Problems

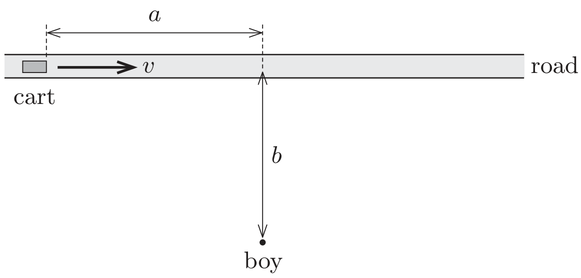
P1 The trajectories of two bodies moving with non-relativistic constant speeds are parallel in a particular inertial reference frame.

a) Is it possible to choose another inertial frame of reference in which the two trajectories cross each other?

b) If such a frame can be found, and the bodies are started with suitable initial conditions, then it could be arranged that they reach the crossing point at the same time. How can this be consistent with the parallel trajectories observed in the first frame of reference?

P2 Ann is sitting on the edge of a carousel that has a radius of 6 m and is rotating steadily. Bob is standing still on the ground at a point that is 12 m from the centre of the carousel. At a particular instant, Bob observes Ann moving directly towards him with a speed of 1 m s^{-1}. With what speed does Ann observe Bob to be moving at that same moment?

P3 A cart is moving on a straight road with constant velocity $\boldsymbol{v}$. A boy, standing in an adjoining meadow, spots the cart and hopes to get a ride on it. In which direction should he run to catch the cart? Solve the problem generally: denote the speed of the cart by v, the maximal speed of the boy by u, and take the initial positions of the cart and boy to be as shown in the figure.



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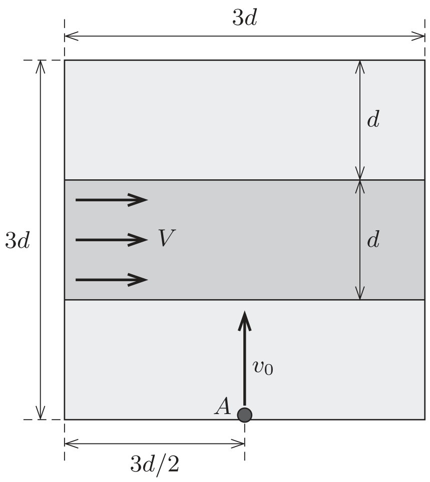
Excerpt
More Information

P4* A group of Alaskan gold prospectors reach a wide straight river that flows with uniform speed v. What immediately catches all their eyes is a huge gold nugget lying on the further bank, directly across the river. The laws governing prospecting in Alaska state that the first person to reach any particular place has the right to establish a mine there; speed is of the essence!

Joe, one of the prospectors, has a canoe, which he can paddle in still water at the same speed u as he can hike along a river bank. What course of action should he take if u/v is (a) smaller than or (b) larger than a certain critical value? Assume that Joe first paddles across the river (in a straight line) and then, if necessary, hikes along the bank to reach the nugget.

P5 The top surface of a horizontal laboratory table is a square of side $3d = 3$ m. Running centrally across the table, and parallel to one of its sides, is a conveyor belt consisting of an endless rubber band of width $d = 1$ m, which moves with a constant velocity $V = 3\ \mathrm{m\ s^{-1}}$. The height of the belt's upper surface exactly matches that of the static part of the table.

A small, flat disc is placed at the middle of one of the edges of the table (at the point A shown in the figure), and the disc is hit so that it starts sliding with velocity $v_0 = 4\ \mathrm{m\ s^{-1}}$ at right angles to the belt. The friction between the disc and the static part of the table is negligible, while the coefficient of kinetic friction between the disc and the rubber band is $\mu = 0.5$.



Where does the disc leave the table?

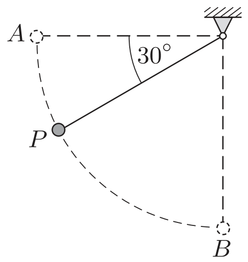
P6* A boy is running north, with a speed of $v = 5\ \mathrm{m\ s^{-1}}$, on the smooth ice cover of a large frozen lake. The coefficient of friction (both kinetic and static) between the tread of his trainers and the ice is $\mu = 0.1$. For the sake of simplicity, assume that the normal force he exerts on the ice, which in reality changes with time, can be substituted by its average value.

a) What is the minimal time that he needs to change direction, so that he is running east with the same speed v?

b) Find the boy's trajectory during the turn in this optimal case.

