1

Introduction – Fundamental definitions – Motivation

1.1 Rotation and global shape of the Earth

At a very elementary level, the Earth is considered to be an axially symmetric ellipsoid, rotating with uniform angular speed about its symmetry axis, which is the polar axis passing through the Earth's center and its north and south poles; under the steady rotation, the direction of this axis maintains a fixed direction in space, i.e., relative to the directions of the "fixed stars." (The celestial objects that come closest to the ideal of remaining "fixed in space" are the quasars, the most distant extragalactic celestial objects.) The direction of the symmetry axis is at an inclination of about 23.5° to the direction of the normal to the plane of the Earth's orbit around the Sun (or more precisely, about the solar system barycenter, i.e. the center of mass of the solar system).

An axially symmetric ellipsoidal shape, bulging at the equator and flattened at the poles, and an internal structure with the same symmetry, would result from the centrifugal force associated with uniform Earth rotation around the polar axis, counterbalanced by gravity. The ellipsoidal structure computed on the basis of this balance of forces, assuming that the material of the rotating body behaves like a fluid under the incessant action of forces acting over very long timescales (i.e., that the resistance of even solid regions to shear deformation is overcome under such conditions), is called the "hydrostatic equilibrium ellipsoid." The Earth's figure (shape) does conform quite closely, though not perfectly, to that of such an ellipsoid. The equatorial radius of the Earth exceeds the polar radius by about 21 km; this is often described as the *equatorial bulge*. This bulge is to be viewed in relation to the mean radius of about 6371 km.

1.2 Orbit of the Earth

The force of gravitational attraction of the Sun on the total mass of the Earth holds the Earth in orbital motion around the Sun; similarly, the Earth's gravitational force maintains the orbital motion of the Moon around the Earth. Each of these two-body orbits in the Sun–Earth–Moon system would be elliptical, in accordance with Kepler's third law, if the third body were absent. In reality, small deviations from the planar elliptical nature of the orbits result from the gravitational attraction of the two individual bodies by the third body, and from the much smaller gravitational forces exerted by the planets.

1.3 Earth orientation – precession and nutation

Earth rotation can be separated into the rotation speed around its symmetry axis (figure axis) and the orientation of this axis (or another axis of the Earth) in space. In reality, Earth rotation and orientation are variable and even yield information on its interior structure. Most of us know that the rotation of a boiled egg noticeably differs from that of a raw egg. This simple observation shows that information on the inside of an egg can be obtained from its rotation. The same idea applies to the observation of the rotation and orientation of the Earth, relative to a "space-fixed" reference frame. (A space-fixed frame is one in which the directions of the most distant sources emitting in the radio frequency spectrum in the sky remain unchanged in time.)

Consider, in particular, the motion of the figure axis in space. It is a composite of two types of motion. The first is a secular motion, called precession, wherein the pole of the axis (which is on the celestial sphere, i.e., on a sphere of unit radius (in arbitrary units) in the "space-fixed" reference frame) traces a circular path at a constant rate (to the first order around J2000) on the celestial sphere, around the normal to the *ecliptic plane*. (The ecliptic plane is the plane of the orbit of the Earth–Moon barycenter (EMB) around the Sun. It is often loosely referred to as the Earth's orbital plane as mentioned in Appendix C.) The axis maintains a constant angle (to a first approximation) of about 23.5° to the normal to the ecliptic in the course of this (precessional) part of the motion, and hence it describes a cone with a half-angle of 23.5° with its axis along the normal.

The second part of the motion is a composite of numerous periodic motions, each of which manifests itself as an ellipse on the surface of the celestial sphere, with its center at the "mean pole," i.e., the spot on the precessional path where the pole would be if the short periodic motions did not exist. This elliptical motion can be resolved into oscillatory motions in two orthogonal directions on the celestial sphere: one over the circular precessional path on the celestial sphere, and the other in the orthogonal direction (towards/away from the normal to the ecliptic). The composite of all the periodic motions constitutes the nutation; it takes the pole of the axis up to a maximum of about 10 arcseconds away from the mean pole.

