Wireless Physical Layer Network Coding

Discover a fresh approach for designing more efficient and cooperative wireless communications networks with this systematic guide. Covering everything from fundamental theory to current research topics, leading researchers describe a new, network-aware coding strategy that exploits the signal interactions that occur in dense wireless networks directly at the waveform level. Using an easy-to-follow, layered structure, this unique text begins with a gentle introduction for those new to the subject, before moving on to explain key information-theoretic principles and establish a consistent framework for wireless physical layer network coding (WPNC) strategies. It provides a detailed treatment of Network Coded Modulation, covers a range of WPNC techniques such as Noisy Network Coding, Compute and Forward, and Hierarchical Decode and Forward, and explains how WPNC can be applied to parametric fading channels, frequency selective channels, and complex stochastic networks. This is essential reading whether you are a researcher, graduate student, or professional engineer.

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Wireless Physical Layer Network Coding

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Preface

About the Book

The book addresses strategies and principles of physical layer coding and signal processing that fully respect and utilize knowledge of the structure of a wireless network. This technique substantially increases the overall network throughput, efficiency, and reliability. Wireless Physical Layer Network Coding (WPNC) (a.k.a. Physical Layer Network Coding (PLNC)) is a general framework for physical (PHY) layer coding and processing strategies in which PHY behavior at a given node depends on its position in the network topology, and the signal-level processing/decoding exploits multiple paths between source and destination. We introduce the concept of Network Coded Modulation (NCM) as a network-structure-aware signal space code, which processes a (hierarchical) joint function of source data. At intermediate nodes NCM utilizes hierarchical decoding, and it is also designed to allow unambiguous decoding at the final destination using multiple hierarchical observations, arriving via different routes. The book addresses the fundamental principles of WPNC in the context of network information theory, and provides a comprehensive classification of the strategies. It also covers advanced design and techniques, including particular coding and processing designs and their respective properties. We also address selected hot research topics and open problems.

Motivation for the Book

It is becoming widely accepted that the most significant future developments in the physical layer of wireless communication systems will not take place in the PHY layer of individual communication links, but rather in the context of complete wireless networks, especially as the density of wireless networks continues to increase. Over the past decade or so there have been significant developments in network information theory; these have shown that very significant overall performance gains are available compared with the conventional paradigm in which PHY techniques are applied to individual links only, leaving network aspects to be dealt with only at higher layers of the protocol stack. One such new research field is network coding, in which coding techniques are applied to multiple data streams at intermediate nodes in a network, rather than only to individual streams on single links. This can exploit network topology to significantly improve
throughput in multi-user networks. However, in its original form it operates at the level
of data streams, rather than signal waveforms, and hence is not well suited to the inher-
ently broadcast nature of wireless networks. Wireless physical layer network coding
(WPNC) allows it to be applied directly to wireless networks, with a further significant
improvement in efficiency. The key advance on conventional PHY techniques is that
the nodes are aware of the network topology and their place within it, and both signal-
ing waveforms and node signal processing exploit this knowledge to improve overall
network throughput.

Book Scope and Organization

The book is carefully balanced, being divided into several “layers” giving different
depths of information for audiences with various levels of background knowledge. Part
I gives a gentle introduction to the key concept with the explanation kept in accessi-
able form. Part II presents fundamental principles in more detail, but still using a “big
picture” global perspective. Part III addresses a mosaic of various particular design tech-
niques and principles that can practically fulfill the general principles of Part II. The
Appendix provides some background material for readers with a weaker background in
communication, signal processing, and information theory.

Throughout the book, we maintain a strong emphasis on the proper classification and
structuring of the problems, techniques, and particular coding, processing, and decod-
ing schemes under discussion. This will help readers to properly orient themselves in
the complex landscape of the different individual approaches. In the currently available
literature these frequently overlap, and suffer from rather “fuzzy” terminology. This may
lead to incorrect comparisons due to the high complexity of the field and the ambigu-
ity and inconsistency of the terminology. (Terminology also changes rapidly due to the
rapid progress of the research community.)

