanics, with specific experience in the field of large strain nonlinear mechanics.

RICHARD D. WOOD is an Honorary Research Fellow in the Zienkiewicz Centre for Computational Engineering at Swansea University. He has over 20 years' experience of teaching the course Nonlinear Solid Mechanics for Finite Element Analysis at Swansea University, which he originally developed at the University of Arizona.

Cambridge University Press
978-1-107-11562-0 — Nonlinear Solid Mechanics for Finite Element Analysis: Dynamics
Javier Bonet , Antonio J. Gil , Richard D. Wood
Frontmatter
More Information

# NONLINEAR SOLID MECHANICS FOR FINITE ELEMENT ANALYSIS: DYNAMICS

**Javier Bonet**
*University of Greenwich*

**Antonio J. Gil**
*Swansea University*

**Richard D. Wood**
*Swansea University*

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314-321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi - 110025, India

103 Penang Road, #05-06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of
education, learning and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781107115620
DOI: 10.1017/9781316336083



First published 2021

*A catalogue record for this publication is available from the British Library*

*Library of Congress Cataloging in Publication data*
Names: Bonet, Javier, 1961– author. | Gil, Antonio J. author. | Wood, Richard D., 1943– author.
Title: Nonlinear solid mechanics for finite element analysis : dynamics / Javier Bonet, University of Greenwich,
Antonio J. Gil, Swansea University, Richard D. Wood, Swansea University.
Description: New York : Cambridge University Press, 2020. | Includes bibliographical references and index.
Identifiers: LCCN 2020020233 (print) | LCCN 2020020234 (ebook) | ISBN 9781107115620 (hardback) |
ISBN 9781316336083 (ebook)
Subjects: LCSH: Continuum mechanics. | Finite element method. | Dynamics.
Classification: LCC QA808.2 .B657 2020 (print) | LCC QA808.2 (ebook) | DDC 531/.3–dc23
LC record available at https://lccn.loc.gov/2020020233
LC ebook record available at https://lccn.loc.gov/2020020234

ISBN 978-1-107-11562-0 Hardback

To Catherine, Clare, Doreen, and our children

**A fragment from the poem**
**"An Essay on Criticism"**
**by Alexander Pope** (1688–1744)

A little Learning is a dang'rous Thing;
Drink deep, or taste not the Pierian Spring:
There shallow Draughts intoxicate the Brain,
And drinking largely sobers us again.
Fir'd at first Sight with what the Muse imparts,
In fearless Youth we tempt the Heights of Arts,
While from the bounded Level of our Mind,
Short Views we take, nor see the lengths behind,
But more advanc'd, behold with strange Surprize
New, distant Scenes of endless Science rise!

# CONTENTS

# PREFACE

This book extends a previous text entitled *Nonlinear Solid Mechanics for Finite Element Analysis: Statics* (referred to in this book as NL-Statics) into the field of transient dynamic processes. As discussed in the NL-Statics volume:

> A fundamental aspect of engineering is the desire to design artifacts that exploit materials to a maximum in terms of performance under working conditions and efficiency of manufacture. Such an activity demands an increasing understanding of the behaviour of the artifact in its working environment together with an understanding of the mechanical processes occurring during manufacture.

Clearly such an assertion includes the time-dependent behavior of solids, be they structures or general solids subject to geometric and material nonlinear effects. The material developed in this book is important in order to understand the fundamental ideas behind, and correct use of, many software packages that are being widely employed in engineering for applications such as: impact and crash simulation by the automotive and aerospace industries; modeling metal forming and manufacturing processes; or, more recently, in virtual reality simulation being developed for computer games or even surgery simulators. For such an understanding, consideration must be given to the concepts of thermodynamics and thermoelastic constitutive modeling, which may become significant for large deformation, large strain processes undergoing transient motion. Recognizing its widespread adoption, particularly as a graduate training platform, this present dynamics text employs MATLAB®* for the implementation of the finite element analyses, the software being freely available at `www.flagshyp.com` or `www.cambridge.org/9781107115620`.

