Proofs and Refutations

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The Logic of Mathematical Discovery

IMRE LAKATOS

Edited by

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Preface to this edition

PAOLO MANCOSU

Proofs and Refutations is one of the undeniable classics of the philosophy of mathematics. Fifty years have passed since the publication of the articles that make up its central core, but the book has lost neither its freshness nor its provocative vitality. It takes the form of a classroom dialogue in which a group of students and their teacher investigate the problem of whether there is a relation that holds between the number of vertices V, the number of edges E, and the number of faces F of regular polyhedra (e.g. the five Platonic solids). At the outset of the dialogues they have arrived at the formula $V - E + F = 2$. They conjecture that the result might extend to any polyhedron (Euler's conjecture), and this is the starting point of a riveting development that carries the reader through the rational reconstruction, as embodied in the class dialogue, of the history of Euler's conjecture, culminating in Poincaré's proof. The reconstruction, in strong contrast to a piece of axiomatic mathematics, features contradictions, monsters, counterexamples, conjectures, concept-stretchings, hidden lemmas, proofs, and a wide range of informal moves meant to account for the rationality of the process leading to concept-formation and conjectures/proofs in mathematical practice.

Yet Euler's conjecture is just a case-study displaying Lakatos's highly original approach to the philosophy of mathematics. A starker contrast with the formalist foundational approach dominant up to the 1960s (and embodied in philosophies of mathematics of neo-positivist inspiration) can scarcely be imagined. Whereas the latter, inspired by Euclid's infallibilist dogmatic style, thought of mathematical theories statically as axiomatic systems, Lakatos was after an account of informal mathematics as a fallible dynamic body of knowledge. Rejecting the positivist distinction between context of discovery and context of justification, he claimed that mathematical practice and its history are not the domain of the irrational but rather display an objectivity and rationality that any philosophy of mathematics worth its name should account for. The tools for addressing the rationality of mathematical growth could not, however, be those of formal logic, whose 'deductivist style' could only address issues of the static variety and was thus unable to account for concept-formation and the rational dynamics driving the development of informal mathematics. Rather, Lakatos found inspiration in Polya's work on mathematical heuristics, Hegel's dialectic, and Popperian conjecture and refutation. This led to Lakatos's dialectical methodology, a 'heuristic style' that reveals the struggle and the adventure of mathematical creation.

There will always be disagreements as to whether or to what extent Lakatos's case studies are paradigmatic and can be extended to mathematics as a whole. Scholars will also continue to disagree about the suitability of the dialectical framework for accounting for mathematical growth and the role of mathematical logic in the history and philosophy of mathematics. But the characteristic trait of a classic is its rich and varied legacy. *Proofs and Refutations* stands up to this test, for it continues to be a source of inspiration to many historians, mathematicians, and philosophers who aspire to develop a philosophy of mathematics that does justice to the static and dynamic complexity of mathematical practice.

Editors' preface

Our great friend and teacher Imre Lakatos died unexpectedly on 2 February 1974. At the time he was (as usual) engaged on many intellectual projects. One of the most important of these was the publication of a modified and extended version of his brilliant essay 'Proofs and Refutations', which appeared in four parts in *The British Journal for the Philosophy of Science,* 14, 1963–4. Lakatos had long had a contract for this book, but had held back publication in the hope of amending and further improving the essay, and of adding to it substantial extra material. This work was considerably delayed by the diversion of his interests to the philosophy of physical science, but in the summer of 1973 he finally decided to go ahead with the publication. During that summer we each discussed plans for the book with him, and we have tried to produce a book which, in the sadly changed circumstances, is as similar as possible to the one then projected by Lakatos.

We have thus included three new items in addition to the original 'Proofs and Refutations' essay (which appears here as Chapter 1). First we have added a second part to the main text. This concerns Poincaré's vector-algebraic proof of the Descartes–Euler conjecture. It is based on chapter 2 of Lakatos's 1961 Cambridge Ph.D. thesis. (The original 'Proofs and Refutations' essay was a much amended and improved version of chapter 1 of that thesis.) A part of chapter 3 of this thesis becomes here appendix 1, which contains a further case-study in the method of proofs and refutations. It is concerned with Cauchy's proof of the theorem that the limit of any convergent series of continuous functions is itself continuous. Chapter 2 of the main text and appendix 1 should allay the doubt, often expressed by mathematicians who have read 'Proofs and Refutations', that, while the method of proof-analysis described by Lakatos may be applicable to the study of polyhedra, a subject which is 'near empirical' and where the counterexamples are easily visualisable, it may be inapplicable to 'real' mathematics. The third additional item (appendix 2) is also based on a part of chapter 3 of Lakatos's thesis. It is about the consequences of his position for the development, presentation and teaching of mathematics.

