

## OPTIMIZATION IN PRACTICE WITH MATLAB<sup>®</sup> FOR ENGINEERING STUDENTS AND PROFESSIONALS

*Optimization in Practice with MATLAB<sup>®</sup>* provides a unique approach to optimization education. It is accessible to **junior and senior undergraduate, and graduate students**, as well as industry practitioners. It provides a strongly practical perspective that allows the student to be **ready to use optimization** in the workplace. It covers traditional materials, as well as important topics previously unavailable in optimization books (*e.g.*, Numerical Essentials – for successful optimization).

### **Outstanding features include:**

- Provides practical applications of real-world problems using **MATLAB**.
- Each chapter includes a **suite of practical examples and exercises** that help students **link the theoretical, the analytical and the computational**. These include a robust set of real-world exercises.
- Provides **supporting MATLAB codes** that offer the opportunity to apply optimization at all levels, from students' term projects to industry applications.
- Offers instructors a comprehensive **solution manual** with solution codes along with **lectures in PowerPoint with animations** for each chapter. The MATLAB m-files are available for download from the book's website.
- **Instructors have the unique flexibility** to structure one- or two-semester courses that may range from gentle introductions to highly challenging, for undergraduate or graduate students.

**Dr. Achille Messac** received his BS, MS and PhD from MIT in Aerospace Engineering. Dr. Messac is a Fellow of the American Institute of Aeronautics and Astronautics (AIAA) and the American Society of Mechanical Engineers. He has authored or co-authored more than 70 journal and 130 conference articles, chaired several international conferences, delivered several keynote addresses, and received the prestigious AIAA Multidisciplinary Design Optimization Award. He has taught or advised undergraduate and graduate students in the areas of design and optimization for more than three decades at Rensselaer Polytechnic Institute, MIT, Syracuse University, Mississippi State and Northeastern University.

Cambridge University Press

978-1-107-10918-6 - Optimization in Practice with MATLAB for Engineering Students and Professionals

Achille Messac

Frontmatter

[More information](#)

---

Cambridge University Press

978-1-107-10918-6 - Optimization in Practice with MATLAB for Engineering Students and Professionals

Achille Messac

Frontmatter

[More information](#)

# Optimization in Practice with MATLAB<sup>®</sup> for Engineering Students and Professionals

Achille Messac, PhD



Cambridge University Press

978-1-107-10918-6 - Optimization in Practice with MATLAB for Engineering Students and Professionals

Achille Messac

Frontmatter

[More information](#)

**CAMBRIDGE**  
UNIVERSITY PRESS

32 Avenue of the Americas, New York, NY 10013-2473, USA

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

[www.cambridge.org](http://www.cambridge.org)

Information on this title: [www.cambridge.org/9781107109186](http://www.cambridge.org/9781107109186)

© Achille Messac 2015

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2015

Printed in the United States of America

*A catalog record for this publication is available from the British Library.*

ISBN 978-1-107-10918-6 Hardback

Additional resources for this publication at [www.cambridge.org/Messac](http://www.cambridge.org/Messac)

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party Internet websites referred to in this publication and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

MATLAB is a registered trademark of The MathWorks, Inc.

## Contents

<i>List of Figures</i>	<i>page</i> xvii
<i>List of Tables</i>	xxiii
<i>Preface</i>	xxv
<i>Contacting the Author Regarding this Book</i>	xxv
<i>Book Website</i>	xxv
<i>Book Organization</i>	xxv
<i>A Message to Students</i>	xxvi
<i>A Message to Industry Practitioners</i>	xxviii
<i>A Message to Instructors</i>	xxviii
<i>Acknowledgements</i>	xxxi
PART I. HELPFUL PRELIMINARIES	1
<b>1 MATLAB<sup>®</sup> as a Computational Tool</b> . . . . .	<b>3</b>
1.1 Overview	3
1.2 MATLAB Preliminaries—Before Starting	4
1.2.1 <i>What Is MATLAB?</i>	4
1.2.2 <i>Why MATLAB?</i>	5
1.2.3 <i>MATLAB Toolboxes</i>	5
1.2.4 <i>How to Use MATLAB in this Book</i>	7
1.2.5 <i>Acquiring MATLAB</i>	7
1.2.6 <i>MATLAB Documentation</i>	7
1.2.7 <i>Other Software for Optimization</i>	8
1.3 Basics of MATLAB—Getting Started	8
1.3.1 <i>Starting and Quitting MATLAB</i>	8
1.3.2 <i>MATLAB Desktop: Its Graphical User Interface</i>	9
1.3.3 <i>Matrices and Variables Operations</i>	14
1.3.4 <i>More MATLAB Expressions</i>	19
1.4 Beyond the Basics of MATLAB	19
1.4.1 <i>Input and Output, Directories and Files</i>	19

	1.4.2	<i>Flow Control, Relational and Logical Operators</i>	20
	1.4.3	<i>M-files</i>	22
	1.4.4	<i>Global and Local Variables</i>	23
	1.4.5	<i>MATLAB Help</i>	23
1.5		Plotting Using MATLAB	24
	1.5.1	<i>Basic Plots</i>	24
	1.5.2	<i>Special Plots: Contour, Scatter, fplot</i>	26
	1.5.3	<i>3-D Mesh and Surface Plots</i>	28
	1.5.4	<i>Using the Plot Editing Mode</i>	29
1.6		Optimizing with MATLAB	30
1.7		Popular Functions and Commands, and More	30
1.8		Summary	30
1.9		Problems	31
		Bibliography of Chapter 1	42
<b>2</b>		<b>Mathematical Preliminaries</b> . . . . .	44
	2.1	Overview	44
	2.2	Vectors and Geometry	44
		2.2.1 <i>Dot Product</i>	44
		2.2.2 <i>Equation of a Line</i>	45
		2.2.3 <i>Equation of a Plane</i>	45
	2.3	Basic Linear Algebra	46
		2.3.1 <i>Preliminary Definitions</i>	47
		2.3.2 <i>Matrix Operations</i>	48
		2.3.3 <i>Determinants</i>	51
		2.3.4 <i>Inverse</i>	53
		2.3.5 <i>Eigenvalues</i>	53
		2.3.6 <i>Eigenvectors</i>	54
		2.3.7 <i>Positive Definiteness</i>	55
	2.4	Basic Calculus: Types of Functions, Derivative, Integration and Taylor Series	55
		2.4.1 <i>Types of Functions</i>	56
		2.4.2 <i>Limits of Functions</i>	59
		2.4.3 <i>Derivative</i>	59
		2.4.4 <i>Partial Derivative</i>	60
		2.4.5 <i>Indefinite Integration</i>	60
		2.4.6 <i>Definite Integration</i>	60
		2.4.7 <i>Taylor Series</i>	61
	2.5	Optimization Basics: Single-Variable Optimality Conditions, Gradient, Hessian	62
		2.5.1 <i>Necessary Conditions for Local Optimum</i>	62
		2.5.2 <i>Stationary Points and Inflection Points</i>	63
		2.5.3 <i>Sufficient Conditions for Local Optima</i>	63
		2.5.4 <i>Gradient and Hessian of a Function</i>	64