The above features are shared by the motions of the rotation and angular momentum axes too, see Chapter 2.

1.4 Primary cause of precession and nutation

As the Earth's rotation axis is tilted with respect to the orbital plane, the equatorial bulge is out of the equatorial plane during the orbital motion. As a result, the Sun and the Moon exert a gravitational torque on the Earth tending to twist the equator towards the orbital plane of the Sun/Moon relative to the Earth. As the Earth is rotating, it reacts to this torque like a spinning top to the gravitational pull on it. The main effect is precession, which is the slow motion of the rotation axis of the Earth around the normal to its orbital plane.

The precessional and nutational motions of the axes are thus primarily due to torques arising from the action of the gravitational potentials of the Moon and the Sun (and of the planets, to a minor extent) on what is loosely called the "equatorial bulge" of the Earth. The gravitational attraction being higher at points nearer to the celestial body than at more distant points, the mass of the equatorial bulge on the side nearer the celestial body gets pulled with a greater force than the bulge on the opposite side. As a result, the Earth gets subjected to a net torque (except at instants when the celestial body is on the equatorial plane), which tends to tilt the axis of the Earth in space; this is illustrated in Fig. 1.1 for the simplified model of a homogeneous and axially symmetric ellipsoidal Earth. It is the gyroscopic response of the rotating Earth to this torque that causes the directions of the various Earth-related axes to keep on varying.

The torque exerted by each of the celestial bodies on the Earth has a time independent part; it is the sum of these constant torques that generates the precessional motion, which is at a constant rate of about 50 arcseconds per year around the circle over the cone mentioned earlier. This motion is more precisely characterized as *luni-solar precession*. The term "luni-solar" recognizes the dominant roles of the Moon and the Sun in the various phenomena considered, but is generally used in the literature as inclusive of the small effects of the planets. We shall encounter later the "planetary precession," which is a misleading term because it does not involve any motion of the Earth's axes in space; it is only a reflection of the extremely slow tilting, caused by the action of the planets, of the reference plane in space (to be defined later) with respect to which the direction of the Earth's axis and hence its precession is defined (see Chapter 5).

The torque due to the solar system bodies has also a huge number of spectral components with frequencies that are related to those of the orbital motions of these bodies relative to the Earth. Each of these components produces a circular motion of the pole of whichever axis might be specified. The superposition of such motions

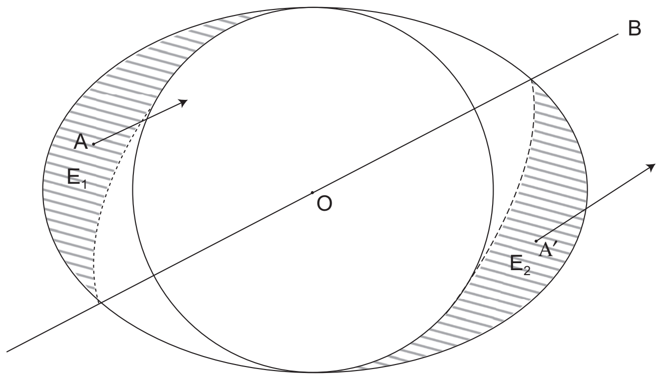

Figure 1.1 Gravitational forcing on the Earth. The figure shows the cross section of a hypothetical homogeneous ellipsoidal Earth with its inscribed sphere, and a celestial body B. Gravitational forces due to B acting on equal mass elements located symmetrically about the line passing through the center (O) and in the direction of B are of equal magnitude and produce equal torques, while in opposite directions, they cancel each other out. But the regions E_1 and E_2 outside the sphere are not symmetrical about the line in the direction of B. So there are net torques on these regions (say at A, A′), which do not cancel out: the torque on E_2 is stronger than that on E_1 because the former region is closer to the gravitating body B.

associated with all the spectral components of nonzero frequency constitutes the full nutation of the specified axis.