P7* A simple pendulum is released from rest with its string horizontal. What kind of curve is the locus of the end of its acceleration vector?

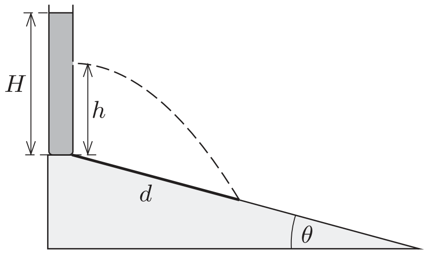
P8 A simple pendulum is released from rest with its string horizontal. Which of the two arcs, AP and PB as defined in the figure, will its bob cover in a shorter time?



P9 The trajectory of a projectile with initial speed v_0 is parabolic in a vacuum (e.g. on the Moon). How far is the focus of this parabola from the launch point? What initial angle of elevation of the projectile is needed if the focus is to be at the same altitude as the launch point?

P10* Point-like objects are thrown with an initial speed of v_0 in various directions from the top of a tower of height h. If the air resistance is negligible, what is the maximum distance from the foot of the tower that they can reach?

P11* At the top of a long incline that makes an angle θ with the horizontal, there is a cylindrical vessel containing water to a depth H. A hole is to be drilled in the wall of the cylinder, so as to produce a water jet that lands a distance d down the incline. How far, h, from the bottom of the vessel should the hole be drilled in order to make d as large as possible? What is this maximum value of d?

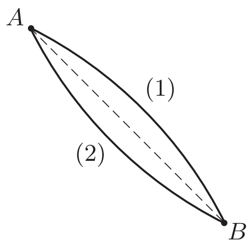


P12 Because of the finite exposure needed, in a side-on photograph of the front wheel of a moving bicycle, the spokes seem blurred. However, there will be some apparently sharp points in the picture. Where are these sharp points? For the sake of simplicity, suppose that the bicycle spokes are radial.

P13* Investigate the form of the image of a spoked bicycle wheel as recorded by a photo-finish camera. Such cameras use very narrow strip photography, electronically capturing a vertical cross-section of the sequence of events only on the finish line; every part of each body is shown as it appeared at the moment it crossed the finish line. The horizontal axis of the image represents time; anything stationary on the finish line appears as a horizontal streak. In a conventional photograph, the image shows a variety of locations at a fixed moment in time; strip photography swaps the time and space dimensions, showing a fixed location at a variety of times. For the sake of simplicity, suppose that the spokes of the bicycle are radial.

P14** A cartwheel of radius 50 m has 12 spokes, assumed to be of negligible width. It rolls along level ground without slipping, and the speed of its axle is 15 m s^{-1}. Use a graphical approach to estimate the minimal speed a crossbow bolt, 20 cm long, must have if it is to pass unimpeded between the spokes of the wheel? Neglect any vertical displacement of the bolt.

P15 A small bob can slide downwards from point A to point B along either of the two different curved surfaces shown in cross-section in the figure. These possible trajectories are circular arcs in a vertical plane, and they lie symmetrically about the straight line joining A to B. During either motion the bob does not leave the curve.



a) If friction is negligible, along which trajectory does the bob reach point B more quickly? How do the final speeds for the two paths compare?

b) What can be said about the final speeds if, although friction is not negligible, the coefficient of friction is the same on both paths?

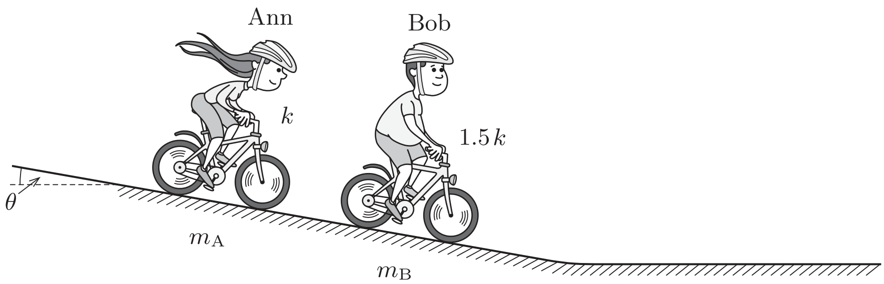
P16 On a windless day, a cyclist ‘going flat out’ can ride uphill at a speed of $v_1 = 12$ km h^{-1}, and downhill at $v_2 = 36$ km h^{-1}, on the same inclined road. What is the cyclist’s top speed on a flat road if his or her maximal effort is independent of the speed at which the bike is travelling?