<i>Contents</i>	vii
2.6 Summary	65
2.7 Problems	66
Bibliography of Chapter 2	69
<b>PART II. USING OPTIMIZATION—THE ROAD MAP</b>	<b>71</b>
<b>3 Welcome to the Fascinating World of Optimization . . . . .</b>	<b>73</b>
3.1 Overview	73
3.2 What Is Optimization? What Is Its Relation to Analysis and Design?	73
3.3 Why Should Junior and Senior College Students Study Optimization?	77
3.4 Why Should Graduate Students Study Optimization?	77
3.5 Why Should Industry Practitioners Study Optimization?	78
3.6 Why Use this Book, and What Should I Expect from It?	78
3.7 How this Book Is Organized	79
3.8 How to Read and Use this Book	80
3.9 Summary	80
3.10 Problems	81
Bibliography of Chapter 3	81
<b>4 Analysis, Design, Optimization and Modeling . . . . .</b>	<b>82</b>
4.1 Overview	82
4.2 Analysis, Design and Optimization	82
4.2.1 <i>What Is Analysis?</i>	83
4.2.2 <i>What Is Design?</i>	84
4.2.3 <i>What Is Optimization?</i>	85
4.2.4 <i>Interdependence of Analysis, Design and Optimization</i>	86
4.3 Modeling System Behavior and Modeling the Optimization Problem	88
4.3.1 <i>Modeling System Behavior</i>	88
4.3.2 <i>Modeling the Optimization Problem</i>	90
4.3.3 <i>Interdependence of System Behavior Modeling and Optimization Modeling</i>	90
4.4 Summary	91
4.5 Problems	91
Bibliography of Chapter 4	92
<b>5 Introducing Linear and Nonlinear Programming . . . . .</b>	<b>93</b>
5.1 Overview	93
5.2 Problem Classes	93
5.3 Single Objective Optimization—An Inclusive Notion	98
5.4 Solution Approaches: Analytical, Numerical, Experimental and Graphical	98
5.4.1 <i>Analytical Optimization</i>	98

5.4.2	<i>Numerical (or Algorithmic) Optimization</i>	99
5.4.3	<i>Experimental Optimization</i>	101
5.4.4	<i>Graphical Optimization</i>	101
5.5	Software Options for Optimization	102
5.5.1	<i>MATLAB Optimization Code—fmincon and linprog</i>	103
5.5.2	<i>Software for Optimization as Stand-Alone (SO-SA)</i>	109
5.5.3	<i>Software for Optimization Within Design Framework (SO-WDF)</i>	111
5.5.4	<i>Software for Optimization Within Analysis Package (SO-WAP)</i>	112
5.6	Summary	114
5.7	Problems	115
	Bibliography of Chapter 5	119
PART III. USING OPTIMIZATION—PRACTICAL ESSENTIALS		121
<b>6</b>	<b>Multiobjective Optimization</b>	123
6.1	Overview	123
6.2	The Multiobjective Problem Definition	123
6.2.1	<i>Example Problem</i>	124
6.2.2	<i>Multiobjective Optimization Problem Statement</i>	124
6.3	Pareto Optimal Solution	125
6.3.1	<i>Introducing the Pareto Solution</i>	125
6.3.2	<i>The Pareto Frontier</i>	126
6.3.3	<i>Obtaining Pareto Solutions</i>	127
6.3.4	<i>Aggregate Objective Function</i>	127
6.4	The Weighted Sum Method	128
6.4.1	<i>Two-Objective Case</i>	128
6.4.2	<i>Addressing More than Two Objectives</i>	129
6.5	Compromise Programming	131
6.6	Generating the Pareto Frontier—with MATLAB	133
6.7	Reaching a Target—Goal Programming	135
6.8	Expressing a Preference—Physical Programming	136
6.9	Multiobjective Optimization Using MATLAB Optimization Toolbox	137
6.10	Summary	138
6.11	Problems	138
	Bibliography of Chapter 6	157
<b>7</b>	<b>Numerical Essentials</b>	158
7.1	Overview	158
7.2	Numerical Conditioning—Algorithms, Matrices and Optimization Problems	158
7.2.1	<i>Reasons Why the Optimization Process Sometimes Fails</i>	159