One of the reasons Lakatos delayed publication was his recognition that some of this extra material, whilst containing many new points and developments of his position, was in need of further consideration and further historical research. This is particularly true of the material (in appendix 1) on Cauchy and Fourier. We also are aware of certain difficulties and ambiguities in this material and of omissions from it. We felt, however, that we should not change the content of what Lakatos had written. As for elaborating on, and adding to, the material, neither of us was in a position to supply the necessary long and detailed historical research. Faced then with the alternatives of not publishing the material at all, or publishing it in an unfinished state, we decided on the latter option. We feel that there is much of interest in it, and hope that it will stimulate other scholars to extend and correct it if necessary.

In general, we did not think it right to modify the content of Lakatos's material, even those parts of it about which we were confident Lakatos had changed his position. We have therefore restricted ourselves to pointing out (in notes marked with asterisks) some of those things we should have tried to persuade Lakatos to change and (which often amounts to the same thing) some of those points we believe Lakatos would have changed in publishing this material now. (His intellectual position had, of course, changed considerably during the thirteen years between completing the Ph.D. thesis and his death. The major changes in his general philosophy are explained in his [1970]. We should mention that Lakatos thought that his methodology of scientific research programmes had important implications for his philosophy of mathematics.)

Our approach to matters of presentation has been to leave the material which Lakatos had himself published (i.e. chapter 1 of the main text) almost entirely unchanged (the only exceptions are a few misprints and unambiguous minor slips). We have, however, rather substantially modified the previously unpublished material – though, to repeat, only in form and not in content. Since this may seem a rather unusual procedure, perhaps a few words of justification are in order.

Lakatos always took a great deal of care over the presentation of any of his material which was to be published, and, prior to publication, he always had such material widely circulated amongst colleagues and friends, for criticism and suggested improvements. We are sure that the material here published for the first time would have undergone this treatment, and that the changes would have been more drastic than those we have dared to introduce. Our knowledge (through personal experience) of the pains Lakatos took to present his position as clearly as possible obliged us to try to improve the presentation of this material as best we could. It is certain that these new items do not read as well as they would have done, had Lakatos himself revised the material on which they are based, but we felt that we were close enough to Lakatos, and involved enough in some of his previous publications, to make a reasonable attempt at bringing the material up to somewhere near his own high standards.

We are very pleased to have had the opportunity to produce this edition of some of Lakatos's important work in the philosophy of mathematics, for it allows us to discharge part of the intellectual and personal debt we both owe him.

JOHN WORRALL
ELIE ZAHAR

Acknowledgments

The material on which this book is based has had a long and varied history, as is in part already indicated in our preface. According to the acknowledgements Lakatos appended to his original essay of 1963–4 (reprinted here as chapter 1), that work began life in 1958–9 at King's College, Cambridge, and was first read at Karl Popper's seminar at the London School of Economics in March 1959. Another version was incorporated in his 1961 Cambridge Ph.D. thesis, on which the rest of this book is also based. The thesis was prepared under the supervision of Professor R. B. Braithwaite. In connection with it, Lakatos acknowledged the financial assistance of the Rockefeller Foundation and that he 'received much help, encouragement and valuable criticism from Dr T. J. Smiley'. The rest of Lakatos's acknowledgements read:

When preparing this latest version at the London School of Economics the author tried to take note especially of the criticisms and suggestions of Dr J. Agassi, Dr I. Hacking, Professors W. C. Kneale and R. Montague, A. Musgrave, Professor M. Polanyi and J. W. N. Watkins. The treatment of the exception-barring method was improved under the stimulus of the critical remarks of Professors G. Pólya and B. L. Van der Waerden. The distinction between the methods of monster-barring and monster-adjustment was suggested by B. MacLennan.

The paper should be seen against the background of Pólya's revival of mathematical heuristic, and of Popper's critical philosophy.

The original essay of 1963–4 carried the following dedication:

For George Pólya's 75th and Karl Popper's 60th Birthday.

In preparing this book, the editors were helped by John Bell, Mike Hallett, Moshé Machover and Jerry Ravetz, who all kindly read drafts of chapter 2 and the appendices, and produced helpful criticisms.

We should also like to acknowledge the work of Sandra D. Mitchell and especially of Gregory Currie, who carefully criticised our reworking of Lakatos's material.

J. W.
E. Z.