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978-1-107-10409-9 - An Introduction to the Theory of Reproducing Kernel Hilbert Spaces

Vern I. Paulsen and Mrinal Raghupathi

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