

Index

- $BC_0(\mathbb{R}^+)$, 236
 $C^d(I)$, 325
 $C^{d,\beta}(I)$, 326
 inclusion relation, 327
 $L^p(0, T)$, 327
 inclusion relation, 328
 $L^1_{loc}(\mathbb{R}^+)$, 227
 $W^{d,p}(0, T)$, 329
 inclusion relation, 329
 $\mathcal{K}(X, Y)$, 332
 $\mathcal{L}(X, Y)$, 330
 \mathcal{V} , 2
 \mathcal{V}_α , 18
 $\mathcal{V}_{\omega,\alpha}$, 175
 $\mathcal{V}_{\theta,\alpha}$, 76
 \mathcal{V}_φ , 242
 \mathcal{V}_φ^1 , 243
 $\mathcal{W}_{\theta,\alpha}$, 76
- Abel, 55
 Abel integral equation, 72
 adjoint, 50, 51, 72
 application, 305
 existence of solution, 38
 generalised, 71, 97
 ill posed, 97
 inversion formula, 36, 43
 non-standard equation, 44
 resolvent kernel, 36
 variation-of-constants formula, 36
 Abel–Volterra integral equation, 19
 abstract VIEs, 53, 302
 adjoint Volterra operator, 13
 auto-convolution operator, 299
 auto-convolution VIE
 adjoint, 151, 173
 basic, 146
 existence of solution, 146, 147
 first kind, 147, 173
 generalised, 147
 regularity of solution, 151
 second kind, 172
 third kind, 172
 Banach algebra, 335
 non-commutative, 335
 non-unital, 336
 point spectrum, 336
 resolvent set, 336
 spectrum, 336
 unital, 335, 336
 Banach fixed-point theorem, 341
 blow-up solution, 121
 blow-up time, 123
 breaking points, 75
 Carathéodory iteration, 110, 168
 Cauchy–Schwarz inequality, 328
 closed graph theorem, 331
 compact mapping, 341
 compact operator, 332
 spectrum, 337
 comparison theorem
 linear VIE, 17
 non-linear VIE, 18, 115
 weakly singular VIE, 27
 completely monotone kernel, 226
 completely monotone solution, 21
 contraction map, 341

- cordial VIE
 - first kind, 255
 - Hammerstein form, 266
 - highly oscillatory kernel, 272, 274
 - non-linear, 266, 267, 270
 - regularity of solution, 253, 270
 - second kind, 252
 - variation-of-constants, 257
- cordial Volterra integral operator, 27, 242
 - basic, 243
 - basic properties, 242
 - composition, 247, 274
 - core, 242
 - highly oscillatory kernel, 271
 - mapping properties, 283
 - non-compact, 245
 - non-linear, 267
 - resolvent set, 252
 - spectrum, 250, 251, 271, 272
- damping factor, 177, 179
- decoupled system of IAEs, 208
- delay
 - constant, 74
 - non-vanishing, 74, 84
 - vanishing, 74
- delay function, 73, 74, 84
 - highly oscillatory, 98
 - non-linear, 79
 - vanishing, 81
- Dirichlet's formula, 5
- Du Bois-Reymond, 51
- Euler beta function, 22
- Euler constant, 41
- Fox–Li oscillator, 183
- Fréchet derivative, 340
- fractional calculus, 54
- Fredholm alternative, 25, 334
- Fredholm integral equation
 - first kind, 72
 - highly oscillatory kernel, 183, 196
 - second kind, 7
 - third kind, 55
- Fredholm integral operator, 7
- Fredholm integral equation
 - second kind, 52
- generalised Abel equation, 71
- Gronwall lemma, 6, 17
 - for cordial VIE, 254
 - generalised, 24, 28
- Hölder constant, 326
- Hölder continuity, 326
- Hölder inequality, 328
- Hölder space, 59, 62, 72, 120, 327
- Hammerstein (1930), 169
- Hammerstein integral equation
 - general theory, 112
- Hammerstein integral operator, 104
- highly oscillatory VIE, 175
 - asymptotic expansion of solution, 177, 181, 188
 - damping of solution, 177, 186
 - resolvent kernel, 176
 - weakly singular kernel, 180, 189
- Hilbert–Schmidt integral operator, 280
- IAE
 - ν -smoothing, 210
 - consistency relation, 208
 - decoupled system, 215
 - fully decoupled system, 208
 - Hammerstein form, 218
 - index-1 system, 215
 - matrix chain, 212
 - non-linear, 218
 - projectors, 212
 - semi-explicit form, 209
 - smooth kernel, 207
 - tractability index, 212, 213
 - weakly singular kernel, 218
- ill-posed problem, 33, 97, 334
- ill-posed VIE, 33
- implicit VIE, 31, 153
 - regularity, 154
 - regularity of solution, 168
- implicitly linear integral equation, 112
- integral inequalities, 53
- integral-algebraic equation: see IAE, 207
- iterated kernels, 3, 11
 - for weakly singular kernel, 23
- kernel
 - completely monotone, 226
 - highly oscillatory, 175
 - see also nullspace, 330
- lag function, 73
- Lalesco, 3, 84
- Laplace transform, 146
- Le Roux (1895), 53

