CHAPTER 1

Introduction

Quantum annealing is a new-generation quantum information technology, which is expected to rapidly solve combinatorial optimization problems with high precision. It is based on quantum statistical physics. The realization of a quantum annealing machine (e.g., D-Wave developed by D-Wave Systems Inc.; Johnson et al., 2011) is a big step in opening the door to new development of quantum information technology as well as a historically significant event in the history of quantum information processing. Nowadays, not only the academic side but also industry have high expectations in the field of quantum information processing.

The target of quantum annealing is to solve combinatorial optimization problems. Combinatorial optimization problems are problems which seek to find the best answer from a vast number of candidates. The most typical example of a combinatorial optimization problem is the traveling salesman problem which is to find the path along which we can visit all locations efficiently. More precisely, the traveling salesman problem is to find the path that minimizes costs such as distance and travel expenses under the condition that all locations are visited once. A naive method is to make a list of all candidates of paths and to find the best solution that minimizes the cost from this list. If we use the naive method, the number of candidates increases exponentially with the number of locations. This situation is called the combinatorial explosion. Thus, the naive method is not efficient in practice. Of course, other combinatorial optimization problems have the same problem.

Combinatorial optimization problems can be expressed by random Ising models. We can map the cost function of a combinatorial optimization problem to the Hamiltonian of a random Ising spin system. Thus, the knowledge of random Ising spin systems is useful to consider combinatorial optimization problems. In order to consider cooperative phenomena in random magnets, Edwards and Anderson considered the Ising model with random interactions, called the Edwards–Anderson model or the EA model (Edwards and Anderson, 1975). The EA model is a prototype with which we consider random spin systems. After the proposition of the EA model, Sherrington and Kirkpatrick proposed the infinite-range interaction version of the EA model, called the Sherrington–Kirkpatrick model or the SK model (Sherrington and Kirkpatrick, 1975). The interactions in the SK model is infinite ranged and the probability distribution of the value of interactions is the Gaussian distribution. Depending on the value of the average and the standard deviation of interactions, spin glass phase appears.

In information science and engineering, particular algorithms to solve each combinatorial optimization problem have been proposed. Since these methods are useful in practice, they are often used in each industrial area. However, a particular algorithm proposed for a combinatorial optimization problem is not necessarily suitable for other combinatorial optimization problems. Thus, if we encounter new types of combinatorial optimization problems, we cannot handle them by using particular algorithms. In statistical physics which is an established discipline in physics, on the other hand, generic algorithms have been developed exhaustively. Simulated annealing, or classical annealing proposed by Kirkpatrick, Gelatt, and Vecchi is a famous algorithm inspired by the concept of statistical physics (Kirkpatrick et al., 1983). Simulated annealing is an algorithm that uses the temperature effect, i.e., the thermal fluctuation effect. The convergence of simulated annealing is guaranteed by the Geman–Geman theorem (Geman and Geman, 1984). Simulated annealing has been widely used for solving combinatorial optimization problems, because of the ease of implementation.

Through the analysis of the EA model and the SK model, the thermal fluctuation effect of random spin systems has been considered. In physics, there is another important fluctuation effect – the quantum fluctuation effect. Chakrabarti first investigated critical behavior of the EA model with a transverse field (Chakrabarti, 1981). After that, phase transition nature and dynamical properties of the random Ising spin models under the transverse field have been studied exhaustively. Ray, Chakrabarti, and Chakrabarti considered dynamical properties of the SK model under the transverse field (Ray et al., 1989). They pointed out that quantum fluctuation lead to some escape routes by tunneling through macroscopically tall but thin barriers which are difficult to scale using classical (thermal) fluctuations. This idea is the basis of quantum annealing.

Quantum annealing is a cousin of simulated annealing. In quantum annealing, we introduce a quantum effect to the combinatorial optimization problem. By decreasing the quantum effect, we can obtain the best solution of the combinatorial optimization problem. This scheme corresponds to simulated annealing. In simulated annealing, we introduce a thermal effect to the combinatorial optimization problem. By decreasing the temperature, we can obtain the best solution of the combinatorial optimization problem. The milestone in the use of quantum annealing is the study done by Kadowaki and Nishimori (Kadowaki and Nishimori, 1998). Their paper is based on the statistical physics of random Ising models. In preceding studies, the quantum effect for random Ising models plays the same role as the thermal one.

As mentioned earlier, quantum annealing is a hot topic in science since quantum annealing is expected to be a powerful and generic algorithm to solve combinatorial optimization problems. Thus, the purpose of this book is to provide an overview of quantum annealing by reviewing famous studies. In addition, related topics of quantum annealing and the background of quantum annealing – quantum statistical physics and information science are also discussed.

The organization of the book is as follows. Part 1 has six chapters. Chapter 1, this chapter, is devoted to introducing the topic of the book. The rest is as follows. In Chapter 2, the physical properties of classical spin models, especially, random Ising models are reviewed. This is a fundamental knowledge needed to study quantum annealing. In Chapter 3, simulated annealing is explained. Simulated annealing is a generic algorithm inspired by statistical physics and developed to find the best solution of combinatorial optimization problems. Simulated annealing has been used in broad areas of science and industry. Quantum annealing which is the main topic of the book is based on simulated annealing. Thus, the knowledge of simulated annealing is useful to study quantum annealing. In Chapter 4, we briefly review the physical properties of quantum random Ising models and compare them with the classical random Ising models explained in Chapter 2. Since the instantaneous Hamiltonian of quantum annealing is described by quantum random Ising models, the knowledge learned in the chapter is used to consider the performance of quantum annealing. In Chapter 5, two typical quantum dynamic properties are shown. One is the Landau–Zener transition and the other is the Kibble–Zurek mechanism. Both of them are used for analysis of the performance of quantum annealing. In Chapter 6, we review quantum annealing that is the main topic of the book from the theoretical to the experimental aspects. In this chapter, we briefly show famous studies on quantum annealing. Part 2 has three chapters written by guest authors. In Chapter 7, adiabatic quantum computers are reviewed by Boaz Tamir and Eliahu Cohen. In this chapter, the architecture of D-Wave, that is a quantum annealing hardware, is also explained. In Chapter 8, the relation between quantum information and quenching dynamics is reviewed by Atanu Rajak and Uma Divakaran. In this chapter, the recent development of theoretical studies on quenching dynamics of a number of models including topological systems is explained. In Chapter 9, the brief history of quantum annealing is presented by Sudip Mukherjee.

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