## READERSHIP

In common with the first volume, NL-Statics, this text is most suited to a postgraduate level of study by those either in higher education or in industry who

* Mathworks, Inc.

have graduated with an engineering or applied mathematics degree. However, the material is equally applicable to first-degree students in the final year of an applied maths course or an engineering course containing some additional emphasis on maths and numerical analysis. A familiarity with elementary dynamics and vibration analysis is assumed, as is some exposure to the principles of the finite element method. It will also be assumed that the reader is already familiar with the nonlinear solid mechanics concepts described in the NL-Statics volume. However, a primary objective of the book is that it be reasonably self-contained with regards to the new material on solid dynamics and thermodynamics.

## LAYOUT

### Chapter 1 Introduction

Here the nature of nonlinear dynamic equilibrium is discussed in the context of very simple one- and two-degrees-of-freedom spring-mass examples. Leap frog and mid-point time integration schemes are introduced as examples of explicit and implicit procedures. As an illustration of physical invariants, conservation of energy is discussed. A very simple two-degrees-of-freedom nonlinear spring-mass model is used by way of an example in order to illustrate these issues. Employing the mid-point rule for this example enables the introduction of the concepts of the directional derivative and the tangent matrix. A short MATLAB code used to implement this example is also presented. In essence, this code is the prototype for the main finite element program discussed later in the book.

### Chapter 2 Dynamic Analysis of Three-Dimensional Trusses

This chapter is largely independent of the remainder of the text and deals with the large strain elastic dynamic behavior of trusses. The chapter first develops the dynamic equilibrium equations, which lead to the common simplification of a lumped mass assumption. In preparation for future chapters, equilibrium is also presented in the context of a variational approach, leading to the introduction of concepts such as the Lagrangian, the action integral, and the derivation of equilibrium via Hamilton's principle of least action. This is equivalent to the common derivation of static equilibrium as the minimum of the total potential energy presented in the NL-Statics volume. Leap-frog and mid-point time integration schemes are re-derived via a discrete approximation of the least action variational principle. Although not derived in the same manner, the trapezoidal time integration scheme is introduced and is shown to be a particular case of the Newmark family of schemes. Global conservation laws dealing with linear and angular momentum and energy are discussed. This is followed by a comprehensive discussion of Hamiltonian formulations involving momentum variables, phase space, and the Hamiltonian map. The notion of simplecticity, the conservation of phase space volume, and the relation to simplectic time integrators is discussed in some detail, drawing on parallels with the deformation gradient described in NL-Statics. A number of illustrative examples are provided using the dynamic version of the FLagSHyP code.

## Chapter 3 Dynamic Equilibrium of Deformable Solids

This chapter describes the dynamic equilibrium of a solid in the large strain context, the key difference between this and static equilibrium being the addition of the inertial forces. The resulting equations are presented in the reference and spatial configurations as well as in differential and integral form via the dynamic version of the principle of virtual work. This chapter also derives the global conservation properties of the dynamic equations, namely momentum and energy, from the principle of virtual work. It finishes by extending the variational concepts of the action integral and Hamilton's principle, presented earlier in the discrete context of trusses, to the continuum.

## Chapter 4 Discretization

This chapter describes the classical finite element discretization of the nonlinear solid dynamic equations when these are expressed in the form of the principle of virtual work. The resulting concept of mass matrix that emerges from discretizing the inertial term and the common practice of nodal mass lumping are described.

The reader is referred to the NL-Statics text for a full exposition of the consistent tangent matrix. The concept of the Lagrangian, previously discussed in Chapter 2 with respect to trusses, is now extended to the discretized continuum. Building on Chapter 2, variational time integrators are employed to re-derive the leap-frog and mid-point time integration schemes. Global conservation properties are again discussed within the context of the discretized equations of the solid.

Chapter 4 also introduces some commonly used time integration schemes and briefly describes the nonlinear solution process involved in their implementation. However, it must be emphasized that it is not the aim of the book to provide a comprehensive study of time or space discretization techniques. The aim is instead to explain the underlying continuum mechanics principles on which such discretizations are based. The chapter, therefore, does not provide an in-depth study of the effects of the discretization, or the errors associated with various time integration schemes or their stability properties.

### Chapter 5 Conservation Laws in Solid Mechanics

This chapter formulates again the laws governing solid dynamics in the form of a system of first-order conservation laws, where both velocities and strains are problem variables. This approach differs from the formulation presented in Chapter 3 in that the equations in this chapter include only first-order derivatives in time, as opposed the second-order derivatives present in the acceleration terms used previously. The chapter begins by establishing a generic conservation law which is then particularized to the various conservation laws such as momentum, energy, or deformation gradient. These are considered in detail for the motion of a solid described in the total Lagrangian, updated Lagrangian, and Eulerian frameworks, including possible thermal effects. Of special interest is the introduction of a geometric conservation law relating to the time evolution of the deformation gradient.