The rather imprecise term ‘effort’ could be interpreted scientifically to mean either *a*) the magnitude of the *force* exerted on the pedals by the rider (which is then transmitted to the wheels via the crank arms, sprockets and chain), or

b) the rider's mechanical *power*.

Solve the problem for both interpretations.

P17* Ann and Bob arrange a 'free-wheel bike race' on a very long slope that makes an angle θ with respect to the horizontal. They have the same type of bicycle, and neither of them pedals during the race. The total mass of Ann and her bike is $m_A = 60$ kg, whereas the corresponding figure for Bob is $m_B = 110$ kg. Because he is overweight (and 'out of shape'), the air drag on Bob is one-and-a-half times larger than that on Ann when they have equal speeds. Which one is going to coast further on the horizontal road at the bottom of the slope?



Assume that their decelerations are due to air drag (proportional to the square of the speed), friction at the bearings hub and rolling friction. The latter two effects should be treated as a sort of kinetic friction, with an 'effective frictional coefficient' of μ.

P18 A small feather with vanishingly small mass is attached to one end of a riding crop by a flimsy thread. The crop is then rotated steadily about an axis passing through its other end and perpendicular to it. What is the trajectory of the feather?

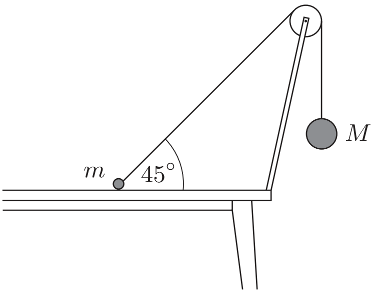
P19* A small pearl moving in deep water experiences a viscous retarding force that is proportional to its speed (Stokes' law). If a pearl is released from rest under the water, then it soon reaches its terminal velocity v_1, and continues sinking with this velocity.

In an experiment, such a pearl is released horizontally with an initial speed v_2.

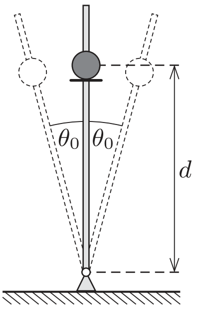
a) What is the minimal speed of the pearl during the subsequent motion?

b) In which direction should the pearl be projected, with the same initial speed v_2 ($< v_1$), in order that its speed increases monotonically during its descent?

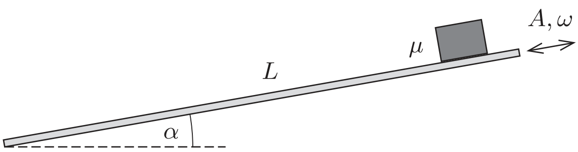
P20 Two spherical bodies, with masses m and M, are joined together by a light thread that passes over a table-mounted pulley of negligible mass. Initially they are held in the positions shown in the figure; then, at a given moment, both of them are released. Mass M is many times – say, one thousand times – larger than mass m. The friction between the smaller ball and the surface of the table is negligible. Will the lighter ball be lifted from the table-top immediately after the release?



P21* A small smooth pearl is threaded onto a rigid, smooth, vertical rod, which is pivoted at its base. Initially, the pearl rests on a small circular disc that is concentric with the rod, and attached to it at a distance d from the rotational axis. The rod starts executing simple harmonic motion around its original position with small angular amplitude θ_0 (*see* figure). What frequency of oscillation is required for the pearl to leave the rod?



P22** The plane of a flat, rigid board of length $L = 6$ m makes an angle of $\alpha = 10°$ with the horizontal, and a small rectangular block is situated at the top of this incline. The board starts vibrating with simple harmonic motion in the direction parallel to a line of steepest descent; the amplitude of the motion is $A = 1$ mm, and its angular frequency is $\omega = 500$ rad s^{-1}. The coefficients of kinetic and static friction between the block and the board are both $\mu = 0.4$. Estimate how long the block, which does not topple over during its motion, takes to reach the bottom of the incline.