The maximum angular displacement due to nutation is about 10 arcseconds or 10 000 milliarcseconds (mas), as mentioned earlier. For this angular displacement, the motion in space of the intersection point of the axis with the mean spherical surface of the Earth is about 310 meters (3.1 cm/mas). The largest of the spectral components, by far, is the so-called Bradley nutation or principal nutation, with a period of about 18.6 years (Bradley, 1748); the major and minor axes of the elliptical motion with this period are about 9200 and 6800 mas, respectively. Other major nutations are already considerably smaller than the principal nutation; they have periods of approximately six months, 9.3 years, two weeks, one year, etc. The combined precession–nutation is illustrated schematically in Fig. 1.2 and as elliptical motions in Fig. 1.3 where nutation in longitude $\Delta\psi$ and obliquity $\Delta\epsilon$ are plotted against each other as functions of time.

1.5 Nutation of a non-rigid Earth

Nutation for a rigid Earth can easily be computed once the time dependence of the torque on the Earth is determined using the ephemerides, which give the relative positions of celestial bodies as functions of time (see Chapter 5). Those theoretical

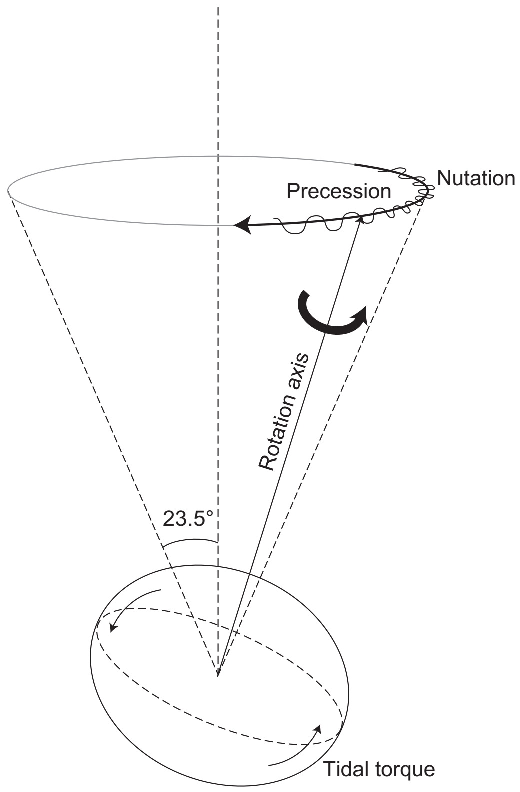

Figure 1.2 Precession and nutation. Precession is a smooth motion of the pole of the Earth's axis along the circle around the normal to the ecliptic plane (shown by the dashed vertical line), and the wiggly excursions from the precessional path represent nutation.

values, however, do not reproduce the observed values since the Earth is deformable and thus non-rigid, and also contains a liquid layer, namely the outer core, as mentioned in the next paragraph (Section 1.6). Thanks to the high precision of the observations by Very Long Baseline Interferometry (see Chapter 4), information on the interior of the Earth can be obtained through appropriate theoretical analyses of the VLBI data, which highly motivates the research in this field. In order to do so, models for the precession and nutation of a non-rigid body have been developed (see Chapter 7). These models are based on knowledge of the Earth's interior gained from seismic studies (see Section 1.6). The response of this Earth to a unit gravitational forcing is then computed. The nutation amplitudes are then obtained using the amplitude of the torque acting on the Earth for any particular forcing period and the response of the deformable Earth to unit forcing at that same forcing period.