<i>Contents</i>		ix
	7.2.2 <i>Exposing Numerical Conditioning Issues—Algorithms and Matrices</i>	160
	7.2.3 <i>Exposing Numerical Conditioning Issues—Optimization Problems</i>	162
7.3	Scaling and Tolerances for Design Variables, Constraints and Objective Functions	163
	7.3.1 <i>Understanding the Accuracy of the Reported Results</i>	165
	7.3.2 <i>Design Variable Scaling—Order of Magnitude (DV-1)</i>	166
	7.3.3 <i>Design Variable Scaling—Tolerance Definition (DV-2)</i>	167
	7.3.4 <i>Design Variable Scaling—Optimization Code Decimal Accuracy Setting (DV-3)</i>	168
	7.3.5 <i>Design Variable Scaling—Combining Order of Magnitude and Desired Tolerance (DV-4)</i>	168
	7.3.6 <i>Design Variable Scaling—Setting Scaling Parameters (DV-5)</i>	169
	7.3.7 <i>Objective Function Scaling</i>	170
	7.3.8 <i>Behavioral Constraints Scaling</i>	171
	7.3.9 <i>Setting MATLAB Optimization Options and Scaling Parameters: Syntax</i>	173
	7.3.10 <i>Simple Scaling Examples</i>	174
7.4	Finite Difference	176
	7.4.1 <i>Fundamentals of Finite Difference</i>	176
	7.4.2 <i>Accuracy of Finite Difference Approximation</i>	179
7.5	Automatic Differentiation	182
7.6	Other Important Numerical and Computational Issues	185
	7.6.1 <i>Sensitivity of Optimal Solutions in Nonlinear Programming</i>	185
	7.6.2 <i>Optimization Termination Criteria and Optimization Termination Causes</i>	186
	7.6.3 <i>Developing Confidence in Optimization Results</i>	187
	7.6.4 <i>Problem Dimension and Computational Burden</i>	187
	7.6.5 <i>Additional Numerical Pitfalls</i>	188
7.7	Larger Scaling Example: Universal Motor Problem	188
	7.7.1 <i>Universal Motor Problem Definition</i>	188
	7.7.2 <i>Design Variable Scaling</i>	190
7.8	Summary	190
7.9	Problems	191
	Bibliography of Chapter 7	198
<b>8</b>	<b>Global Optimization Basics</b> . . . . .	200
	8.1 Overview	200
	8.2 Practical Issues in Global Optimization	200
	8.3 Exhaustive Search	202
	8.4 Multiple Start	203

8.5	Role of Genetic Algorithms in Global Optimization	205
8.6	MATLAB Global Optimization Toolbox	209
8.7	Summary	210
8.8	Problems	211
	Bibliography of Chapter 8	212
<b>9</b>	<b>Discrete Optimization Basics</b> . . . . .	213
9.1	Overview	213
9.2	Defining Discrete Optimization	213
9.3	Exhaustive Search	214
9.4	Relaxation Approach	215
9.5	Advanced Options: Genetic Algorithms, Simulated Annealing, and Branch and Bound	217
	9.5.1 <i>Genetic Algorithms</i>	217
	9.5.2 <i>Simulated Annealing</i>	218
	9.5.3 <i>Branch and Bound</i>	218
9.6	Summary	221
9.7	Problems	221
	Bibliography of Chapter 9	222
<b>10</b>	<b>Practicing Optimization—Larger Examples</b> . . . . .	223
10.1	Overview	223
10.2	Mechanical Engineering Example	223
	10.2.1 <i>Structural Example</i>	223
	10.2.2 <i>Tolerance Allocation Problem</i>	225
10.3	Aerospace Engineering Example	229
	10.3.1 <i>Ground Controllability</i>	230
	10.3.2 <i>Ground Stability</i>	230
	10.3.3 <i>Structural Integrity</i>	231
10.4	Mathematical Example	232
	10.4.1 <i>Data Fitting</i>	232
	10.4.2 <i>Least Squares Data Fitting</i>	233
10.5	Civil Engineering Example	234
10.6	Electrical Engineering Example	236
	10.6.1 <i>Introduction to Thermoelectric Window Design</i>	236
	10.6.2 <i>Brief Introduction to the Trust Region Method</i>	237
	10.6.3 <i>Modeling TE Units</i>	238
	10.6.4 <i>Solving Optimization Problem</i>	239
	10.6.5 <i>Results</i>	241
10.7	Business Example	241
10.8	Summary	242
10.9	Problems	242
	Bibliography of Chapter 10	246

<i>Contents</i>	xi
<b>PART IV. GOING DEEPER: INSIDE THE CODES AND THEORETICAL ASPECTS</b>	249
<b>11 Linear Programming</b> . . . . .	251
11.1 Overview	251
11.2 Basics of Linear Programming	251
11.3 Graphical Solution Approach: Types of LP Solutions	253
11.3.1 <i>The Unique Solution</i>	253
11.3.2 <i>The Segment Solution</i>	253
11.3.3 <i>No Solution</i>	254
11.3.4 <i>The Solution at Infinity</i>	255
11.4 Solving LP Problems Using MATLAB	255
11.5 Simplex Method Basics	257
11.5.1 <i>The Standard Form</i>	257
11.5.2 <i>Transforming into Standard Form</i>	258
11.5.3 <i>Gauss Jordan Elimination</i>	259
11.5.4 <i>Reducing to a Row Echelon Form</i>	261
11.5.5 <i>The Basic Solution</i>	263
11.6 Simplex Algorithm	264
11.6.1 <i>Basic Algorithm</i>	264
11.6.2 <i>Special Cases</i>	271
11.7 Advanced Concepts	271
11.7.1 <i>Duality</i>	272
11.7.2 <i>Primal-Dual Relationships</i>	273
11.7.3 <i>Interior Point Methods</i>	274
11.7.4 <i>Solution Sensitivity</i>	274
11.8 Summary	276
11.9 Problems	276
Bibliography of Chapter 11	277
<b>12 Nonlinear Programming with No Constraints</b> . . . . .	279
12.1 Overview	279
12.2 Necessary and Sufficient Conditions	279
12.3 Single Variable Optimization	280
12.3.1 <i>Interval Reduction Methods</i>	281
12.3.2 <i>Polynomial Approximations: Quadratic Approximation</i>	285
12.4 Multivariable Optimization	287
12.4.1 <i>Zeroth-Order Methods</i>	287
12.4.2 <i>First-Order Methods</i>	293
12.4.3 <i>Second-Order Methods</i>	298
12.5 Comparison of Computational Issues in the Algorithms	303
12.5.1 <i>Rate of Convergence</i>	303

