Almost All About Unit Roots
Foundations, Developments, and Applications

Many economic theories depend on the presence or absence of a unit root for their validity, and econometric and statistical theories undergo considerable changes when unit roots are present. Thus, knowledge about unit roots has become very important, necessitating an extensive, compact, and nontechnical book on this subject. This book introduces the literature on unit roots in a comprehensive manner to both students and empirical and theoretical researchers in economics and other areas. In providing a clear, complete, and critical discussion of the unit root literature, In Choi covers a wide range of topics, including uniform confidence interval construction, unit root tests allowing structural breaks, mildly explosive processes, exuberance testing, fractionally integrated processes, seasonal unit roots, and panel unit root testing. Extensive, up to date, and readily accessible, this book is a comprehensive reference source on unit roots for both students and researchers.

In Choi is a professor of economics at Sogang University in Seoul, Korea. His research focus has been on time series and panel data analysis, and he has published numerous articles in leading journals in economics and statistics. He is a Fellow of the *Journal of Econometrics* and has received the Plura Scripsit Award from *Econometric Theory* and the Chongram Award from the Korean Economic Association. He is currently an associate editor of the *Journal of Business and Economic Statistics*.

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In memory of

my father Mr. Suk Whan Choi

Contents

Foreword

Socioeconomic trends figure prominently in media discussion and the financial pages of newspapers. They dominate data, guide policy decisions, and attract intense interest that extends well beyond the subject matter of economics and finance. In spite of this widespread influence trends are poorly understood. They are the inscrutable Hamlet of econometrics. No one really knows what they will do next.

During the 1980s econometrics embarked on a revolutionary journey that opened up a new understanding of the stochastic properties of trend. The unit root revolution changed the way the profession thought about trend by emphasizing the role of random elements in the trend mechanism and by formulating a technically well-defined concept of long-run behavior that did not remove randomness. By the 1990s, functional limit laws, stochastic integrals, and functionals of stochastic processes had overtaken econometrics in a firestorm that swept away earlier methods. New symbols of limit theory forever changed the pages of the mainline economics journals. New thinking penetrated econometric teaching and empirical practice. And a vast literature of applied economic analysis was born that demonstrated surprising sophistication in its use of modern econometric technology and nonstandard limit theory.

The implications of this unit root revolution have been enormous. The methodology exported itself throughout the social and business sciences with concomitant changes in thinking that acknowledged the ubiquitous presence of nonstationarity in data. The methods now reach into the natural sciences in areas as diverse as paleoclimatology and biodiversity with datasets that span hundreds of millions of years rather than the decades and centuries of economic data.

The single most important tool in the development of a theory for nonstationary time series was the use of limit laws on function spaces, first brilliantly exposited by Patrick Billingsley in his monograph *Convergence of Probability Measures,* published in 1968. Measure theoretic principles in metric spaces

underpinned all the foundations of this major work. Appropriately, Billingsley's author index reference to the mathematician Paul Halmos (who wrote a classic treatise on measure theory) read quite simply as "a.e." (almost everywhere).

Fittingly too, Choi's volume on nonstationarity in the Themes in Modern Econometrics series is entitled *Almost All About Unit Roots.* The world of unit roots has exploded since the 1990s when the last generation of textbooks on the subject were written. Choi escorts us into this excitingly complex arena of research and empirical findings with a text that reaches out to the non-specialist and practitioner, while providing for the specialist a guidebook to the expanding universe of literature that has come to define the discipline of unit root econometrics.

The unit root revolution was a revolution in thinking about economic time series and trends that massively changed the conduct of empirical research and supported the emergence of the field of financial econometrics. The excitement of this transformation of econometrics lives on in the latest work of the profession, which this volume seeks to reveal.

One of the laws of modern econometrics is that "no one understands trends, but everyone sees them in the data." This volume by Choi will help readers learn how unit root econometrics enabled us to tackle the curiously inscrutable phenomenon of trend by marshalling new scientific methods of function space limit theory and inference.

Peter C. B. Phillips
January 2015

Preface

There has been much research conducted on nonstationary time series in the last few decades, and the related literature continues to expand. Research on nonstationary time series can be categorized into two areas: unit roots and cointegration. The literature on unit roots dates back to White (1958), and methods related to unit roots are now popular among economists and other social scientists. The concept of cointegration was developed by Engle and Granger (1987), and the techniques for cointegration have been accepted as standard tools in economics and other areas.

The aim of this book is to introduce the literature on unit roots in a comprehensive manner to both students and empirical and theoretical researchers in economics and other areas. The literature on nonstationary time series is now so huge that it seems difficult, if not impossible, to include all the related topics in a single monograph. Therefore, this book focuses on unit roots.