- Lighthill's equation, 267, 311
 Lighthill's Volterra integral operator, 251
- maximal solution, 170
 minimal solution, 170
 Mittag-Leffler function, 19, 28, 54
 completely monotone, 20
- Nemytzkiĭ (1934), 170
 Nemytzkiĭ operator, 111
 Neumann series, 4, 292
 convolution kernel, 12
 system of VIEs, 15
 nilpotency, 299
 non-linear second-kind VIE
 existence of solution, 105
 uniqueness of solution, 105
 non-linear VFIE
 state-dependent delay, 163
 non-linear VIE
 auto-convolution equation, 146
 continuation of solution, 170
 first kind, 157
 multiple solution, 116
 non-standard form, 145
 non-trivial solution, 117
 Picard iteration, 106
 standard form, 103
 trivial solution, 117
 weakly singular kernel, 107
 non-vanishing delay, 74, 84
 norm of operator, 330
 null space, 330
- ODE
 quenching of solution, 166
 open mapping theorem, 331
 operator
 adjoint, 338
 bounded, 330
 closed, 330
 compact, 332, 333
 graph, 330
 inverse, 331
 nilpotent, 284
 norm, 330
 projection, 332
 quasi-nilpotent, 284
 self-adjoint, 289, 338
 substitution, 111
 orthogonal projection, 332
 oscillator
- Fox–Li, 183
 other types, 192
 separable, 179
- Paley–Wiener theorem, 225
 partial VIEs
 semi-discretisation, 14
 PDE
 quenching of solution, 137
 quenching time, 135
 Picard iteration, 3, 11, 53, 78
 auto-convolution VIE, 148
 non-linear VIE, 104
 VFIE, 82
 weakly singular VIE, 23
 point spectrum, 337
 population growth VFIE
 state-dependent delay, 309
 primary discontinuity points, 75, 85
 separation property, 85
 projection operator, 332
 proportional delay, 159
 non-vanishing, 75
 VFIE, 76
- quasi-nilpotent operator, 339
 quenching
 differential equation, 134
 VIE, 139
 quenching time, 134, 139
- regularity of solution
 cordial VIE, 254
 second-kind VIE, 58
 weakly singular VFIE, 90
 weakly singular VIE, 58
 renewal equation, 11, 13
 vector form, 16
 resolvent equation, 5, 88
 abstract, 286
 complementary, 5, 286
 convolution kernel, 12
 resolvent kernel, 4, 88
 abstract, 286
 first kind, 287
 for non-smooth kernel, 64
 for weakly singular kernel, 22
 integrability, 225
 resolvent theory, 301
- Schauder fixed point theorem, 342
 Schauder–Leray fixed point theorem, 342

- Shea–Wainger theorem, 227
 singular value decomposition, 290
 singular values, 289
 singularly perturbed VIE, 198
 boundary layer, 200
 Hammerstein equation, 207
 highly oscillatory kernel, 184
 inner solution, 200
 outer solution, 200
 reduced equation, 199
 weakly singular kernel, 206
 Sobolev space, 329
 spectral radius, 339
 spectral theorem, 338
 state-dependent delay, 309
 substitution operator, 111
 superposition principle, 61
 system of VIEs
 representation of solution, 15
 resolvent equations, 14
 resolvent kernel, 14
 viscoelastic problems, 14

 time transformation, 169
 tractability index
 definition, 213
 Tricomi's equation, 72

 Uryson integral operator, 103

 variation-of-constants formula, 232
 linear VIE, 10
 VFIE, 87
 VFIE
 blow-up solution, 161
 existence of solution, 86
 highly oscillatory kernel, 195
 initial function, 85
 iterated kernel, 77, 78, 80
 non-vanishing delay, 74, 84, 85, 90, 160, 161
 primary discontinuity points, 75
 proportional delay, 76, 237
 quenching theory, 166
 regularity of solution, 79, 80, 86, 89, 90, 98, 99
 representation of solution, 77, 87, 89
 smoothing of solution, 90
 state-independent delay, 159
 vanishing delay, 74, 81, 159, 161
 variation-of-constants, 87
 weakly singular kernel, 80, 90