12.5.2	<i>Line Search Methods</i>	304
12.5.3	<i>Comparison of Different Methods</i>	305
12.6	Summary	306
12.7	Problems	306
	Bibliography of Chapter 12	308
<b>13</b>	<b>Nonlinear Programming with Constraints</b>	310
13.1	Overview	310
13.2	Structure of Constrained Optimization	310
13.3	Elimination Method	312
13.4	Penalty Methods	313
13.5	Karush-Kuhn-Tucker Conditions	317
13.6	Sequential Linear Programming	323
13.7	Sequential Quadratic Programming	326
13.8	Comparison of Computational Issues	328
13.9	Summary	329
13.10	Problems	329
	Bibliography of Chapter 13	331
PART V. MORE ADVANCED TOPICS IN OPTIMIZATION		333
<b>14</b>	<b>Discrete Optimization</b>	335
14.1	Overview	335
14.2	Problem Classes, Examples and Definition	335
14.2.1	<i>Problem Classes</i>	336
14.2.2	<i>Popular Example Problems</i>	336
14.2.3	<i>Problem Definition and Computational Complexity</i>	337
14.3	Solution Approaches	339
14.3.1	<i>Brute Force Method: Exhaustive Search</i>	340
14.3.2	<i>Graphical Method</i>	341
14.3.3	<i>Relaxation Approach: Solve as Continuous Problem</i>	341
14.3.4	<i>Branch and Bound Method</i>	342
14.3.5	<i>Cutting Plane Method</i>	343
14.3.6	<i>Evolutionary Algorithms</i>	351
14.3.7	<i>Software Options for Discrete Optimization</i>	351
14.4	Summary	352
14.5	Problems	352
	Bibliography of Chapter 14	353
<b>15</b>	<b>Modeling Complex Systems: Surrogate Modeling and Design Space Reduction</b>	355
15.1	Overview	355
15.2	Modeling Challenges in Complex Optimization Problems	355
15.3	Impact of Problem Dimension	357
15.3.1	<i>Design Variable Linking</i>	357

<i>Contents</i>		xiii
	15.3.2 <i>Design of Experiments</i>	359
15.4	Surrogate Modeling	363
	15.4.1 <i>Surrogate Modeling Process</i>	364
	15.4.2 <i>Polynomial Response Surface Methodology</i>	365
	15.4.3 <i>Radial Basis Function Method</i>	367
	15.4.4 <i>Kriging Method</i>	371
	15.4.5 <i>Artificial Neural Networks (ANN)</i>	371
15.5	Summary	372
15.6	Problems	372
	Bibliography of Chapter 15	373
<b>16</b>	<b>Design Optimization Under Uncertainty</b> . . . . .	376
16.1	Overview	376
16.2	Chapter Example	377
16.3	Generic Components/STEPS of Design Under Uncertainty	378
16.4	STEP 1: Identifying Types of Uncertainty	380
16.5	STEP 2: Uncertainty Quantification	382
	16.5.1 <i>Sufficient Data Available: Probability Theory</i>	382
	16.5.2 <i>Insufficient Data: Non-Probabilistic Methods</i>	384
16.6	STEP 3: Uncertainty Propagation	385
	16.6.1 <i>Sampling Methods</i>	385
	16.6.2 <i>First-Order and Second-Order Reliability Methods (FORM and SORM)</i>	387
	16.6.3 <i>Polynomial Approximation Using Taylor Series</i>	387
	16.6.4 <i>Advanced Methods Overview</i>	389
	16.6.5 <i>An Important Note on Uncertainty: Analysis vs. Optimization</i>	389
16.7	STEP 4: Embedding Uncertainty into an Optimization Framework	389
	16.7.1 <i>Reliability-Based Design Optimization (RBDO)</i>	391
	16.7.2 <i>Use of Approximation Methods Under Uncertainty</i>	392
	16.7.3 <i>Robust Design Optimization (RDO)</i>	393
16.8	STEP 5: How to Analyze the Results	396
	16.8.1 <i>Mean Performance and Robustness Trade-off</i>	397
	16.8.2 <i>Deterministic vs. Robust Solutions</i>	397
	16.8.3 <i>Constraint Trade-offs</i>	398
	16.8.4 <i>Final Design Choice</i>	398
	16.8.5 <i>Multiobjective Problems Under Uncertainty: Decision-Making Problem</i>	398
16.9	Other Popular Methods	399
	16.9.1 <i>Taguchi's Robust Design Methods</i>	399
	16.9.2 <i>Stochastic Programming</i>	399
16.10	Summary	399

16.11	Problems	400
	Bibliography of Chapter 16	402
<b>17</b>	<b>Methods for Pareto Frontier Generation/Representation</b> . . . . .	<b>406</b>
17.1	Overview	406
17.2	Mathematical Preliminaries	406
17.3	Normal Boundary Intersection Method	408
17.4	Normalized Normal Constraint Method	410
17.5	Pareto Filter	416
17.6	Examples	419
17.7	Summary	427
17.8	Problems	427
	Bibliography of Chapter 17	428
<b>18</b>	<b>Physical Programming for Multiobjective Optimization</b> . . . . .	<b>429</b>
18.1	Overview	429
18.2	Linear Physical Programming (LPP)	430
	18.2.1 <i>Classification of Preferences: Soft and Hard</i>	430
	18.2.2 <i>Ranges of Desirability for Various Classes</i>	430
	18.2.3 <i>Inter-Criteria Preferences: OVO Rule</i>	432
	18.2.4 <i>LPP Class Function Definition</i>	433
	18.2.5 <i>LPP Weight Algorithm</i>	435
	18.2.6 <i>LPP Problem Formulation</i>	435
18.3	Nonlinear Physical Programming (NPP)	436
	18.3.1 <i>LPP vs. NPP</i>	436
	18.3.2 <i>NPP Class Function Definition</i>	437
	18.3.3 <i>NPP Problem Model</i>	437
18.4	Comparison of LPP with Goal Programming	439
18.5	Numerical Example	439
	18.5.1 <i>Goal Programming Solution</i>	442
	18.5.2 <i>Linear Physical Programming Solution</i>	442
18.6	Summary	443
18.7	Problems	444
	Bibliography of Chapter 18	444
<b>19</b>	<b>Evolutionary Algorithms</b> . . . . .	<b>445</b>
19.1	Overview	445
19.2	Genetic Algorithms	446
	19.2.1 <i>Basics of Genetic Algorithms</i>	446
	19.2.2 <i>Options in MATLAB</i>	449
19.3	Multiobjective Optimization Using Genetic Algorithms	451
	19.3.1 <i>Example</i>	451
19.4	Other Evolutionary Algorithms	453
	19.4.1 <i>Ant Colony Optimization</i>	454
	19.4.2 <i>Simulated Annealing</i>	454