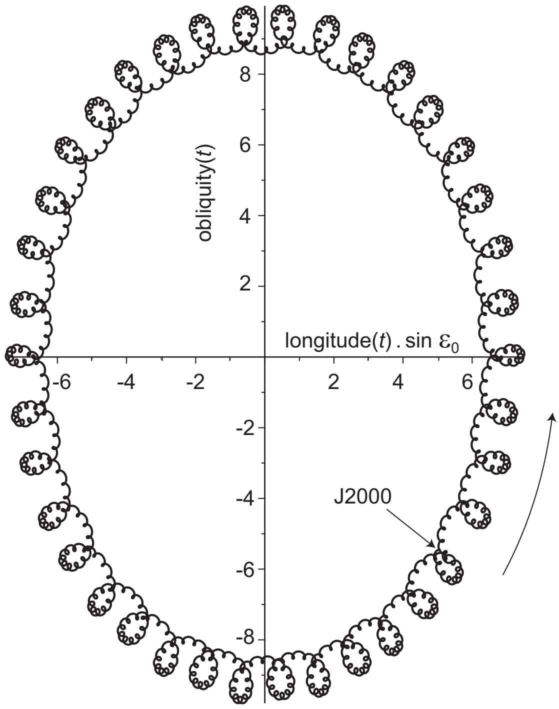

Figure 1.3 Nutation in longitude and obliquity as a function of time over 18.6 years. The elliptical feature represents the 18.6 year nutation, and the wiry loops around it are contributed by the semi-annual nutation and other nutations of still smaller amplitudes. Units: arcseconds.

It is apt to draw attention at this point to the existence of normal modes of the non-rigid Earth, which have a large influence on forced nutations at nearby frequencies (see Section 2.5).

1.6 Models of the Earth's interior

The Earth system consists of what is commonly referred to as the "solid Earth," plus the fluid layers at the surface, namely, the oceans and the atmosphere. The so-called solid Earth is, in reality, far from being wholly solid. It is made up of three major regions or layers: the outermost solid layer called the mantle, extending down from the mean outer radius of 6371 km to a mean radius of about 3480 km; a solid region with a mean radius of about 1220 km around the Earth's center, called the solid inner core (SIC); and a fluid region in between, called the fluid outer core (FOC). In constructing models for the structure and characteristics of the interior of the Earth (density, elastic properties, etc., as functions of position), time dependent deformations of all kinds are of course disregarded.

Information obtained from data on the travel times of seismic waves to various stations spread out over the Earth's surface, and from other data on the frequencies of seismic normal modes, is used to construct spherically symmetric models of the variation of density and elastic parameters of the matter from the center to the surface. This process involves the use of an appropriate procedure to remove the effects of the ellipticity of equidensity surfaces in the Earth's interior from the observational data. Of the many spherically symmetric models constructed in this fashion, the Preliminary Reference Earth Model (PREM; Dziewonski and Anderson, 1981) has been most widely used in recent times, superseding earlier models (e.g., 1066A of Gilbert and Dziewonski, 1975). These models are presented in the form of tables of values of the density, elastic parameters, gravity, etc., at various radial distances from the geocenter. Given such a model, hydrostatic equilibrium theory is employed to obtain the corresponding axially symmetric *ellipsoidal* model, in which the surfaces of equal density and equal elastic parameters (including, in particular, the Earth's exterior surface and internal boundaries such as the Core Mantle Boundary (CMB) and the Inner Core Boundary (ICB)) are axially symmetric and ellipsoidal.

Hydrostatic equilibrium Earth models have been of great importance to nutation theory, though the deviations from the hydrostatic equilibrium structure are not ignorable and have been taken into account in recent theoretical treatments. The deviations are of two kinds. The first kind is revealed through a slightly higher value for the dynamical ellipticity of the Earth, as inferred from precession–nutation studies, than the value that hydrostatic equilibrium theory would lead to. The excess ellipticity may be explained as the effect of the viscous forces resulting from the convective flow within the mantle as a consequence of the temperature gradient between the upper and lower boundaries of the mantle, the latter (the CMB) being at a much higher temperature than the former. The mantle material yields to these forces, persisting over millions of years, as if it were a highly viscous fluid. Secondly, the actual Earth is not strictly ellipsoidal. Deviations from the ellipsoidal structure are reflected in deviations of the Earth's own external gravitational potential from ellipsoidal symmetry, which manifest themselves through their effects on the orbital motions of near-Earth satellites; they are quantified by analyses (1) of Satellite Laser Ranging (SLR) data on these orbits, (2) of microwave data of relative position changes between twin satellites as from the GRACE[1] mission, (3) of data from accelerometers aboard satellites, as in the CHAMP[2] mission, (4) exploiting GPS satellite-to-satellite tracking data as well, and (5) of gradiometer data of acceleration differences over short distances between an ensemble of test masses inside a satellite, as in the GOCE[3] mission.

