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Multiscale Methods for Fredholm Integral Equations

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Preface

Fredholm equations arise in many areas of science and engineering. Consequently, they occupy a central topic in applied mathematics. Traditional numerical methods developed during the period prior to the mid-1980s include mainly quadrature, collocation and Galerkin methods. Unfortunately, all of these approaches suffer from the fact that the resulting discretization matrices are dense. That is, they have a large number of nonzero entries. This bottleneck leads to significant computational costs for the solution of the corresponding integral equations.

The recent appearance of wavelets as a new computational tool in applied mathematics has given a new direction to the area of the numerical solution of Fredholm integral equations. Shortly after their introduction it was discovered that using a wavelet basis for a singular integral equation led to numerically sparse matrix discretization. This observation, combined with a truncation strategy, then led to a fast numerical solution of this class of integral equations.

Approximately 20 years ago the authors of this book began a systematic study of the construction of wavelet bases suitable for solving Fredholm integral equations and explored their usefulness for developing fast multiscale Galerkin, Petrov–Galerkin and collocation methods. The purpose of this book is to provide a self-contained account of these ideas as well as some traditional material on Fredholm equations to make this book accessible to as large an audience as possible.

The goal of this book is twofold. It can be used as a reference text for practitioners who need to solve integral equations numerically and wish to use the new techniques presented here. At the same time, portions of this book can be used as a modern text treating the subject of the numerical solution of integral equations, which is suitable for upper-level undergraduate students as well as graduate students. Specifically, the first five chapters of this book are designed for a one-semester course, which provides students with a solid background in integral equations and fast multiscale methods for their numerical solutions.

An early version of this book was used in a summer school on applied mathematics sponsored by the Ministry of Education of the People's Republic of China. Subsequently, the authors used revised versions of this book for courses on integral equations at our respective institutions. These teaching experiences led us to make many changes in presentation, resulting from our interactions with our many students.

We are indebted to our many colleagues, who gave freely of their time and advice concerning the material in this book, and whose expertise on the subject of the numerical solution of Fredholm equations, collectively, far exceeds ours. We mention here that a preliminary version of the book was provided to Kendall Atkinson, Uday Banerjee, Hermann Brunner, Yanzhao Cao, Wolfgang Dahmen, Leslie Greengard, Weimin Han, Geroge Hsiao, Hideaki Kaneko, Rainer Kress, Wayne Lawton, Qun Lin, Paul Martin, Richard Noren, Sergei Pereverzyev, Reinhold Schneider, Johannes Tausch, Ezio Venturino and Aihui Zhou. We are grateful to them all for their constructive comments which improved our presentation.

Our special thanks go to Kendall Atkinson for his encouragement and support in writing this book. We would also like to thank our colleagues at Sun Yat-Sen University, including Bin Wu, Sirui Cheng, Xianglin Chen as well as the graduate student Jieyang Chen for their assistance in preparing this book.

Finally, we are deeply indebted to our families for their understanding, patience and continued support throughout our efforts to complete this project.

Symbols

a.e.	almost everywhere; §1.1
$\mathcal{A}^*$	adjoint operator of $\mathcal{A}$; §2.1.1
$\mathbf{A}[\mathbf{i},\mathbf{j}]$	minor of matrix $\mathbf{A}$ with lattice vectors $\mathbf{i}$ and $\mathbf{j}$; §1.2
$\mathcal{B}(\mathbb{X},\mathbb{Y})$	normed linear space of all bounded linear operators from $\mathbb{X}$ into $\mathbb{Y}$; §2.1.1
$\mathbb{C}$	set of complex numbers; §1.1
$C(D)$	linear space of all real-valued continuous functions on D; §2.1
$C^m(D)$	linear space of all real-valued m-times continuously differentiable functions on D; §2.1
$C^\infty(D)$	linear space of all real-valued infinitely differentiable functions on D; §2.1
$C_0(D)$	subspace of $C(\bar{D})$ consisting of functions with support contained inside D; §A.1
$C_0^\infty(D)$	subspace of $C^\infty(\bar{D})$ consisting of functions with support contained inside D and bounded; §A.1
$c_\sigma(D)$	positive constant defined in §2.1.2
card T	cardinality of T; §2.2.2
$\mathrm{cond}(\mathcal{A})$	condition number of $\mathcal{A}$; §2.2.3
$D(\lambda)$	complex-valued function at λ defined by (1.18)
$\det(\mathbf{A})$	determinant of matrix $\mathbf{A}$; §1.2
$\mathrm{diag}(\cdot)$	diagonal matrix; §1.2
$\mathrm{diam}(S)$	diameter of set S; §1.1
$H^m(D)$	Sobolev space; §A.1
$H_0^m(D)$	Sobolev space; §A.1
$L^p(D)$	linear space of all real-valued pth power integrable functions $(1 \le p < \infty)$; §2.1

