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978-1-107-09254-9 - Precession, Nutation, and Wobble of the Earth

V. Dehant and P. M. (Sonny) Mathews

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## PRECESSION, NUTATION, AND WOBBLE OF THE EARTH

Earth rotation has long been used as a measure of time, together with using the stars as reference points, to determine travellers' whereabouts on the globe. However, the rotation of the Earth is not uniform and its orientation with respect to the sky goes on changing.

Covering both astronomical and geophysical perspectives, this book describes changes in the Earth's orientation, specifically precession and nutation, and how they are observed and computed in terms of tidal forcing and models of the Earth's interior. Following an introduction to key concepts and elementary geodetic theory, the book describes how precise measurements of the Earth's orientation are made using observations of extragalactic radio sources by very long baseline interferometry (VLBI) techniques. It demonstrates how models are used to accurately pinpoint the location and orientation of the Earth with reference to the stars and how to determine variations in its rotation speed (length-of-day variations). A theoretical framework is also presented that describes the role played by the structure and properties of the Earth's deep interior.

Incorporating suggestions for future developments in nutation theory for the next generation models, this book is ideal for advanced-level students and researchers in the fields of solid Earth geophysics, planetary science, and astronomy.

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## Preface

For several years (about 20) both authors have been working in the domain of Earth rotation, and in particular on nutation. The Working Group on nutation, established by the International Astronomical Union and the International Union of Geodesy and Geophysics in 1994, was a starting point for bringing together scientists thinking about what was missing in the nutation series adopted by the International Astronomical Union. Collaboration between the scientists of the WG was very successful and, in particular, the authors' collaboration began at that time. Recently, it has appeared to us that there were no existing books yet dedicated to the subject, and the scientific community is looking for a suitable publication. The literature contains a lot of relevant articles, but many of them rely on previous work and do not give full details. This book aims at bringing everything together for the first time. The book is addressed to students or scientists who want to understand nutations. The aim of this book is to give a reasonably comprehensive introduction to the fundamental concepts, mathematical formalism, and methodology of the Earth's nutation. It is only assumed that the student or reader is familiar with the elementary principles of calculus, although we might have used in some parts short-cuts for reasons of simplicity, and with the underlying physical principles in the foreground. Another important aim of this book is to make a comprehensive list of the geophysical and astronomical processes involved in nutation, in order to be able to investigate the "next decimal place."

*The authors wish to take this opportunity to acknowledge all those who have aided in the preparation of this book.*

Abbreviations

Notation	Definition
AAM	Atmospheric Angular Momentum
AU	Astronomical Units
BCRF	Barycentric Celestial Reference Frame
BCRS	Barycentric Celestial Reference System
BIH	Bureau International de l’Heure
CEP	Celestial Ephemeris Pole
CHAMP	CHAllenging Minisatellite Payload
CIO	Celestial Intermediate Origin
CIP	Celestial Intermediate Pole
CMB	Core Mantle Boundary
cpsd	cycle per sidereal day
CRF	Celestial Reference Frame
CRS	Celestial Reference System
CW	Chandler Wobble
DExxx	Development Ephemeris where xxx is a number
DSN	Deep Space Network
ECRF	Ecliptic Celestial Reference Frame
ECRS	Ecliptic Celestial Reference System
ELP20yy	Ephéméride Lunaire Parisienne where 20yy is a year
EMB	Earth–Moon Barycenter
EOPs	Earth Orientation Parameters
EOT	Empirical Ocean Tide Model
ERA	Earth Rotation Angle
ESA	European Space Agency

Notation	Definition
ESTRACK	ESA Tracking station
FCN	Free Core Nutation
FES	Finite Element Solution
FICN	Free Inner Core Nutation
FOC	Fluid Outer Core
GCM	General Circulation Model
GCRF	Geocentric Celestial Reference Frame
GCRS	Geocentric Celestial Reference System
GMST	Greenwich Mean Sidereal Time
GNSS	Global Navigation Satellite System
GOCE	Gravity field and steady-state Ocean Circulation Explorer
GOT	Goddard Ocean Tide model
GPS	Global Positioning System
GRACE	Gravity Recovery And Climate Experiment
GSH	Generalized Spherical Harmonics functions
GST	Greenwich Sidereal Time
GTRF	Geocentric Terrestrial Reference Frame
GTRS	Geocentric Terrestrial Reference System
IAG	International Association of Geodesy
IAU	International Astronomical Union
IBO	Inverted Barometric Ocean
ICB	Inner Core Boundary
ICRF	International Celestial Reference Frame
ICRS	International Celestial Reference System
ICW	Inner Core Wobble
IERS	International Earth rotation and Reference frame Service
IMCCE	Institut de Mécanique Céleste et de Calcul des Ephémérides
INPOP20xx	Intégrateur Numérique Planétaire de l’Observatoire de Paris where 20xx is a year
InSIGHT	Interior exploration using Seismic Investigations, Geodesy and Heat Transport
IRF	Inertial Reference Frame
IRF	Intermediate Reference Frame
ITRF	International Terrestrial Reference Frame
ITRS	International Terrestrial Reference System
IUGG	International Union for Geodesy and Geophysics

Notation	Definition
J2000	event (epoch) at date 2000 January 1.5
JGM	Joint Gravity Model
JPL	Jet Propulsion Laboratory
LaRa	Lander Radioscience experiment
LExxx	Lunar Ephemeris where xxx is a number
LLR	Lunar Laser Ranging
LOD	Length-Of-Day
LTE	Laplace Tidal Equations
mas	milliarcsecond
$\mu$ as	microarcsecond
MHB	Nutation model of Mathews, Herring, and Buffett
MHB2000	Nutation model of Mathews, Herring, and Buffett (Mathews <i>et al.</i> , 2002)
MOP	Mars Orientation Parameters
NAIF	Navigation and Ancillary Information Facility
NASA	National Aeronautics and Space Administration
NDFW	Nearly Diurnal Free Wobble
NIBO	Non-Inverted Barometric Ocean
NNRC	No-Net-Rotation Condition
NRO	Non-Rotating Origin
OAM	Ocean Angular Momentum
OD	Ocean Dynamics
PFCN	Prograde Free Core Nutation
PM	Polar Motion
PN	Precession Nutation
POD	Precise Orbit Determination
PREM	Preliminary Reference Earth Model
RMS	Root Mean Square
SIC	Solid Inner Core
SLR	Satellite Laser Ranging
SOS	Sasao, Okubo, Saito (Sasao <i>et al.</i> , 1980)
SPICE	Spacecraft ephemeris, Planet, satellite, comet, or asteroid ephemerides, Instrument description kernel, Pointing kernel, Events kernel
TGP	Tide Generating Potential
TIO	Terrestrial Intermediate Origin
TOM	Tilt-Over Mode

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Notation	Definition
TRF	Terrestrial Reference Frame
TRS	Terrestrial Reference System
UT	Universal Time
VLBI	Very Long Baseline Interferometry
VSOP	Variations Séculaires des Orbites Planétaires