[1] GRACE stands for Gravity Recovery And Climate Experiment.
[2] CHAMP stands for CHAllenging Minisatellite Payload.
[3] GOCE stands for Gravity field and steady-state Ocean Circulation Explorer.

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Excerpt
More information

1.7 Precession, nutation, and geodynamics

The three regions/layers of the Earth's interior are not "locked together" in their rotational motions; they have considerable freedom to rotate about differing axes and at different rates. Therefore, it is necessary to clarify here that what is called the rotation vector in Section 1.3 was meant to refer to the angular velocity vector of rotation as observed at the surface of the Earth. It closely reflects the rotation of the mantle region as a whole. There are different angular velocity vectors for the fluid core region and for the SIC. Further elaboration of the concept of the rotation vector becomes necessary in the case of the fluid core region wherein the fluid flow deviates from being purely rotational in nature, primarily because the boundaries of the region are essentially ellipsoidal rather than spherical. This aspect and other refinements will be deferred to Chapter 7. See the paragraphs containing Eqs. (7.71) to (7.74) of that chapter.

1.8 The Earth's normal modes

The free motions are not driven by any external torque and depend on the interior constitution of the Earth. The free normal modes of high interest for nutation are the manifestations of compensatory variations of the rotation vectors of the different layers inside the Earth related to internal physics.

The three-layer Earth has four such modes: the Chandler wobble (CW) with a period of about −435 days in an Earth-fixed reference frame (the period −435 days means that the rotational motion has a period of 435 days in the sense opposite to that of the Earth's rotation); the free core nutation (FCN) with a period of about −430 days in the celestial frame, along with its associated wobble (the nearly diurnal free wobble or NDFW) which has nearly diurnal frequency in the terrestrial frame as its name implies. We use the term wobble in a very broad sense for any periodic or quasi-periodic motion of the Earth's rotation axis with respect to an Earth-fixed reference frame, irrespective of the frequency or the physical origin of the motion (see Section 2.6). For nutation computation it is also necessary to consider the less prominent free inner core nutation (FICN) and inner core wobble (ICW) that are explained in Section 2.20.

There are various mechanisms by which the rotation of one region influences the rotations of the others: the fluid pressure on the ellipsoidally shaped boundaries between regions; the torque due to the action of the gravitational field of the mantle (which is an ellipsoidal shell) on the ellipsoidal inner core when their symmetry axes are out of alignment; electromagnetic forces induced by differential rotation of adjoining regions in the presence of magnetic fields crossing their common boundary; viscous drag exerted by the core fluid on the solid regions; local topography on the solid side of the fluid–solid boundaries; and so on. The coupling of the three regions in this manner is an important aspect of the rotational response of the Earth to external torques as well as of the free rotational motions in the absence of external torques. In particular, while a wholly solid Earth has only a single rotational normal mode, the three-layer Earth has four: the Chandler wobble (which is the counterpart of the single mode of the wholly solid Earth) (CW); the free core nutation (FCN); the free inner core nutation (FICN) also called the prograde free core nutation (PFCN); and the inner core wobble (ICW). These modes are defined in Section 2.20. The resonance associated with the FCN is a prominent feature of the low frequency forced nutations, and the FICN/PFCN resonance too is not insignificant. The FICN and ICW modes would not arise if a SIC were not present.