$L^\infty(D)$	linear space of all real-valued essentially bounded measurable functions; §2.1
$m(D)$	positive constant defined in §2.1.2
$\mathbb{N}$	set of positive integers $\{1, 2, 3, \ldots\}$; §1.1
$\mathbb{N}_0$	set of integers $\{0, 1, 2, \ldots\}$; §1.1
$\mathbb{N}_n$	set of positive integers $\{1, 2, \ldots, n\}$ for $n \in \mathbb{N}$; §1.1
$P_{\mathbf{A}}$	characteristic polynomial of matrix $\mathbf{A}$; §1.2
$\mathbb{R}$	set of real numbers; §1.1
$\mathbb{R}^d$	d-dimensional Euclidean space; §1.1
$\mathrm{Re}\, f$	real part of f; §1.6
$r_q(\mathbf{A})$	minor equation of $\mathbf{A}$; (1.4)
R_λ	resolvent kernel; §1.4
$\mathrm{rank}\, \mathbf{A}$	rank of matrix $\mathbf{A}$; §3.3.5
$s(n)$	dimension of space $\mathbb{X}_n$; §3.3
$\mathrm{span}\, S$	span of set S; §3.3.1
$\mathbb{U}$	index set $\{(i,j) : i \in \mathbb{N}_0, j \in \mathbb{Z}_{w(i)}\}$; §4.1
$\mathbb{U}_n$	index set $\{(i,j) : i \in \mathbb{Z}_{n+1}, j \in \mathbb{Z}_{w(i)}\}$; §4.5.1
$\mathrm{vol}(S)$	volume of set S; §1.1
$W^{m,p}(D)$	Sobolev space; §A.1
$W_0^{m,p}(D)$	Sobolev space; §A.1
$w(n)$	dimension of space $\mathbb{W}_n$; §4.1
$\mathbb{Z}$	set of integers $\{0, \pm1, \pm2, \ldots\}$; §1.1
$\mathbb{Z}_n$	set of integers $\{0, 1, 2, \ldots, n-1\}$ for $n \in \mathbb{N}$; §1.1
$\Gamma(\cdot)$	gamma function; §2.1.2
∇	gradient operator; §2.1.3
Δ	Laplace operator; §2.1.3
$\rho(\mathcal{T})$	resolvent set of operator $\mathcal{T}$; §11.2
$\sigma(\mathcal{T})$	spectrum of operator $\mathcal{T}$; §11.2
ω_{d-1}	surface area of unit sphere in $\mathbb{R}^d$; §2.1.3
$\omega(K, h)$	modules of continuity of K; §1.3
!	factorial; for example (1.4)
$\bigcup^\perp$	union of orthogonal sets; §4.1
$\bigoplus$	direct sum of spaces; §4.1
$\bigotimes$	tensor product (direct product); §1.3
$\circ$	functional composition; §3.3.1
$\lvert\alpha\rvert$	sum of components of lattice vector α; §2.1
$\lvert s - t\rvert$	Euclidean distance between s and t; §2.1.2

$\|\mathcal{A}\|$	norm of operator $\mathcal{A}$; §2.1.1
$\|\cdot\|_{m,\infty}$	norm of $C^m(\bar{D})$; §2.1
$\|\cdot\|_p$,	norm of $L^p(D)$ $(1 \leq P \leq \infty)$; §2.1
$(\cdot,\cdot)$	inner product; §2.1
$\langle\cdot,\cdot\rangle$	value of a linear functional at a function; §2.1.1
$\sim$	same order; §5.1.1
$\xrightarrow{s}$	pointwise converge; §2.1.1
$\xrightarrow{u}$	uniformly converge; §2.1.1