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During the past two decades there has been active interplay between geometric measure theory and Fourier analysis. This book describes part of that development, concentrating on the relationship between the Fourier transform and Hausdorff dimension.

The main topics concern applications of the Fourier transform to geometric problems involving Hausdorff dimension, such as Marstrand type projection theorems and Falconer's distance set problem, and the role of Hausdorff dimension in modern Fourier analysis, especially in Kakeya methods and Fourier restriction phenomena. The discussion includes both classical results and recent developments in the area. The author emphasizes partial results of important open problems, for example, Falconer's distance set conjecture, the Kakeya conjecture and the Fourier restriction conjecture. Essentially self-contained, this book is suitable for graduate students and researchers in mathematics.

Pertti Mattila is Professor of mathematics at the University of Helsinki and an expert in geometric measure theory. He has authored the book *Geometry of Sets and Measures in Euclidean Spaces* as well as more than 80 other scientific publications.

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Fourier Analysis and Hausdorff Dimension

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To John Marstrand

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Preface

This is a book on geometric measure theory and Fourier analysis. The main purpose is to present several topics where these areas meet including some of the very active recent interplay between them. We shall essentially restrict ourselves to questions involving the Fourier transform and Hausdorff dimension leaving many other aspects aside.

The book is intended for graduate students and researchers in mathematics. The prerequisites for reading it are basic real analysis and measure theory. Familiarity with Hausdorff measures and dimension and with Fourier analysis is certainly useful, but all that is needed will be presented in Chapters 2 and 3. Although most of the material has not appeared in book form, there is overlap with several earlier books. In particular, Mattila [1995] covers part of Chapters 4–7, Wolff [2003] of Chapters 14, 19, 20 and 22, and Stein [1993] of 14 and 19–21. Several other overlaps are mentioned in the text. The surveys Iosevich [2001], Łaba [2008], [2014], Mattila [2004], Mitsis [2003a] and Tao [2001], [2004] are closely related to the themes of the book.

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This book grew out of several graduate courses I have taught at the Department of Mathematics and Statistics of the University of Helsinki. I am grateful to the department for excellent facilities and for the students and post docs attending the courses for inspiration and for their comments and questions. For financial support I am also indebted to the Academy of Finland. Many mathematicians have been of great help in the preparation of this book. In particular, my special thanks are due to Vasilis Chousionis, who helped me in many aspects, and to Kenneth Falconer, who read large parts of the manuscript and made a great number of valuable comments on mathematics, style and language. Terence Tao's blogs and lecture notes have been very useful as well as Ana Vargas's master's thesis, which she kindly sent to me. For many comments and corrections I would like to thank Anthony Carbery, Marton Elekes, Burak Erdoğan, Risto Hovila, Tuomas Hytönen, Alex Iosevich, Tamás Keleti, Sangyuk Lee, José María Martell, Tuomas Orponen, Keith Rogers, Tuomas Sahlsten, Andreas Seeger, Pablo Shmerkin, Ville Suomala, Terence Tao, Ana Vargas and Laura Venieri. Finally I am much obliged to Jon Billam, Clare Dennison, Samuel Harrison and others at the Cambridge University Press for their help and for accepting the book for publication.