INCOME DISTRIBUTION DYNAMICS
OF ECONOMIC SYSTEMS

Econophysics has been used to study a range of economic and financial systems. This book uses the econophysical perspective to focus on the income distributive dynamics of economic systems. It focuses on the empirical characterization and dynamics of income distribution and its related quantities from the epistemological and practical perspectives of contemporary physics. Several income distribution functions are presented which fit income data and results obtained by statistical physicists on the income distribution problem. The book discusses two separate research traditions: the statistical physics approach; and the approach based on nonlinear trade-cycle models of macroeconomic dynamics. Several models of distributive dynamics based on the latter approach are presented, connecting the studies by physicists on distributive dynamics with the recent literature by economists on income inequality. As econophysics is such an interdisciplinary field, this book will be of interest to physicists, economists, statisticians, and applied mathematicians.

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INCOME DISTRIBUTION DYNAMICS
OF ECONOMIC SYSTEMS

An Econophysical Approach

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To my mother
and the memory of my father
When my information changes, I alter my conclusions.

*John Maynard Keynes*

It is a capital mistake to theorize before one has data. Insensibly one begins to twist facts to suit theories, instead of theories to suit facts.

*Arthur Conan Doyle,* “A Scandal in Bohemia” (1892)

the test of a first-rate intelligence is the ability to hold two opposed ideas in the mind at the same time, and still retain the ability to function.

*F. Scott Fitzgerald,* “The Crack-Up” (1936)
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Preface

I

The empirical characterization of the individual income distribution is an old topic in economics. The first systematic studies on this subject are generally attributed to the Italian economist Vilfredo Pareto (1848–1923) who, at the end of the nineteenth century, investigated this problem in a quantitative way and empirically noted the power-law nature of the income distribution among the richest persons in a society (Pareto, 1897, pp. 299–345). This result became known as the Pareto power law and is still valid today. Shortly afterwards, Max Otto Lorenz (1876–1959) and Corrado Gini (1884–1965) proposed income inequality measures which are still widely used today and also bear their names, the Lorenz curve (Lorenz, 1905), and the Gini coefficient (Gini, 1912).

Despite this promising start, the empirical characterization and dynamics of income distribution did not receive from mainstream economics the attention it deserves. Except for isolated initiatives, which did not go much further than the essentially descriptive work of these pioneers, mainstream economics has basically left this problem to the sidelines of economic research during most of the twentieth century (Atkinson, 1997).

It is uncertain why economists became so uninterested by such an important subject. Perhaps, they found the problem of the dynamic characterization of income distribution too difficult to deal with in view of the analytical and conceptual tools used by most economists during this period. Another possible reason could lie in the ideological choices that shape one’s views as to what is considered an important research theme. As such, economists may have seen this problem as an unimportant one, perhaps because they may have really believed that economic development would by itself solve economic inequality, and then assumed that there was no
need to do any research on this topic. Or, maybe, the reason for leaving this subject out in the cold for so long possibly lies in the fact that income distribution in modern economies is such a socially sensitive subject that rendered itself unattractive to one’s career advancement during a politically tumultuous century mostly dominated by ideological rivalry. Indeed, income distribution is a hot topic, both economically and politically, since it goes to the heart of society’s views on issues like opportunity, egalitarianism and the gap between rich and poor, which means that politics is never far behind when one deals with this subject. So, due to its inherently sensitive and potentially controversial nature, it also seems possible to suppose that several generations of economists assumed that their careers would be bettered by avoiding this topic, and then chose to concentrate on other problems.

Whatever the reasons, income distribution resurfaced as an important research subject in the early 2000s in part at least due to the activity of physicists who became interested in economic problems and started to deal with issues which until then were the almost exclusive domain of economists (Eugene Stanley et al., 2001; Doyne Farmer et al., 2005). Income distribution became then a research topic of the emerging field of *econophysics*, a term coined by the physicist Harry Eugene Stanley in the middle 1990s to name the use of physical concepts and methods to study economic problems. The work of econophysicists on the income distribution problem bore fruits in the form of new and important results which started to emerge just after the appearance of econophysics itself (e.g. Drăgulescu and Yakovenko, 2001a, 2001b, 2003).

