

TOPOLOGICAL PHASES OF MATTER

Topological phases of matter are an exceptionally dynamic field of research: several of the most exciting recent experimental discoveries and conceptual advances in modern physics have their origins in this field. These have generated new – topological – notions of order, interactions, and excitations. This text provides an accessible, unified, and comprehensive introduction to the phenomena surrounding topological matter, with detailed expositions of the underlying theoretical tools and conceptual framework, alongside accounts of the central experimental breakthroughs. Among the systems covered are topological insulators, magnets, semimetals, and superconductors. The emergence of new particles with remarkable properties such as fractional charge and statistics is discussed alongside possible applications such as fault-tolerant topological quantum computing. Suitable as a textbook for graduate or advanced undergraduate students, or as a reference for more experienced researchers, the book assumes little prior background, providing self-contained introductions to topics as varied as phase transitions, superconductivity, and localization.

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Roderich Moessner , Joel E. Moore
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Contents

<i>List of Tables</i>	<i>page</i> viii
<i>List of Boxes</i>	ix
<i>Preface</i>	xi
<i>Acknowledgments</i>	xiii
1 Introduction	1
2 Basic Concepts of Topology and Condensed Matter	11
2.1 Berry Phases in Quantum Mechanics	12
2.2 One Electron in a Magnetic Field: Landau Levels	20
2.3 One Electron in a Crystal: Bloch's Theorem	24
2.4 The Simplest Tight-Binding Model	26
2.5 Dirac Band Structure of the Honeycomb Lattice	29
2.6 Landau Theory of Symmetry-Breaking Phases	32
2.7 Two Mathematical Approaches to Topology	40
2.8 Topological Defects in Symmetry-Breaking Phases	50
3 Integer Topological Phases: The Integer Quantum Hall Effect and Topological Insulators	58
3.1 IQHE: Basic Phenomena and Theory	60
3.2 Two Lattice Models of the IQHE, and Chern Number	69
3.3 Time-Reversal Symmetry in Classical and Quantum Physics	74
3.4 Topological Insulators in 2D: Basic Phenomena and Theory	76
3.5 A Lattice Model of the 2D Topological Insulator	82
3.6 3D Topological Insulators: Basic Phenomena	84
3.7 Skyrmions in the Quantum Hall Effect	88

vi	<i>Contents</i>	
4	Geometry and Topology of Wavefunctions in Crystals	95
4.1	Inversion Symmetry, Electrical Polarization, and Thouless Pumping	97
4.2	The Integer Quantum Hall Effect and Topological Invariants of Energy Bands	104
4.3	Many-Particle Interpretation of Topological Invariants	107
4.4	Time-Reversal Invariance and \mathbb{Z}_2 Invariants	109
4.5	Axion Electrodynamics, Non-Abelian Berry Phase, and Magnetoelectric Polarizability	118
5	Hydrogen Atoms for Fractionalization	124
5.1	The Fractional Quantum Hall Effect	126
5.2	Fractionalization, Order, and Topology in $d = 1$	145
5.3	The Resonating Valence Bond Liquid	156
5.4	Spin Ice	167
6	Gauge and Topological Field Theories	179
6.1	Pure Ising Gauge Theory and Absence of Local Order	181
6.2	Ising Gauge Theory with Matter	187
6.3	Kitaev's Toric Code	192
6.4	Maxwell Electromagnetism	194
6.5	Tensor Gauge Theories and Fractons	198
6.6	Long-Wavelength and Topological Field Theories	201
6.7	Mutual Statistics and the Quantum Hall Hierarchy	214
6.8	<i>BF</i> Theory	215
7	Topology in Gapless Matter	218
7.1	Geometric Quantities in the Semiclassical Theory of Metals	220
7.2	Dirac and Weyl Semimetals	225
7.3	Electromagnetic Response of Topological Semimetals	229
7.4	Kitaev Honeycomb Model	233
8	Disorder and Defects in Topological Phases	239
8.1	Introduction to Disorder and Localization	241
8.2	A Semiclassical Model of Quantum Hall Transitions	247
8.3	Adding Quantum Mechanics: Network Models	251
8.4	Basic Ideas of Random Matrix Theory and the Tenfold Way	253
8.5	Vortices in Conventional Superconductors	260