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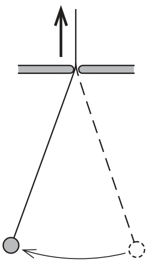
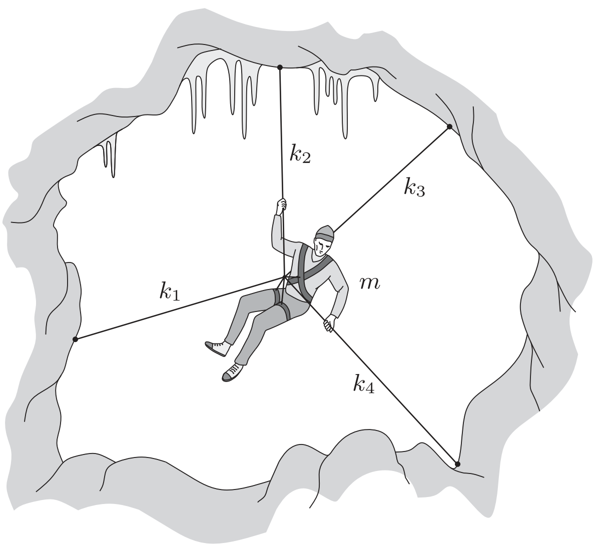
More Information

P23** The cord of a swinging simple pendulum passes through a small hole in a ceiling and into a loft above. There, a scientist's assistant holds the loose end of the cord and pulls it up very slowly (*see* figure). Does the linear amplitude (the largest horizontal excursion) of the pendulum change? If so, how?



P24* A mountaineer (a former circus artist) has to spend the night on the (vertical) side of a high mountain. So, as shown in the figure, he clamps himself to four carabiners fixed to the rock face using four extraordinarily flexible springs. The masses of the springs and their unstretched lengths are negligible, and their spring constants are $k_1 = 150\ \mathrm{N\ m^{-1}}$, $k_2 = 250\ \mathrm{N\ m^{-1}}$, $k_3 = 300\ \mathrm{N\ m^{-1}}$ and $k_4 = 400\ \mathrm{N\ m^{-1}}$. The mountaineer can be considered – for the sake of simplicity – as a point-like body with a mass $m = 70$ kg.



What is the mountaineer's period of oscillation if he is displaced from his equilibrium position and then released?

P25 A small rubber eraser is placed at one edge of a quarter-circle-shaped track of radius R that lies in a vertical plane and has its axis of symmetry vertical (*see* figure); it is then released. The coefficient of friction between the eraser and

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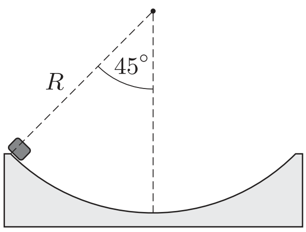
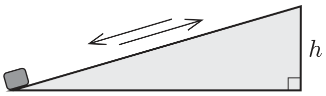
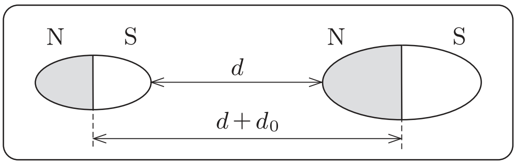
the surface of the track is $\mu = 0.6$. Will the eraser reach the lowest point of the track?



P26 A box of mass 1 kg is placed on an incline on which it does not spontaneously start to slide. It is pulled up, and then down, the incline very slowly in such a way that the traction force is always parallel to the slope (*see* figure). The total work done is 10 J. What is the maximum height h of the incline? Assume that the coefficients of static and kinetic friction are equal.

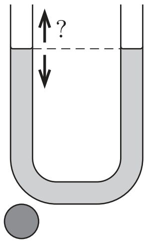


P27* Two permanent magnets are aligned on a horizontal, very slippery tabletop with a gap of length d between them; because of their finite sizes, their centres of mass are $d + d_0$ apart (*see* figure). The magnets are held in such a position that the net force between them is attractive, and there are no torques generated.