Cambridge University Press

978-1-107-10918-6 - Optimization in Practice with MATLAB for Engineering Students and Professionals

Achille Messac

Frontmatter

[More information](#)

<i>Contents</i>	xv
19.4.3 <i>Tabu Search</i>	454
19.4.4 <i>Particle Swarm Optimization (PSO)</i>	455
19.5 Summary	455
19.6 Problems	455
Bibliography of Chapter 19	459
<i>Author Index</i>	461
<i>Subject Index</i>	465

Cambridge University Press

978-1-107-10918-6 - Optimization in Practice with MATLAB for Engineering Students and Professionals

Achille Messac

Frontmatter

[More information](#)

---



## List of Figures

1.1	MATLAB Desktop	<i>page</i> 9
1.2	MATLAB Command Window	10
1.3	MATLAB Command History	10
1.4	MATLAB Current Directory	11
1.5	MATLAB Workspace Browser	11
1.6	MATLAB Variable Editor	12
1.7	MATLAB Editor or Debugger	13
1.8	MATLAB Help Browser	13
1.9	Single and Multiple Plots	25
1.10	Printing and Saving	26
1.11	Contour and Scatter Plots	27
1.12	Function and Mesh Plots	28
1.13	Plot Editing Mode	29
2.1	Equation of a Plane	46
2.2	Type of Functions	56
2.3	Monotonic Functions	57
2.4	Unimodal and Multimodal Functions	58
2.5	Convex and Concave Functions	58
2.6	Local and Global Minima	63
3.1	Motivation for Optimization: Creating New Possibilities	74
3.2	Traditional vs. Optimal Design Process	75
3.3	Generic Car Optimization	76
3.4	Evolutionary and Revolutionary Transformation of the Airplane	77
4.1	Multi-stage Design Process	84
4.2	Relationship Between Design, Analysis and Optimization (A Representative Example)	86

5.1	Classification of Optimization Methods	94
5.2	Numerical Illustration of Optimization	100
5.3	Graphical Illustration of Optimization	102
5.4	Optimization Flow Diagram	104
5.5	Nonlinear Optimization	106
5.6	Linear Optimization	109
5.7	Structure of Water Tank	117
5.8	Input and Output for the Function <code>wtower.m</code>	118
5.9	Schematic of Cantilever Beam	118
6.1	A Simple Beam Design Problem	124
6.2	Multiobjective Optimization	125
6.3	Pareto Frontier	127
6.4	Weighted Sum Method	129
6.5	The Plane of Weights for a Three-Objective Optimization Problem	130
6.6	Weighted Sum for Non-convex Pareto Frontier	131
6.7	Compromise Programming	132
6.8	Generating the Pareto Frontier Using MATLAB	134
6.9	Pareto Frontier for the Example	135
6.10	Expressing Preference Using Physical Programming	137
6.11	Problem 6.3—Water Tower	139
6.12	Golinski Speed Reducer	144
6.13	Sandwich Beam Designed with Vibrating Motor	146
6.14	Schematic of Simple Two-Bar Truss	149
6.15	Pressure Vessel	150
6.16	Proposed Schematic of Active Window	152
6.17	Ten-Bar Truss	154
7.1	Optimization Flow Diagram—with Scaling	164
7.2	Graphical Representation of Finite Difference Approximation	178
7.3	Finite Difference—Number of Digits of Accuracy	182
7.4	Automatic Differentiation vs. Finite Difference Approximation	184
7.5	Input and Output for the Program <code>umotor.m</code>	189
7.6	Pareto Frontier	191
7.7	Scaling of Design Variables	192
7.8	Schematic of Two-Bar Truss	193
7.9	Tubular Column	194
7.10	Schematic of Simply Supported I-Beam	196
7.11	Schematic of Cantilever Beam	197
8.1	Unimodal and Multimodal Objective Functions	201
8.2	The 3D Surface Plot of a Multimodal Function	201
8.3	Procedure for a Genetic Algorithm	206

<i>List of Figures</i>	xix
8.4 MATLAB Genetic Algorithm Solver from the Global Optimization Toolbox	209
9.1 The Actual Optimum Solution	217
9.2 Branch and Bound Method Flowchart	219
9.3 Branch and Bound Method Example	220
10.1 Ten-Bar Truss	224
10.2 Black Box	224
10.3 Tolerance Design Example	226
10.4 Normal Distribution	227
10.5 Pareto Frontier for the Tolerance Allocation Problem	229
10.6 An Aircraft in a Ground Turn	230
10.7 Aircraft Side Area and Its Centroid	231
10.8 Front View of the Aircraft Structure Modeled as a Beam with Two Simple Supports	231
10.9 Data Fitting Through Three Points	233
10.10 Least Squares Data Fitting Through Four Points	234
10.11 Schematic of TE Window	236
10.12 Electric Network	237
10.13 Heat Exchanger Network Design Problem	243
11.1 Types of LP Solutions	254
11.2 Simplex Algorithm	265
11.3 Simplex Method Example	267
11.4 Sensitivity of Optimum to Slope of Linear Objective Function	275
12.1 Interval Updates of the Bisection Method	281
12.2 Plot of the Objective Function	282
12.3 Line Segments Divided According to the Golden Ratio	283
12.4 Interval Updates of the Golden Section Search	284
12.5 Reflection of the Vertex with the Highest Function Value	289
12.6 Conjugate Directions for a Diagonal Matrix A	296
12.7 Conjugate Directions for a Non-diagonal Matrix A	296
13.1 An Example of a Constrained Nonlinear Optimization Problem	311
13.2 Parabolic Penalty	314
13.3 Generic Penalty Function for Double-Sided Boundaries	314
13.4 The LICQ Does Not Hold	320
13.5 Using the KKT Conditions to Solve an Optimization Problem	321
14.1 Discrete Optimization Overview	336
14.2 Graphical Solution for Example 1	338