The onset of a serious economic crisis in 2008 caught most economists by surprise (Krugman, 2009), a fact that brought about a major theoretical crisis in the mainstream economic theory (Buiter, 2009; Colander et al., 2009; Kirman, 2009; Keen, 2011b; Soos, 2012) and the process that this economic crisis entailed brought back to the spotlight renewed concerns about the income distribution and inequality problems (Krugman, 2014b). And if there were still doubts about the importance and sensitivity of these topics, the influential book by Thomas Piketty (2014) on the rising inequality worldwide, and the controversy that it immediately started (Irwin, 2014; Krugman, 2014a; Rankin, 2014; Wolfers, 2014) left no margin for such doubts (Palley, 2014). Indeed, Piketty greatly helped to bring this issue back to the spotlight not only among economists, but also to the society at large by extensively dealing with income and wealth distributions and proposing possible ways of counteracting the increasing polarization of both distributions in several societies.

It is in this context that this author felt that a book dealing with the distributive dynamics of economic systems in a broad theoretical and empirical sense from the viewpoint of econophysics is not only important, but also timely.
This book deals with the empirical characterization and dynamics of income distribution and its related quantities from the epistemological and practical perspectives of contemporary physics. In other words, economic problems are approached here from the methodological point of view of econophysics. The goal is to present a set of tools that may contribute in shedding light on the distributive dynamics of economic systems. Hence, it aims at presenting a set of approaches and methodologies that may allow us to go beyond the simple descriptive, often solely verbal, level when discussing this subject. Due to the youth of such an approach, the theories and models the reader will encounter in the next pages do not form a single, logically intertwined, set. Some of them are complementary to each other without a clear logical sequence, while others are even contradictory to one another. But, since most of them are empirically based, they offer a particular view of the problem of the dynamic description of income distribution and its related quantities, like wealth, both empirically and theoretically.

The fact that the personal income distribution is not described by an unique theory is not a handicap, but an advantage at this stage of our knowledge of the problem, and also reflects the fact that after being in practice excluded from the mainstream of economic research for such a long time, this issue has only recently resurfaced as an important research problem. As it will be thoroughly discussed in Chapter 1, science is not ever complete, but always limited. Hence, what this book aspires to offer is a set of viewpoints about income distribution characterization and dynamics as they are seen today from the perspective of econophysics, hopefully serving, therefore, as starting points for further analyses.

If the theories and models shown here prove useful, they will remain part of our common body of knowledge and will be further developed. By useful, I mean those that advance concepts and results that bring new perspectives to the problem at hand and can be developed and submitted to empirical verification. If they are not useful, they will be abandoned or superseded by other theories and models. This is the way science evolves. Physicists know this process very well since physics has gone through the steps of proposal, testing, refinement and, not infrequently, the effective abandonment of theories and models several times in its history. Physics has in fact a quite large collection of superseded theories and models which proved untenable empirically, and serve as a reminder of how slippery, misleading, and treacherous scientific research can be. So, physicists are unimpressed by such developments, but economists still seem to react very uneasily to this. One of the aims of this book is to argue that they should accept this
process as the natural way in which science evolves, however unpleasant it might be to the individuals who possess long held, but empirically flawed, theoretical convictions.

Another aim of this book is to bring together two separate research traditions, namely the approach to income distribution based on statistical physics and the one on nonlinear trade cycle models of macroeconomic dynamics. In both of these approaches to distributive dynamics the core variables are the income distribution components, expressed either as a result of trading, mostly kinetic exchange, or of class dynamics, in fact distributive competition.

This undertaking is, nevertheless, made by grounding the whole discussion on the epistemological viewpoints that have been used by physicists for over a century. Hence, Chapter 1 is considered essential reading on this respect, because it presents in general terms the epistemological perspective of the whole book in the context of the research object of both economics and econophysics, as well as some basic notions of the economic thought and a few general aspects of the history of both physics and economics.

III

This book is organized in three parts.

Part I discusses the basic topics and tools on the subject of income distribution required in most of the remaining parts of the book, namely the methodological basis of econophysics, its similarities and differences from economics, several distribution functions used to model the income data, including the Pareto power law, the basic inequality measures of the income distribution such as the Lorenz curve and the Gini coefficient. It also presents the evidence collected by both economists and physicists that bring empirical support to those distribution functions. A review of some topics of Thomas Piketty’s (2014) landmark book that this author regards as important to the econophysical approach to income distribution dynamics can also be found in this part.

