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# Optimization for Chemical and Biochemical Engineering

# Theory, Algorithms, Modeling and Applications

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# Notation

AFM	atomic force microscope
AKPZ	anisotropic KPZ equation
$a_0$	lattice constant
$c_q(\ell)$	q-th order correlation function
$d_{ m E}$	embedding dimension
$d_f$	fractal dimension
Ľ	system size
≡	<i>defined</i> to be equal
$\sim$	<i>asymptotically</i> equal (in scaling sense)
$\approx$	approximately equal (in numerical value)

### Preface

This book is the result of a decade of teaching the Masters course "Optimization" at the University of Cambridge, during which period a lot of material has been collected and taught. The philosophy behind the lecture notes and now this book is that teaching and research are strongly connected and should never be separated. As such, upcoming research topics in literature have to be included always, especially in such a fast-moving area of research as the one dealt with in this book.

In addition to the standard lecture notes of the Masters course, a deeper consideration of optimal control, global optimization, optimization under uncertainty, multiobjective optimization, mixed-integer programming and model predictive control are included here, which are the areas of increased interest in recent years. The mathematics of the topics covered in this book can be complex in their own right, but it is attempted to give enough information to be applied to chemical engineering problems.

The use of optimization techniques in chemical engineering has a long history, with a profound impact by Professor Roger Sargent. Since the inception of Process Systems Engineering (PSE) by Professor Roger Sargent, the field has expanded to areas outside classical chemical engineering, e.g. biotechnology. This is possible due to the flexible nature with which the concepts of PSE can be used for problems with real-world application. This book tries to teach these fundamental concepts.

The preparation of this book took longer than initially planned, but nevertheless it is with great pleasure that we can now share what we think is an exciting and extremely interesting area of research, for industry and academia. Each chapter presenting new concepts and ideas is followed by exercises for the reader to test the understanding. None of the topics shown in this book can be covered in extensive detail, as this would exceed the scope. As such, further reading recommendations are given to guide the reader. In addition, the material covered is backed with references from recent literature to put into context how the concepts presented in this book are used in "the real world."