The book is not primarily intended as a university course textbook but rather as a ref-
erence source for researchers, PhD students, and engineers who would like to understand
the principles of WPNC in the context of other techniques or would like to start their
own research work in this field. Therefore the book is a highly structured set of Parts–
Chapters–Sections, which are intended, as far as possible, to be read in a self-contained
manner.

Jan Sykora and Alister Burr
Mathematical Symbols

Basic Symbols, Sets

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>( \mathbb{N} )</td>
<td>positive integers</td>
</tr>
<tr>
<td>( \mathbb{N}_0 )</td>
<td>non-negative integers</td>
</tr>
<tr>
<td>( \mathbb{R} )</td>
<td>real numbers</td>
</tr>
<tr>
<td>( \mathbb{C} )</td>
<td>complex numbers</td>
</tr>
<tr>
<td>( \mathbb{Z} )</td>
<td>integer numbers</td>
</tr>
<tr>
<td>( \mathbb{Z}_j )</td>
<td>complex (Gaussian) integers</td>
</tr>
<tr>
<td>( \mathcal{A}_1 \times \mathcal{A}_2 )</td>
<td>Cartesian product of sets</td>
</tr>
<tr>
<td>( \mathcal{A}^N )</td>
<td>Cartesian product of sets ( N ) times</td>
</tr>
<tr>
<td>(</td>
<td>\mathcal{A}</td>
</tr>
<tr>
<td>( \emptyset )</td>
<td>empty set</td>
</tr>
<tr>
<td>( \cup ), ( \cap )</td>
<td>union and intersection operators for the sets</td>
</tr>
<tr>
<td>( \setminus )</td>
<td>set difference (set minus)</td>
</tr>
<tr>
<td>( {c_i}_i )</td>
<td>set of variables ( c_i ) for all feasible indices ( i )</td>
</tr>
<tr>
<td>( \tilde{a} = {a_1, \ldots, a_n} )</td>
<td>set of all components</td>
</tr>
<tr>
<td>( [a, b) = {x : a \leq x &lt; b} )</td>
<td>semiopen interval</td>
</tr>
<tr>
<td>( [k_1 : k_2] )</td>
<td>integer interval ( {k_1, k_1 + 1, \ldots, k_2}, k_1, k_2 \in \mathbb{Z} )</td>
</tr>
<tr>
<td>( f : A \mapsto B )</td>
<td>( f ) mapping from domain ( A ) to codomain (range) ( B )</td>
</tr>
<tr>
<td>( F[], \ldots, ]</td>
<td>operator ( F )</td>
</tr>
<tr>
<td>( \exists )</td>
<td>there exists</td>
</tr>
<tr>
<td>( \forall )</td>
<td>for all</td>
</tr>
<tr>
<td>( \triangleq )</td>
<td>equal by definition</td>
</tr>
<tr>
<td>( \equiv )</td>
<td>equivalent, defines equivalence class</td>
</tr>
<tr>
<td>( \approx )</td>
<td>approximately or asymptotically equal</td>
</tr>
<tr>
<td>( \lesssim, \gtrsim )</td>
<td>approximately less than and greater than</td>
</tr>
<tr>
<td>( \lesssim, \gtrsim )</td>
<td>asymptotically less than and greater than</td>
</tr>
<tr>
<td>( \Rightarrow, \Leftrightarrow )</td>
<td>implication and equivalence</td>
</tr>
<tr>
<td>( {a} )</td>
<td>the set of all values the variable ( a ) can take</td>
</tr>
<tr>
<td>( \delta(t) )</td>
<td>Dirac delta function (continuous time)</td>
</tr>
<tr>
<td>( \delta[k] )</td>
<td>Kronecker delta (discrete time)</td>
</tr>
<tr>
<td>sup</td>
<td>supremum</td>
</tr>
<tr>
<td>sinc(x) = \sin(\pi x)/(\pi x)</td>
<td>sampling function</td>
</tr>
</tbody>
</table>
Mathematical Symbols