1.9 Motivation for the book

The motivation for this book stems from the major advances that have taken place, in the very recent past, in the precision of observations of Earth rotation variations, and the advances made, in response to this development, in the theoretical modeling of the variations. The main focus is on developments in the theory of precession–nutation and polar motions. The precision of observations of the time dependent nutation has reached the sub-milliarcsecond level (of the order of 0.2 mas). The matching of these observations by the theoretical model adopted recently by the IAU (International Astronomical Union) and IUGG (International Union for Geodesy and Geophysics) is also at about the same level. (The precision of observations in the case of individual spectral components of the nutation is of the order of 10 microarcseconds or so, except for nutations of very long periods such as the 18.6 year nutation which have larger uncertainties.) Now, the contribution to nutation from the "non-rigidity" of the Earth – a term which is often used conveniently (if rather loosely) to include not only the deformability of the Earth but also the existence of the fluid core – is of the order of 60 mas. This is over a hundred times the current uncertainty in the observational and theoretical results indicated above. The potential for deriving information from nutation studies about aspects of the Earth's interior which influence nutations is then evident. This is a powerful motivation for such studies. Another motivation, not less important, arises from the fact that nutations leave their imprint on a variety of quantities of importance in geophysics, which are now measurable with very high precision through space geodetic and other means, and that the nutation contributions have to be separated out in order to bring out clearly the other geophysical information contained in the data. Examples are movements of observational sites, variations of the geopotential outside the Earth, gravity variations recorded by superconducting gravimeters, etc., caused by phenomena other than Earth rotation variations.

1.10 Organization of the book

The book is organized as follows. In Chapter 2 we introduce the concepts, but we have dedicated the complete Chapter 3 to the reference systems and frames in use in our book. It is indeed of high importance to define these frames precisely, as Earth rotation, precession, nutation, and polar motion relate the body-fixed terrestrial frame to the celestial frame "fixed in space." Recent concepts such as the *non-rotating origin* are shifted to the end of the book (see Chapter 12, more addressed to specialists in the domain).

In order to bring out clearly some of the essential aspects of nutations and wobbles, we provide in Chapter 2 the explicit solutions for a very simple model of the motion of the celestial body (see Section 2.24). This is very helpful for students beginning to study precession and nutation. Chapter 4 is dedicated to the observational methods in the study of precession and nutation as well as polar motion. Once the concepts and observational data are set up, we set the basis for the precession and nutation computation. Chapter 5 summarizes the rigid Earth nutation theories. Parts of this chapter may look quite complicated in addressing the Hamiltonian theory. However, we provide the necessary basis for the understanding of this theory. These parts may be left for specialists, in particular when the reader is a geophysicist. Then, the next chapters (Chapter 6 on the deformation of the Earth, Chapter 7 on the nutation of a non-rigid Earth, and Chapter 8 on anelasticity contribution) provide the theory of the non-rigid Earth as used for the last precession–nutation model adopted by the international community. As the Earth is not only a "solid body" but possesses fluid layers above its surface, it is necessary to correct the nutation for the existence of oceans and of an atmosphere, which is performed in Chapter 9. Chapter 10 provides the reader (specialized in the domain of nutation) with recent refinements of the non-rigid Earth nutation theory, which also provides clues to understanding the future steps in nutation theory (the chapter is addressed to those who want to go further in the modeling of precession and nutation). Chapter 11 treats the comparison of observation and theory.

Precession is a very useful observable for internal geophysics of the Earth because its rate is proportional to its dynamical flattening, reflecting the global mass repartition inside the Earth (different with respect to the equator than with respect to the direction of the rotation axis). Similarly to the Earth, Mars is a terrestrial planet (with an internal structure like the Earth's, with an iron core, a silicate mantle, a lithosphere, and a rocky crust), but smaller (its radius is about half the radius of the Earth), and rotating a little bit slower (24h 37min 23s or 1.026 days). It is thus flattened at the pole and its inclination is 25.19°, which is similar to the 23.439° of the Earth. Mars is further away from the Sun (about 1.5 times the