 VFIE of the first kind
 boundedness condition, 92
 existence of solution, 92
 non-vanishing delay, 90
 smoothing of solution, 92
 unbounded solution, 92
 vanishing delay, 82, 99
 VHIE, 111
 VIDE
 variation-of-constant formula, 8
 VIDE: Volterra integro-differential equation, 8
 VIE
 Abel–Volterra, 19
 abstract equations, 53, 302
 adjoint equation, 13, 48
 blow-up of solution, 108
 blow-up profile, 132
 blow-up theory, 127
 blow-up time, 109, 121, 123, 126, 165
 boundary singularities, 64
 comparison theorem, 17, 28
 convolution kernel, 11, 30
 cordial, 252
 diagonal singularity, 58
 finite-time blow-up, 108, 121
 first kind, 2, 3, 29, 54, 67
 Hammerstein form, 111, 233
 highly oscillatory kernel, 175
 identification of memory kernel, 317
 ill-posed, 33
 implicit, 31, 153, 173
 kernel singularities, 18
 linear oscillator, 176
 maximal solution, 113
 minimal solution, 113
 multiple solution, 132
 non-classical, 54
 non-linearly perturbed, 231
 non-standard, 145
 non-standard forms, 104
 optimal control problem, 316
 optimal control problems, 316
 quadratic, 146
 quenching of solution, 109, 135, 139
 quenching theory, 138
 quenching time, 110, 137
 regularity of solution, 58, 60, 61, 63, 64, 119
 renewal equation, 13
 representation of solution, 5
 second kind, 2, 3, 18

- separable oscillator, 176
- singular kernel, 18, 26, 58, 65
- singularly perturbed, 198
- state-dependent delay, 163
- system, 14
- third kind, 2, 31, 46, 51, 55, 56, 261, 320
- two variable limits of integration, 15
- variation of constants, 10
- Volterra–Stieltjes, 300, 302, 303
- Volterra–Stieltjes equations, 52
- weakly singular kernel, 19
- VIE of the first kind
 - ν -smoothing problem, 34
 - adjoint, 97
 - cordial, 255
 - degree of ill-posedness, 34
 - Hammerstein form, 30
 - highly oscillatory kernel, 184
 - logarithmic kernel, 41
 - mildly ill-posed, 290
 - regularity of solution, 40, 44, 67, 68, 70
 - resolvent kernel, 256, 286
 - resolvent representation, 256
 - severely ill-posed, 290
 - singular kernel, 36, 42, 68, 70
 - system, 35, 50, 97
 - variation-of-constants, 256
- VIE of the second kind
 - non-linear, 103
- VIE of the third kind, 45, 46
 - connection with cordial VIE, 261
 - non-linear, 268, 270
- VIE with weakly singular kernel
 - ν -smoothing, 42
- Volterra
 - biography, 53
 - Nota I, 1, 3–5, 29, 52
 - Nota II, 1, 39, 50, 52, 55, 99
 - Nota III, 2, 46
 - Nota IV, 2
- Volterra functional integral equation: see VFIE, 73
- Volterra inequality, 17
- Volterra integral equation: see VIE, 2
- Volterra integral operator
 - \mathcal{V} , 2
 - \mathcal{V}_α , 18
 - \mathcal{V}_φ^1 , 243
 - ν -smoothing, 34
 - adjoint, 13, 43, 276, 289, 294
 - boundary singularity, 244
 - compact, 279
 - cordial, 242
 - delay argument, 76
 - eigenfunction, 45
 - eigenvalue, 45
 - finite rank, 11
 - Hammerstein, 111
 - linear, 2
 - mapping properties, 282
 - non-compact, 27, 45, 242
 - norm of powers, 292
 - quasi-nilpotency, 284
 - singular value, 288
 - spectrum, 7
 - two variable limits, 298
- Volterra–Abel integral operator
 - non-standard, 42
- Volterra–Hammerstein integral equation, 104
- Volterra–Hammerstein integral operator, 111
- weakly singular kernel
 - ν -smoothing, 42
- weakly singular VFIE
 - regularity of solution, 90
- weakly singular VIE
 - representation of solution, 22, 59
- well-posed problem, 33
- Young inequality, 328