14.3	Branch and Bound Method Example: Integer Bounds on $x_1$	344
14.4	Branch and Bound Method Example: Integer Bounds on $x_2$	345
15.1	A Column Subject to Buckling	358
15.2	Rectangular Cross Section	358
15.3	A 3-Level and 3-Dimensional Full Factorial Design (27 Points)	360
15.4	Latin Hypercube Sampling (LHS) with 10 Points	361
15.5	A LHS for a 2-Variable Problem, Generated Using MATLAB	362
15.6	Applications of Surrogate Modeling	363
15.7	Fitted Function	370
15.8	A Generic Topology of Neural Networks	372
15.9	Cantilever Beam Subject to a Tip Force	373
15.10	Rectangular Cross Section	373
16.1	Two-Bar Truss Example	377
16.2	Overview of Design Optimization Under Uncertainty	379
16.3	Normal Distribution	383
16.4	Finding the Underlying Distribution of the Given Cross-Sectional Area Data Set	384
16.5	Uncertainty Problems in Engineering Design	390
16.6	Inequality Constraints: Deterministic vs. Robust	394
16.7	Formulation for Type II Constraints	394
16.8	Two-Bar Truss Results	396
17.1	General Design Objective Space for a Bi-Objective Case	408
17.2	Translated Design Objective Space for a Bi-Objective Case	409
17.3	Normalized Design Objective Space for a Bi-Objective Case	410
17.4	A Set of Evenly Spaced Points on the Utopia Line for a Bi-Objective Problem	411
17.5	Graphical Representation of the Normalized Normal Constraint Method for Bi-Objective Problems	411
17.6	Utopia Hyperplane for a Three-Objective Case	414
17.7	Evenly Spaced Points on the Utopia Plane for a Three-Objective Case	416
17.8	Normal Constraint Generates a Non-Pareto Solution Under a Contrived Feasible Region	417
17.9	Pareto Filter	418
17.10	Flow Diagram of Pareto Filter	418
17.11	Pareto Frontier Using Normalized Normal Constraint	421
17.12	Pareto Frontier Generated Using Normal Boundary Intersection and Normalized Normal Constraint	424
17.13	Three-Bar Truss Under Static Loading	428
18.1	LPP Ranges of Preferences for Soft Classes	431

<i>List of Figures</i>	xxi
18.2 NPP Ranges of Preferences for Soft Classes	438
18.3 GP vs. LPP—Comparison	440
18.4 GP vs. LPP—AOF Visualization	441
18.5 GP vs. LPP—Example Results	443
19.1 Basics of Genetic Algorithms	447
19.2 The MATLAB Genetic Algorithm and Direct Search Toolbox	452
19.3 Pareto Front for Multiobjective Genetic Algorithm Example	453
19.4 Sandwich Beam Designed with Vibrating Motor	456
19.5 Traveling Salesman Problem: Locations of Cities	458

Cambridge University Press

978-1-107-10918-6 - Optimization in Practice with MATLAB for Engineering Students and Professionals

Achille Messac

Frontmatter

[More information](#)

---

## List of Tables

1.1	Arithmetic Operators (help ops)	page 37
1.2	Logical Operators (help ops)	37
1.3	Special Characters (help ops)	37
1.4	Program Control Flow Constructs (help lang)	38
1.5	Scripts, Functions and Variables (help lang)	38
1.6	Argument Handling (help lang)	38
1.7	Message Display (help lang)	38
1.8	Elementary Matrices (help elmat)	39
1.9	Basic Array/Matric Information (help elmat)	39
1.10	Special Variables and Constants (help elmat)	39
1.11	Trigonometric Functions (help elfun)	40
1.12	Exponential and Complex Functions (help elfun)	40
1.13	Rounding and Remainder (help elfun)	41
1.14	Specialized Math Functions (help specfun)	41
1.15	Matrix Analysis (help matfun)	41
1.16	Basic Statistical Operations (help datafun)	41
1.17	Optimization Functions	42
1.18	Equation Solving Functions	42
1.19	Least Squares (Curve Fitting)	42
5.1	Broad Classification of Software for Optimization	103
6.1	Problem 6.4—Results	141
6.2	Problem 6.6—Results	142
6.3	Constants and Design Variable Constraints	148
6.4	Geometrical Properties for the Different Thermoelectric Units	153
7.1	Ill-Conditioned Matrices and Algorithms ( $\  M \  = \max_{ij} \{   m_{ij}   \}$ )	162
7.2	Numerical Accuracy Definitions	166
7.3	Example of Finite Difference Accuracy ( $f(x) = 4x^4 + 2x^3 + 1/x$ )	181

xxiv	<i>List of Tables</i>
7.4	Constraints for Universal Motor 190
7.5	Optimum Design Results for Different Objective Weights 197
8.1	Design Variable Combinations and Their Objective Function Values 202
8.2	Local Optima for 11 Starting Points 204
9.1	Design Variable Combinations and Their Objective Function Values 215
10.1	Loading Conditions 224
10.2	Material Properties 235
10.3	Ranges of Desirability for the Objectives 244
11.1	Initial Simplex Tableau 267
11.2	Simplex Method Example: Identifying the Pivotal Column 269
11.3	Simplex Method Example: Identifying the Pivotal Row 270
11.4	Simplex Method Example: Tableau in Canonical Form 271
11.5	Simplex Method Example: The Second Iteration 271
11.6	Simplex Method Example: Final Simplex Tableau 271
12.1	The End Points and the Derivatives at Each Midpoint 282
12.2	Endpoints, Golden Section Points and Function Values at Each Iteration 285
12.3	The Iterations for the Quadratic Approximation Method 287
12.4	The Iterations for the Conjugate Gradient Method 298
12.5	Comparison of Optimization Methods for Unconstrained Nonlinear Problems 305
12.6	Results Summary for Given Search Methods 308
12.7	Results Summary for Given Search Methods 308
13.1	Optimal Results for Different $R$ Values 316
13.2	Optimal Results for Different $R$ Values 317
14.1	Brute Force Method: Six Elements 340
14.2	Brute Force Method: Twenty Combinations 340
14.3	Initial Simplex Tableau Before Adding Cutting Planes 348
14.4	Final Simplex Tableau Before Adding Cutting Planes 348
14.5	Initial Simplex Tableau After Adding First Cutting Plane (Eq. 14.55) 349
14.6	Final Simplex Tableau After Adding the First Cutting Plane (Eq. 14.55) 350
14.7	Initial Simplex Tableau After Adding Second Cutting Plane (Eq. 14.62) 350
14.8	Final Simplex Tableau After Adding Second Cutting Plane (Eq. 14.62) 351
14.9	Broad Classification of Software for Optimization—with Discrete Case 352
16.1	Sample Set of One Hundred Cross-Sectional Areas for a Batch-Produced Truss Bar (in <sup>2</sup> ) 381
18.1	Preference Ranges for $\mu_1$ and $\mu_2$ 440