Contents

vii

8.6	Flux and Crystalline Defects in Integer Topological Phases	266
8.7	Vortices in Quantum Hall States and Composite Fermions	268
8.8	Spin Liquids and Disorder	271
9	Topological Quantum Computation via Non-Abelian Statistics	285
9.1	Quantum Computation: Universality and Complexity	286
9.2	Error Correction versus Fault-Tolerance	289
9.3	Nonlocal Operations for Quantum Computing	292
9.4	Majoranas in One Dimension: The Kitaev Chain	297
9.5	Majoranas in Two Dimensions	301
9.6	Universal Computation and the Read–Rezayi States	310
9.7	Experimental Implementations of Majorana Modes	311
10	Topology out of Equilibrium	316
10.1	Time-Dependent and Time-Periodic (Floquet) Hamiltonians	317
10.2	Floquet Basics	318
10.3	Floquet Topological Insulators	324
10.4	Anomalous Floquet–Anderson Insulator	326
10.5	Driven Kitaev Chain and π -Majorana Fermions	329
10.6	Many-Body Floquet Discrete Time Crystal	334
11	Symmetry, Topology, and Information	340
11.1	Symmetry-Protected Topological Phases	341
11.2	Entanglement Entropy in Topological States	349
11.3	The Universe of Topological Materials; Closing Remarks	352
	<i>Appendix: Useful Sources, Quantities, and Equations</i>	355
	<i>References</i>	358
	<i>Index</i>	371

Tables

4.1	Comparison of Berry phase theories of polarization and magnetoelectric polarizability	<i>page</i> 121
8.1	Ten symmetry classes of free-fermion Hamiltonians in dimensions 1–4 and their topological possibilities	259
A.1	Some useful quantities and equations	357

Boxes

2.1	The Berry Phase of the Adiabatic Dynamics of a Spin	<i>page</i> 17
2.2	Topology from Geometry: The Gauss–Bonnet Theorem	40
2.3	The Berezinskii–Kosterlitz–Thouless Transition	53
3.1	One Particle on a Ring Pierced by Magnetic Flux	62
3.2	Modulation Doping	67
4.1	Tight-Binding Chain with Two Orbitals per Unit Cell	102
4.2	The Wess–Zumino–Witten Model	110
5.1	Single-Mode Approximation	134
5.2	Fractional Statistics of Particles in Two Dimensions	140
5.3	Fractional Quantum Numbers	146
5.4	Klein Models	151
5.5	Classical Dimer Models and Their Correlations	162
6.1	Quantum IGT in d Dimensions and Classical IGT in $d + 1$	182
6.2	Bound States of the Dirac Equation: Jackiw–Rebbi Model	202
7.1	Semiclassical Equilibrium	224
8.1	One-Parameter Scaling Approach to Anderson Localization	243
8.2	Bogoliubov–de Gennes Formalism of Superconductivity	254
8.3	The Josephson Effect and Gauge Invariance	263
9.1	The No-Cloning Theorem	290
9.2	What Is a Majorana Fermion or Zero Mode?	294
9.3	The Jordan–Wigner Transformation and Statistics in 1D	299
9.4	Solution and Phase Diagram of the Kitaev Honeycomb Model	306
10.1	Phase Structure in and out of Equilibrium	320

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Preface

Topological condensed matter physics presents an embarrassment of riches, both in the bewildering variety of phenomena that fall under this heading and in the sheer volume of publications in the field. In addition, the breadth of the field is reflected in a diversity of backgrounds of its practitioners: it is a place where immigrants from high-energy physics collaborate with materials chemists. This poses the twin challenges of selection and organization of material for a book.

Against this backdrop, the content of this book reflects our vision of the field, in particular what we feel might in the long run form part of the canon of many-body physics. We have tried to emphasize conceptual and historical milestones. We focus on topological phases in solids, rather than on similar phases of neutral atoms in either helium or ultra-low-density atomic gases, which couple very differently to electromagnetic fields, even though these have certainly contributed to central developments of the field.

We have limited ourselves to a relatively small number of references. Given that the material of the book corresponds to such a vast body of work, we would otherwise have ended up with an unpalatably long but necessarily still woefully incomplete list of references. We have tried to include scholarly review articles where hundreds more references can be found in a more structured fashion. We apologize to those objecting to this or any other aspect of the presentation for our shortcomings; they are warmly encouraged to contribute their own versions, as condensed matter physics does not have the literature it deserves.