This book takes the approach of discussing as many papers as possible in presenting developments in the literature on unit roots. Yet it emphasizes important works that either contain novel ideas or have been cited often. By reading this book, the student or researcher can understand major developments in the literature on unit roots and related areas. The papers covered in this book were published in more than 30 major journals in econometrics, statistics, and other branches of social science up to 2013. A few unpublished papers and book chapters are also included.

This book is ideal for graduate students and researchers in economics, finance, political science, sociology, statistics, and other areas who want to learn about unit roots as they conduct their empirical research projects or theoretical research on unit roots. Material in this book can be taught in graduate-level courses on time series analysis along with more conventional textbooks such as Brockwell and Davies (1991), Hamilton (1994), and Fuller (1976). This book is also useful as a reference for researchers interested in nonstationary time series analysis. For those who are interested in theoretical aspects of unit roots,

this book provides an up-to-date literature survey and suggests some open questions. To understand this book fully, the reader must have some knowledge of time series analysis at the level of, for example, Brockwell and Davies (1991).

There are excellent books that deal with unit roots, such as Banerjee, Dolado, Galbraith, and Hendry (1993); Hatanaka (1996); Maddala and Kim (1998); and Patterson (2010, 2011). However, the first three of these books were written more than 15 years ago and do not reflect recent contributions to the literature on unit roots. The books by Patterson focus on conventional methods of unit root testing. In contrast, this book contains up-to-date and comprehensive research results about unit roots and therefore is complementary to them.

I am thankful to many people who have guided me in my development as a researcher and helped me while I was writing this book. I am grateful to Professors Peter Phillips, Don Andrews, Matthew Shapiro, Vassilis Hajivassiliou, Benedikt Pötscher, and David Pollard, who taught me econometrics, statistics, and probability at Yale. In particular, Professor Peter Phillips, as my thesis advisor, led me into the research area of nonstationary time series. He showed me how to do research, how to teach, and how hard academicians should work to achieve their goals. I owe much to him for what I am now as a researcher and educator. In fact, without his encouragement, I would not have dared to start writing this book. He, as well as four anonymous reviewers, provided very helpful comments on previous versions of this book. Those comments greatly improved the book, for which I am truly thankful. I also thank him for his insightful foreword for this book. I first learned econometrics from Professor Ki-Jun Jeong, then at Seoul National University. His course was challenging for most undergraduate students at that time, but I became interested in econometrics despite its difficulty. I appreciate his serious teaching. Part of this book was written while I was visiting the University of Leeds and the University of Bonn. I thank Professors Jörg Breitung, Martin Carter, Matei Demetrescu, Christian Pigorsch, Kevin Reilly, and Yongcheol Shin for their hospitality. Minchul Yum (now a graduate student at the Ohio State University) gathered the papers covered in this book, and I thank him for his diligent efforts. John Morris read the entire manuscript carefully and suggested grammatical corrections and stylistic improvements, which I deeply appreciate. The research reported in this book was supported by the National Research Foundation of Korea (project #NRF-2010-342-B00006), which I gratefully acknowledge. Last, but most important, I thank my wife Joanne Jung-un Han for her enduring support for the last 25 years. Indeed, I can live a happy personal life and a productive professional one thanks to her presence.

In Choi
Seoul, Korea
September 2013

Abbreviations and Notation

The following abbreviations and notations are used throughout this book.

AR	autoregressive
ARCH	autoregressive conditional heteroskedasticity
ARIMA	autoregressive integrated moving average
ARMA	autoregressive moving average
cdf	cumulative density function
CLT	central limit theorem
DGP	data-generating process
FCLT	functional central limit theorem
GARCH	generalized autoregressive conditional heteroskedasticity
GLS	generalized least squares
GMM	generalized method of moments
HPD	highest posterior density
i.i.d., iid	independent and identically distributed
IV	instrumental variables
LAD	least absolute deviation
LM	Lagrange multiplier
LR	likelihood ratio
MA	moving average
MLE	maximum likelihood estimator
OLS	ordinary least squares
pdf	probability density function
SUR	seemingly unrelated regression
VAR	vector autoregression
$[x]$	integer not greater than x
$\mathbf{1}\{A\}$	indicator function taking value one when A is true and zero otherwise

$\Rightarrow$	weak convergence
$\xrightarrow{p}$	convergence in probability
B	backward-shift operator (i.e., $B^m X_t = X_{t-m}$ for an integer m)
Δ^d	$\Delta^d X_t = (1-B)^d X_t$
$X_t = \mathrm{I}(d)$	$\Delta^d X_t$ is a stationary process