## Preface

### Contacting the Author Regarding this Book

I am delighted that you are using this book in your study of, or involvement with, optimization. I would very much welcome your comments, particularly regarding this first edition, and any suggestions that you might have for the next edition. For your comments regarding this book, I will be happy to receive your direct email at [OptimizationInPracticeMessac@google.com](mailto:OptimizationInPracticeMessac@google.com).

### Book Website

The website [www.cambridge.org/Messac](http://www.cambridge.org/Messac) will be maintained for this book. Information for instructors and students will be separately provided. Software for various problems will be provided in this website. I expect it to be a dynamic website, where the information available will evolve over time to be responsive to readers' requests and feedback.

### Book Organization

This book is intended to be used by undergraduate and graduate students in the classroom, or by industry practitioners learning independently. Its organization suits these objectives. Following are messages specifically tailored for students, for industry practitioners and for instructors. The book has five parts:

Part I *Helpful Preliminaries*

Part II *Using Optimization—The Road Map*

Part III *Using Optimization—Practical Essentials*

Part IV *Going Deeper: Inside the Codes and Theoretical Aspects*

Part V *More Advanced Topics in Optimization*

**Part I** has two chapters that present prerequisite material explaining how to use MATLAB and some useful mathematical information. Most of this book assumes knowledge of undergraduate calculus and elementary linear algebra. The second

chapter provides a brief review of the math needed. In **Part II**, three chapters introduce the world of optimization in the form of a road map. This part provides the basics of what should be known about optimization before attempting to use it. In **Part III**, there are five chapters that teach the basic use of optimization. In fact, learning the material up to Part III provides the practitioner with sufficient information for solving practical problems. In doing so, student users will not be experts on how optimization *works under the hood*, so to speak, but will have the ability to use optimization in general practical contexts. Stated differently, while not being an expert about what is *under the hood*, the student will be a pretty good *driver* and should be able to *drive* to the desired destination (*i.e.*, the optimal design). In **Part IV**, three chapters provide a meaningful understanding of the computational and theoretical optimization process for linear programming and nonlinear programming with and without constraints. This is equivalent to learning the basics of what is **under the hood** of a car. **Part V** builds on the first parts of the book to provide a foundation for more advanced studies in optimization. In Part V, we learn optimization at a deeper level, where advanced topics are introduced over six chapters.

### A Message to Students

As this book reaches your hands, you may have already signed up to take an optimization course, you may be in your junior or senior year or a graduate student, or you may already be in industry and feel that optimization may be useful to you. Regardless of what the case may be, you probably have two important questions in your mind: (i) Why should I learn optimization? and (ii) Is this a good book for me to use to learn optimization? Let me provide you with some objective comments, together with some subjective thoughts.

Why should you learn optimization? The truth is this. If you are an engineer or about to become one, if you are a financial analyst, or if you deal with numbers to determine how desirable or undesirable the performance of a system or a design is, then optimization will almost certainly be able to help you do a better job—without fail. Optimization applies to most engineering activities, management operations activities and numerous other fields where performance (or *goodness*) can be numerically quantified.

More importantly, if you are trying to design a system that must perform a certain way, you have unsuccessfully tried all that you know, and you have already asked the experts for help without success, then there is a good chance that optimization will help you succeed. Interestingly, it would not be at all surprising that the expert may find it too difficult to obtain an adequate answer, when optimization can be successfully used to find one. This is, in part, because optimization can intelligently examine thousands of design alternatives in less time than it takes the expert to examine a single design alternative. Optimization can also perform an intelligent search in a complex environment that may not be clear to the human mind. Optimization essentially makes it obsolete to engage in the typical trial-and-error process, as we search for a good design.

The following are some important comments regarding the rapidly growing popularity of optimization in recent years, and about how this book offers a unique approach to bringing optimization to a broader audience.

Until recently, *the cost of computing* was a critical issue that hindered the broad application of optimization. Fortunately, with the revolutionary decrease in the cost of computing in recent years, a desktop computer is often all that is needed to solve many practical optimization problems—making the application of optimization dramatically more practical.

The application of optimization in practical settings is increasingly becoming commonplace. A growing number of software developers have begun to include optimization capabilities as they respond to growing demand. They realize that it is not sufficient to determine how a structure deforms under given loads. It is also important, and of interest, to *discover* how to change the design in order to reduce that deformation. Optimization provides a reliable and systematic way to obtain this reduction. Importantly, this powerful benefit of optimization applies to a plethora of analysis software in engineering, management, finance and numerous other fields.

Let us now turn our attention to the second question: Is this a good book for you to use to learn optimization? This book will provide you with a unique combination of desirable attributes. It will provide you with the knowledge to start using optimization software in general. In particular, you will have the ability to use MATLAB for this purpose with effectiveness and efficiency. *Note that any other optimization software could have been used, but we use MATLAB for its broad popularity, effectiveness and convenience.*

This book focuses primarily on the material that is required for the practical application of optimization: more time spent on the computational analysis and optimization of real-world problems, and less time on the inner theory and mathematics of optimization. This approach is beneficial to students who wish to initiate research studies in optimization, as well as to those students who primarily wish to use optimization for practical purposes. This book is explicitly intended to teach you *how to use optimization successfully*, and to do so while avoiding unnecessary mathematics. The first parts of this book can be used for a one-semester course that is fully accessible to junior-level undergraduate students. Those interested in advanced topics are referred to Part V of this book.