Few are the book projects that are swiftly concluded, and ours is not one of them. Without the collapse of our travel schedules due to the present pandemic, perhaps this manuscript would still not be finished. Of course, had we completed the book more swiftly, we would have missed a number of exciting developments, such as the discoveries of Floquet time crystals and Weyl and Dirac semimetals. At any rate, we do not expect a letup of the sequence of discoveries any time soon.

Indeed, overall condensed matter physics continues to have a refreshingly unmodern flavor to it. Most of its discoveries are made by small collaborations of creative individuals, often entirely serendipitously – just think of the integer quantum Hall effect, the cuprate superconductors, the isolation of graphene, or the prediction of topological insulators. This is a far cry from purportedly goal-oriented large-scale research programs, the organization of which both of us are admittedly also guilty of. At the same time, it is undeniably true that the use of large-scale facilities like modern neutron and light sources has advanced the field tremendously.

The passage of time has also asserted itself in several other ways. Topological condensed matter physics has in the meantime been recognized by the 2016 Nobel Prize for Duncan Haldane, Michael Kosterlitz, and David Thouless. Neither was sad news in short supply, with Thouless passing away only a few months after the untimely death of Shoucheng Zhang. The passing of Phil Anderson in March 2020 concluded the extraordinary career of arguably the most influential scientist of the second half of the twentieth century.

Finally, it might be useful to explain our approach to pedagogy, since it is intended that this book will be useful also for courses of self-study. Our feeling is that an encyclopedic list of results without derivations is unlikely to help readers understand the material for themselves, while too detailed a presentation tends to obscure the underlying conceptual structure of the material. In compromise, we have tried to explain a moderate number of central results, with a key example where appropriate; most of these do not require a great deal of technical background or impose greatly on the reader's patience. We would like to think, of course, that the book can thus also be useful as an initial reference that can be consulted for the basics of a subject and as a source for more comprehensive reading.

For the nonexpert reader, although much interesting material has had to be omitted, what has been included may still be hard to navigate initially. To lower the entry bar, besides providing background in Chapter 2 to make the book reasonably self-contained, we have collated the most fundamental material in two chapters, which we recommend as an entry point. These are Chapter 3, on integer topological phases, and Chapter 5, on fractionalization.

Ideally, the book will be useful for active practitioners in the field as well as for newcomers, to whom we would like to extend a heartfelt welcome.

Acknowledgments

The embarrassment of riches of topological condensed matter physics mentioned in the preface is matched by the number and generosity of the people to whom we are indebted on our scientific journey to date. The first mention should be of those scientists who guided our steps during our scientific childhood and adolescence. Added to this are the collaborators with whom we began and sustained a research effort in the field.

In this spirit, J.E.M. thanks Duncan Haldane and Xiao-Gang Wen for explaining the beauty of topological order; Leon Balents and Cenke Xu for collaborations in the early days of topological materials; and Joseph Orenstein and David Vanderbilt for adding a dose of realism, in their distinct ways, later on. Similarly, R.M. is grateful to John Chalker and Shivaji Sondhi, especially for the early work on frustrated magnetism that was finding its place in the fabric of topological condensed matter physics as it was being woven. Since then, he has benefited greatly from the time and enthusiasm of many collaborators. Feeling distinctly uncomfortable singling out any individually, he would like to hide behind the Europhysics Prize Committee to signal his gratitude for satisfying theory–experiment collaborations like the one involving the discovery of emergent magnetic monopoles with Claudio Castelnovo and the groups of Alan Tennant and Santiago Grigera.

Both of us have had the privilege of watching junior scientists from our research groups develop into independent contributors to the field of topological physics. We particularly thank our students and postdocs, too many to list, not just for their collaboration on research but for making the job of professor a worthwhile one. Early versions of this material were inflicted upon students, and refined based on their questions, not just at our institutions but at a number of advanced schools. J.E.M. acknowledges Oxford University Press for permission to reuse material in Section 2.7, Chapter 4 and Section 6.6.1 from his Les Houches lecture notes (Chamon et al., 2017).

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R.M. dedicates this book to the memory of Philip W. Anderson. As the founding pioneer of much of condensed matter physics, his intellectual influence pervades this field, and this book, on many levels. We mourn the loss of an outstanding person who was a role model for many of us, not just as a scientist, and feel privileged to have known him.

J.E.M. dedicates this book to another native of Illinois, William Moore, for his contributions in other spheres, including as a father.