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AN INTRODUCTION TO THE THEORY OF REPRODUCING KERNEL HILBERT SPACES

Reproducing kernel Hilbert spaces have developed into an important tool in many areas, especially statistics and machine learning, and they play a valuable role in complex analysis, probability, group representation theory, and the theory of integral operators. This unique text offers a unified overview of the topic, providing detailed examples of applications, as well as covering the fundamental underlying theory, including chapters on interpolation and approximation, Cholesky and Schur operations on kernels, and vector-valued spaces. Self-contained and accessibly written, with exercises at the end of each chapter, this unrivaled treatment of the topic serves as an ideal introduction for graduate students across mathematics, computer science, and engineering, as well as a useful reference for researchers working in functional analysis or its applications.

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An Introduction to the Theory of Reproducing Kernel Hilbert Spaces

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Preface

This book grew out of notes for seminars and courses on reproducing kernel Hilbert spaces taught at the University of Houston beginning in the 1990s.

The study of reproducing kernel Hilbert spaces grew out of work on integral operators by J. Mercer in 1909 and S. Bergman's [4] work in complex analysis on various domains. It was an idea that quickly grew and found applications in many areas.

The basic theory of reproducing kernel Hilbert spaces (RKHS) goes back to the seminal paper of Aronszajn [2]. In his paper, Aronszajn laid out the fundamental results in the general theory of RKHS. Much of the early part of this book is an expansion of his work.

The fascination with the subject of RKHS stems from the intrinsic beauty of the field together with the remarkable number of areas in which they play a role. The theory of RKHS appears in complex analysis, group representation theory, metric embedding theory, statistics, probability, the study of integral operators, and many other areas of analysis. It is for this reason that in our book the theory is complemented by numerous and varied examples and applications.

In this book we attempt to present this beautiful theory to as wide an audience as possible. For this reason we have tried to keep much of the book as self-contained as we could. This led us to rewrite considerable parts of the theory, and experts will recognize that many proofs that appear here are novel.

Our book is composed of two parts.

In Part I we present the fundamental theory of RKHS. We have attempted to make the theory accessible to anyone with a basic knowledge of Hilbert spaces. However, many interesting examples require some background in complex variables and measure theory, so the reader might find that they can follow the theory, but then need further knowledge for the details of some examples.

In Part II we present a variety of applications of RKHS theory. We hope that these applications will be interesting to a broad group of readers, and for this reason we have again tried to make the presentation as accessible as possible. For example, our chapter on integral operators gives a proof of Mercer's theorem that assumes no prior knowledge of compact operators. Similarly, our presentation of stochastic processes assumes less probability and measure theory than is generally used. We do not go into great depth on any of these particular topics, but instead attempt to convey the essence of why the theory of RKHS plays a central role in each of these topics. We try to show the ways in which a knowledge of the RKHS theory provides a common theme that unifies our understanding of each of these areas.

Regrettably, one of the authors' favorite topics, Nevanlinna-Pick interpolation, is missing from Part II. The choice was deliberate. There is already an outstanding book on the subject by Agler and McCarthy [1] and a serious presentation of the Nevanlinna-Pick theory requires more function theory than we had previously assumed. Since RKHS theory is so broad, it is likely that the reader's favorite topic has been omitted.

This book can be used in three ways. First, it can be used as a textbook for a one-semester course in RKHS. Part I, together with some of the applications in Part II, has been the foundation of a one-semester graduate topics course at the University of Houston on several occasions and can easily be covered in that time. Second, this book can be used as a self-study guide for advanced undergraduates and beginning graduate students in mathematics, computer science and engineering. Third, the book can serve as a reference on the theory of RKHS for experts in the field.

A number of people have provided useful feedback on the book that has (we hope!) improved our exposition. In particular, we would like to thank Jennifer Good from the University of Iowa for carefully reading many parts of a nearly final draft of this book and providing detailed comments. In addition, we would like to thank our many students and colleagues at the University of Houston who sat through the earlier versions of this book and pointed out omissions and errors. Ultimately we, the authors, take responsibility for any errors or oversights that remain.

The authors would also like thank the institutions that provided support during the writing of this book, namely The University of Houston, The United States Naval Academy and Vanderbilt University.

Finally, none of this would have been possible without the patience of our wives and children. Special thanks go to our dining room table, which gave up its traditional role in life to serve as a book project center.