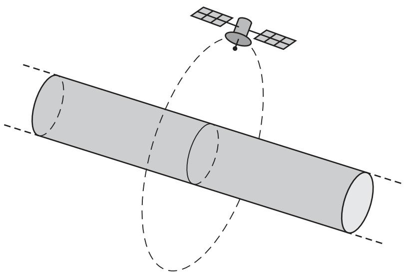
If one of the magnets is held firmly in position and the other is released, then the two collide after 0.6 s. If the roles are reversed, then the time interval between the release and the collision is 0.8 s. How long would it take the two magnets to collide, if both were released simultaneously?

P28* A U-shaped tube contains liquid that initially is in equilibrium. If a heavy ball is placed below the left arm of the tube, how do the liquid levels in the two arms change?



P29** We wish to produce the maximum possible gravitational acceleration at a given point in space, using a piece of plasticine[1] of uniform density and given volume. Into what shape should the plasticine be moulded?

P30 One of the planets of a star called 'Noname' has the shape of a long cylinder. The average density of the planet is the same as that of the Earth, its radius R is equal to the radius of the Earth, and the period of its rotation around its long axis is just one day.



a) The first cosmic velocity $v_{c,1}$ is the speed of a satellite in a stable orbit just above the planet's surface. How large is it for this planet?

b) What is the altitude of a 'geostationary' communications satellite above the surface of this 'sausage' planet?

c) What can be said about the second cosmic velocity (the escape velocity) for this planet?

P31 The Examining Institute for Cosmic Accidents (EXINCA) sent the following short report to one of its experts:

> One of the exploration space ships belonging to the titanium-devouring little green people has found a perfectly spherical, homogeneous asteroid; it has no atmosphere, but is made of pure titanium. As part of the preparations for mining, a straight tunnel was constructed, and railway lines were laid in it. The length of the tunnel was equal to the radius of the asteroid, with both ends on the latter's surface. Unfortunately, although its braking system was on and locked, one of the mine wagons slipped into the shaft at one end of the tunnel. Initially it speeded up, but later it gradually slowed down, reversed and finally stopped exactly in

[1] Although this word has passed into common usage, technically it is a registered trademark for Plasticine.

the middle of the tunnel. Just before it reversed, the wagon came very close to running down the mine captain, who was standing on the track.

EXINCA asked the expert (you) to obtain numerical values for the following:

a) how far along the tunnel the mine captain was standing,

b) the coefficient of kinetic friction between the wheels of the mine wagon and the rails,

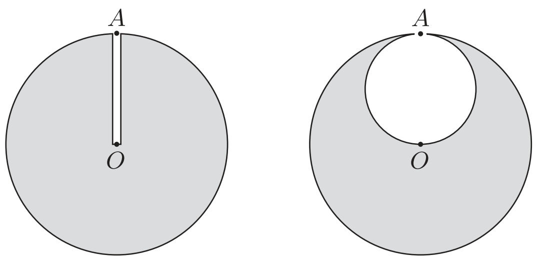
c) the total time of the wagon's motion.

Assume that the volume of the tunnel is negligible compared to that of the asteroid.

P32 A space ship carrying titanium-devouring little green men has found and landed on a perfectly spherical planet of radius R. A narrow trial shaft has been bored from point A on the surface of the planet to O, its centre; this has confirmed that the whole planet is made of homogeneous (edible) titanium. In addition, according to the measurements made, the temperature inside the narrow shaft is constant, and equal to T_0. The planet has an atmosphere with molar mass M, and the atmospheric pressure at its surface is p_A.

a) Find the air pressure at the bottom of the shaft.

After the exploratory drilling, work has continued, and the little green men have started secret excavation of the titanium, as a result of which they have formed a spherical cavity of diameter AO inside the planet, as illustrated in the figure. The excavated titanium is being transported away using expendable cargo space craft. Air from the atmosphere has moved to fill the cavity and, as a consequence, the pressure at the access point A has decreased from p_A to p'_A.



b) Assuming that the temperature everywhere inside the cavity is the same as it was in the shaft, how has the atmospheric pressure at O changed?

P33 Mr Tompkins[2] visited Wonderland in his dream, where the laws of physics are almost the same as we know them – except that gravity deviates 'slightly' from Newton's well-known law. Awakening with a start, he remembered

[2] He is the eponymous main character in the physicist George Gamow's book, *Mr Tompkins in Wonderland*, first published in 1940.