To make this practical learning possible, after the preparatory material of Part I, realistic solution approaches that involve the use of a computer are provided in Part II. With our ability to use an optimization and modeling code, such as MATLAB, we will be able to readily apply what we learn to real-life problems. The approaches do not fundamentally change with different optimization codes. The changes that occur from problem to problem are readily handled with any optimization code, such as the size and other generic features of the problem. MATLAB is an easy-to-use and very popular software that is useful in all areas of engineering. It is also used in a growing number of non-engineering fields. If you don't yet know MATLAB, that is fine. You will learn it as we go along. The first chapter of this book provides an introduction to MATLAB that focuses on the material that we will need.

Your prospective learning of optimization in a way that focuses on practical applications is timely in view of the increasingly computational world. It will prove to be an important component of your education. Ultimately, learning optimization will provide you with a truly powerful tool to do things more successfully than could be done without optimization.

### **A Message to Industry Practitioners**

To obtain an overall idea about the objectives and intent of this book, I suggest that you also read the messages to students and instructors. Regarding the particular needs of industry practitioners, this book is deliberately designed to quickly get to the point. Many software products are coming to market with optimization capabilities, and it is wise to acquire the appropriate background to start using these capabilities. This book directly provides the required knowledge in a way that is unencumbered by unnecessary math, and focuses on such practical aspects as: (i) How do I make my design lighter, stronger and cheaper? (ii) Once I get an answer from my analysis code, what is the next step to improving that answer?

In addition, even if you are not using an analysis software (but can compute the performance of your system), you may ask how to modify your system to improve its performance, and how to do so systematically without the usual manual trial and error. Optimization is a powerful way to accomplish these objectives, and this book provides the required knowledge in a practical and accessible way. As mentioned earlier, those interested in more advanced topics are referred to Part V of this book.

### **A Message to Instructors**

This book takes a novel pedagogical approach to the teaching of optimization. It takes a different perspective regarding what material should be included in a first optimization course; and it provides the means to teach optimization to juniors, seniors, and graduate students who are taking their first course in optimization. The material in this book can be divided into two self-contained one-semester courses. The first course can be offered at the undergraduate or graduate level. At the undergraduate level, Parts I, II, and III could be covered. At the graduate level, (i) Parts II, III, and IV could be nominally covered, (ii) Part II can be covered quickly, (iii) Part V can be covered as part of a challenging first course or as part of a second course, and (iv) a term project could be assigned potentially based on some advanced problems in the book. The problems at the end of each chapter are divided into “Warm-up,” “Intermediate,” and “Advanced,” reflecting their respective levels of difficulty. Some “Graduate Level” problems are provided as well. This flexibility can be exploited to address the diverse skills of students cohorts and of undergraduate/graduate students. The book is structured to provide full flexibility to accommodate the objectives of the instructor, as well as the skills and interests of the students.

Importantly, this book is also structured with the instructor in mind to facilitate the pedagogical process. In particular, the book's website ([www.cambridge.org/Messac](http://www.cambridge.org/Messac)) provides a comprehensive set of materials that support the instructor's needs. A 270-page solution manual is provided. A comprehensive set of lecture materials is provided *in editable form*, which comprises approximately 850 PowerPoint slides, thereby providing the flexibility to suit the instructor's goals and pedagogical style. The book's website is also provided to enrich the coverage of certain topics and to allow us to pose larger practical problems that might not otherwise be considered. The book MATLAB codes are also available in the book's website. The instructor is invited to visit the website and to contact me at [OptimizationInPracticeMessac@google.com](mailto:OptimizationInPracticeMessac@google.com) for any suggestions for the website or any other aspects of the book.

This book provides an introduction to the practical application of optimization as a potentially last optimization course. Alternatively, this book can be used as a more accessible introduction to the subject of optimization, to be followed by courses that cover more advanced topics.

Regarding the philosophy of the material covered in this book, we critically assess the need to include candidate topics by asking the question: Is this material needed for someone who primarily wishes to *use* optimization, or is it primarily required for someone who intends to *develop* optimization algorithms? Priority is given to topics that directly contribute to the successful *use* of optimization. For example, is it necessary for an introductory user to learn sequential quadratic programming? Is this knowledge necessary for the successful application of optimization, when robust implementations are broadly available? The book is structured to easily allow the instructor to potentially leave that topic to a second optimization course (using this book), where more theoretically advanced subjects can be presented. This singular philosophy helped prioritize the candidate topics for inclusion (*and where*) in the book. The net result is a text that appreciably departs from tradition, but that we believe makes a novel contribution to the teaching of optimization. It provides the material to teach optimization in the traditional way, but with the flexibility/option to employ a more pragmatic approach that may be more inviting and effective in an introductory undergraduate or graduate first or second course.

A top-level view of the five parts of the book is provided as follows:

**Part I** *Helpful Preliminaries*

1. MATLAB as a Computation Tool
2. Mathematical Preliminaries

**Part II** *Using Optimization - The Road Map*

3. Welcome to the Fascinating World of Optimization
4. Analysis, Design, Optimization, and Modeling
5. Introducing Linear and Nonlinear Programming

**Part III** *Using Optimization - Practical Essentials*

6. Multiobjective Optimization
7. Numerical Essentials
8. Global Optimization Basics
9. Discrete Optimization Basics
10. Practicing Optimization – Larger Examples

**Part IV** *Going Deeper: Inside the Codes and Theoretical Aspects*

11. Linear Programming
12. Nonlinear Programming with No Constraints
13. Nonlinear Programming with Constraints

**Part V** *More Advanced Topics in Optimization*

14. Discrete Optimization
15. Modeling Complex Systems: Surrogate Modeling and Design Space Reduction
16. Design Optimization Under Uncertainty
17. Methods for Pareto Frontier Generation/Representation
18. Physical Programming for Multiobjective Optimization
19. Evolutionary Algorithms