\[ \lg x = \log_2 x \] binary logarithm
\[ a^* \] complex conjugation
\[ U(x) \] unit step function
\[ (x)^+ = \max(0, x) \] positive part function
\[ \land, \lor \] Boolean “and”, “or”
\[ j = \sqrt{-1} \] imaginary unit
\[ e \] base of the natural logarithm
\[ \angle z \] angle of complex number
\[ \frac{\partial f}{\partial x} \] standard partial derivative of the function \( f \) over variable \( x \)
\[ \tilde{\frac{\partial f}{\partial z}} \] generalized partial derivative of complex valued function over complex valued variable
\[ \int (\cdot) \, d x \] abbreviated form for the integration over the whole domain of variable \( x \), e.g. \( \int_{-\infty}^{\infty} (\cdot) \, d x \)
\[ \sum_{x \in \mathcal{G}(x) = y} f(x) \] sum over the set of all \( x \) consistent with explicit condition \( g(x) = y \)

Number Theory, Vectors, Matrices, Inner-Product Spaces, Lattices

\[ \langle \cdot, \cdot \rangle \] inner product
\[ a \] vector (all vectors are column vectors)
\[ \mathbf{1} = [1, \ldots, 1]^T \] unity vector
\[ \mathbf{I}, \mathbf{I}_N \] identity matrix with size defined by context, \( N \times N \) identity matrix
\[ \text{diag}(a) \] diagonal matrix with the components of \( a \) on the main diagonal
\[ A \in \mathbb{C}^{m \times n} \] \((m,n)\) matrix of complex numbers
\[ [A]_{i,j} \] element of the matrix on the \( i \)th row and \( j \)th column
\[ A \succeq B \] \( A - B \) matrix is positive semi-definite
\[ (\cdot)^T \] transposed matrix or vector
\[ (\cdot)^H \] Hermitian transpose
\[ A^{-1} \] matrix inverse
\[ A^\dagger = (A^H A)^{-1} A^H \] matrix pseudoinverse
\[ \det A \] determinant of matrix \( A \)
\[ \boxtimes \] Kronecker matrix product
\[ \circ \] element-wise Hadamard product of two matrices/vectors
\[ \mathbb{S}_M \] finite ring
\[ \mathbb{F}_p^n \] Galois (finite) extended field with characteristic \( p \)
\[ \mathbb{F}_M^{N_1 \times N_2} \] \( N \)-dimensional vector and \( N_1 \times N_2 \) matrix on \( \mathbb{F}_M \) GF
\[ \oplus, \otimes \] addition and multiplication on GF (this explicit notation is used only when we need explicitly to distinguish it, otherwise ordinary “plus” and “times” operators are also used)
\[ \mathcal{E} \] energy
### Mathematical Symbols

- \( c(t) = a(t) \ast b(t) \) convolution in continuous time \( \int a(t - \tau)b(\tau) \, d\tau \)
- \( c = a \circ b \) convolution in discrete time \( c_n = \sum_{k=0}^{\infty} a_{n-k}b_k \), \( a = [\ldots, a_0, a_1, \ldots ]^T \) and similarly for \( b, c \)
- \( c = a \odot b \) cyclic convolution \( c_n = \sum_{k=0}^{N-1} a_{n-k} \mod N b_k \), \( a = [a_0, a_1, \ldots, a_{N-1}]^T \) and similarly for \( b, c \)
- \( x \perp y \) orthogonal \( x \) and \( y \), i.e. \( \langle x; y \rangle = 0 \) for some inner product definition
- \( \Lambda \) lattice
- \( \mathcal{V}(\Lambda_x) \) fundamental Voronoi cell of lattice \( \Lambda_x \)
- \( \Lambda_c / \Lambda_s \) quotient group for lattices \( \Lambda_c, \Lambda_s \)