### Chapter 6 Thermodynamics

Nonlinear dynamic transient processes are often accompanied by changes in temperature and heat flow. This chapter aims to introduce the basic concepts in thermodynamics that are necessary to understand such thermomechanical processes. The approach followed is based on the concept of energy conjugacy; that is, entropy is introduced as the conjugate variable to temperature. The energy conservation law is re-expressed as the first law of thermodynamics and in terms of internal energy rate as well as entropy rate. The chapter also introduces the entropy conservation inequality. A number of simple thermo-elastic material models are described, which include the Mie–Grüneisen equation of state widely used for metals and the simple entropic elasticity model used for rubbers.

### Chapter 7 Space and Time Discretization of Conservation Laws in Solid Dynamics

This chapter presents the finite element solution of finite deformation solid dynamic equilibrium equations presented in the form of a system of first-order conservation laws. This approach leads to a more accurate representation of strains and stresses as they become primary variables rather than obtained through derivatives of the geometry. Both isothermal and thermoelastic constitutive models will be explored. The topic is introduced using a simple one-dimensional linear elastic example. The well established Petrov–Galerkin stabilized methodology, widely used in computational fluid dynamics, is described for this purpose. This is combined with a very simple explicit Runge–Kutta time integration scheme, which is also widely used in the literature. A solution algorithm is presented which is implemented as a MATLAB program discussed in Chapter 9.

### Chapter 8 Computer Implementation for Displacement-Based Dynamics

In this chapter information is presented on the nonlinear finite deformation finite element computer program FLagShyP (**F**inite Element **Large** **S**train **H**yperelastoplastic **P**rogram). This program, thoroughly introduced in NL-Statics, is extended in this chapter to the case of dynamic loading by accommodating the various time integration schemes discussed in Chapter 4. Usage and layout of the MATLAB program is discussed together with the function of the various key subroutines.

This free program is available from `www.flagshyp.com` or `www.cambridge.org/9781107115620`. Alternatively, it can be obtained by `e-mail` request to the authors `j.bonet@greenwich.ac.uk` or `a.j.gil@swansea.ac.uk`. The authors would like to acknowledge the assistance given by Dr. Rogelio Ortigosa in the development of this computer program.

### Chapter 9 Computer Implementation for Conservation-Law-Based Explicit Fast Dynamics

This chapter presents a new computer implementation for the analysis of explicit fast dynamics using a first-order conservation-law-based formulation. Specifically, a new program entitled PG_DYNA_LAWS (**P**etrov–**G**alerkin Explicit **Dyna**mics for Conservation **Laws**) is introduced for the solution of finite deformation problems expressed in the form of a system of first-order conservation laws as described in Chapter 7.

This free program is available from `www.flagshyp.com` or `www.cambridge.org/9781107115620`. Alternatively, it can be obtained by `e-mail` request to the authors `j.bonet@greenwich.ac.uk` or `a.j.gil@swansea.ac.uk`. The authors would like to acknowledge the assistance given by Dr. Chun Hean Lee in the development of this computer program.

### Appendix – Shocks

Dynamic problems sometimes involve solutions where by the variables experience sudden discontinuities across moving surfaces. These are known as shocks and are common in problems involving impact or the sudden application of loads, such as as a result of explosions. The appendix describes the conditions that the main dynamic variables need to satisfy across shock surfaces in order to meet the global conservation laws. In fluid dynamics these are known as Rankine–Hugoniot equations, and the appendix will derive similar conditions in the context of Lagrangian solid mechanics. The use of these conditions, together with experimental measurements of shock speeds in order to derive constitutive models, will also be explored.

Finally, the appendix will derive equations for the traction and velocity at contact points between solids impacting against each other.

## Bibliography

A bibliography is provided that enables the reader to access the background to the more standard aspects of solid dynamics and thermodynamics, from Hamiltonian and conservation law standpoints. Also listed are texts and papers that have been of use in the preparation of this book or that cover similar material in greater depth.