### Random Variables, Processes, and Information Theory

- \( X, x \) strict notation for random variable and its particular realization
- \( y, y^{(j)} \) alternative (relaxed) form of notation (identified by its context) for random variable and its particular realization
- \( x^K = \{x_1, x_2, \ldots, x_K\} \) a sequence (a tuple) of variables
- \( X^K = \{X_1, X_2, \ldots, X_K\} \) a sequence (a tuple) of random variables
- \( x(S) = \{x_k : k \in S\} \) set (a tuple) of variables with indices given by \( S \)
- \( \Pr(\cdot) \) probability
- \( p(x), p_X(x), p_x(x) \) PDF (PMF) with implicit and explicit denotation of random variable
- \( p(x|z), p_XZ(x|z), p_{xz}(x|z) \) conditional PDF (PMF) with implicit and explicit denotation of random variables
- \( x \sim p(x) \) drawn according to the given PDF/PMF
- \( A \perp B \) independent random variables
- \( A \perp B | C \) (or \( A \perp B | C \)) \( A \) and \( B \) conditionally independent given \( C \)
- \( \mathcal{U}(S) \) uniform distribution over the set \( S \)
- \( \mathcal{N}(\mathbf{m}, \mathbf{C}) \) Gaussian distribution with mean vector \( \mathbf{m} \) and variance matrix \( \mathbf{C} \)
- \( \mathcal{E}[] \) ensemble domain expectation operator
- \( \mathcal{E}_v[\cdot], \mathcal{E}_{p(v)}[\cdot] \) expectation over explicit random variable or distribution
- \( \mathcal{H}(X) \) entropy of random variable \( X \)
- \( \mathcal{H}(X|Y) \) conditional entropy of \( X \) conditioned by \( Y \)
- \( I(X; Y) \) mutual information between \( X \) and \( Y \)
- \( I(X; Y|Z) \) conditional mutual information between \( X \) and \( Y \) given \( Z \)
- \( \mathcal{H}(p) \) binary entropy function
- \( A \mapsto B \mapsto C \) Markov chain variables
- \( (x, y) \in T \) \( x \) and \( y \) are jointly typical
- \( \mathcal{R}(S_1, S_2) \) rate region for independent codebooks with \( S_1, S_2 \) random channel symbols
### Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>2WRC</td>
<td>2-Way Relay Channel</td>
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<tr>
<td>AF</td>
<td>Amplify and Forward</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
</tr>
<tr>
<td>BC</td>
<td>Broadcast Channel</td>
</tr>
<tr>
<td>BPSK</td>
<td>Binary Phase Shift Keying</td>
</tr>
<tr>
<td>CF</td>
<td>Compute and Forward</td>
</tr>
<tr>
<td>CpsF</td>
<td>Compress and Forward</td>
</tr>
<tr>
<td>CRLB</td>
<td>Cramer–Rao Lower Bound</td>
</tr>
<tr>
<td>CSE</td>
<td>Channel State Estimation</td>
</tr>
<tr>
<td>DF</td>
<td>Decode and Forward</td>
</tr>
<tr>
<td>DFT</td>
<td>Discrete Fourier Transform</td>
</tr>
<tr>
<td>GF</td>
<td>Galois Field</td>
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<tr>
<td>H-</td>
<td>Hierarchical</td>
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<tr>
<td>H-BC</td>
<td>Hierarchical BC</td>
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<tr>
<td>H-constellation</td>
<td>Hierarchical Constellation</td>
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<td>H-decoding</td>
<td>Hierarchical Decoding</td>
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<td>HDF</td>
<td>Hierarchical Decode and Forward</td>
</tr>
<tr>
<td>HI</td>
<td>Hierarchical Information</td>
</tr>
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<td>H-Ifc</td>
<td>Hierarchical Interference</td>
</tr>
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<td>H-MAC</td>
<td>Hierarchical MAC</td>
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<tr>
<td>HNC map</td>
<td>Hierarchical Network Code map</td>
</tr>
<tr>
<td>H-NTF</td>
<td>Hierarchical Network Transfer Function</td>
</tr>
<tr>
<td>H-NTM</td>
<td>Hierarchical Network Transfer Matrix</td>
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<tr>
<td>H-PEP</td>
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<td>Hierarchical Successive CF Decoding</td>
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<td>Hierarchical Soft-Output Demodulator</td>
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<td>MAC</td>
<td>Multiple Access Channel</td>
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<td>MAP</td>
<td>Maximum A posteriori Probability</td>
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<td>Abbreviations</td>
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<td>MIMO</td>
<td>Multiple-Input Multiple-Output</td>
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<td>Signal-to-Noise Ratio</td>
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<td>Uniformly Most Powerful</td>
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