Part II discusses income and wealth distributions from the viewpoint of statistical physics. After a brief discussion on uncertainty and risk in physics and economics, Part II reviews a series of models of the income distribution characterization and dynamics advanced mostly by physicists. The list of models is modest, but representative of current econophysics literature on the subject. The exponential income distribution takes a prominent role as it made it possible to connect the empirical income distributions to the Boltzmann–Gibbs distribution of energy and to present the problem under the viewpoint of money conservation. Other models based on
statistical mechanics are also reviewed, including those that discuss trade as elastic and inelastic collision of scattering particles in a typical physical setting. Most models presented in Part II of the book are essentially kinetic exchange models, which, nevertheless, offer a variety of interesting insights into the problem, as well as severe modeling limitations, both of these points being discussed at length. Such kind of modeling was also extended to other aspects of inequality of an economic system, such as the effective econophysical definition of income classes and the inequality of energy consumption. All economic theories, both orthodox and heterodox, are applied to the problem, provided they are able to bring new insights and open new perspectives to the distributive dynamics of economic systems.

Part III deals with income distribution under the realization that economic systems have circular flows. This dynamic viewpoint connects income classes to macroeconomic trade cycle theories, whose contributions were mainly due to economists. The initial sections present a typical economic viewpoint on issues like uncertainty, confidence, investment, and stability so that the models discussed afterwards are presented on solid conceptual foundations. The Goodwin (1967) macroeconomic dynamics of growth with cycles is viewed as an important start-up model, whose qualities and severe limitations, both theoretical and empirical, are discussed in detail. Then several other models based, or inspired, on this macrodynamics are presented and discussed. In this final part of the book some tools of dynamic systems theory are briefly used.

IV

The book is structured so as to be read in sequence, since several concepts discussed in previous chapters are used in the next ones. Nevertheless, some chapters are, to a reasonable extent, independent from the others and can be skipped by the hurried reader depending on his/her interests and level of knowledge. Thus, if the reader’s sole interest is in the models’ technicalities, Chapter 1 can be skipped, going directly to Chapter 2 where the most important income distribution functions are presented and discussed; Chapter 2 offers essential material for those whose interest is focused on income distribution models of statistical econophysics. On the other hand, if the interest lies exclusively in the guiding methodology and concepts on which the approach of the whole book is grounded, Chapter 1 offers a discussion on economics and econophysics that does not require any mathematics.

Alternatively, the hurried reader can go straightaway to Chapter 5 if the interest is limited to models based on trade cycles. The results of Chapter 2 are, nonetheless,
essential for the models presented in Chapter 4, and although Chapters 5 and 6 were written as a single logical unity to be read in sequence, they are mostly independent from the previous chapters. Chapter 3 on Thomas Piketty’s (2014) work can be read by and large independently from the rest of the book. The arrows diagram below indicates the logical dependency of the chapters. The full lines mean required dependency, whereas dashed lines stand for optional dependency.

Works that discuss subjects closely related to society’s political organization and inner workings frequently offer proposals and advice aimed at influencing state policy, as there are those who argue that advancing public policy recommendations is the purpose of such works. Notwithstanding, the reader will find no such advice or recommendations in this book, let alone proposals about how to “change the world.” This is so because this book was written with the intention of discussing how things are, rather than how they should, or must, be. Such stance arises from the realization that the scholars of past generations who dealt with these issues and felt the need to influence the political events of their times lived in social and cultural environments very different from ours, and, naturally, were driven by different motivations. But times change, and as for the researchers of the early twenty-first century, the time of writing, there are three important points that cannot be dismissed by anyone who analyzes these matters.

First, as it will be thoroughly discussed in Chapter 1, science by its very nature is incomplete and limited, so every scientific analysis and conclusion can be revised or even refuted. That may happen even to well-established theories as their domains of validity are oftentimes established well after their initial formulation and testing. Scientific analyses and conclusions must be subject to constant theoretical criticism and empirical testing and only time will tell if they survive such close scrutiny and reveal their strengths, weaknesses, domains of validity, and limitations, that is, to
what extent they correspond to the truth and how, when, and if they can be safely applied to the benefit of society. And it is impossible to predict beforehand the amount of time necessary for this process.

Thus, if one is aware of science’s own limitations this should be reason enough to exercise sober restraint and refrain from providing advice and recommendations that may affect society at large, as such advice could be based on theories and models which may not have been fully tested or whose conclusions may prove to be not as far reaching as initially thought, or even wrong at later times. So, premature proposals and recommendations based on not fully tested theories might lead to bad outcomes that have the potential to provoke loss of public confidence in the scientists who had gone public in favor of what later becomes discredited theories or, worse, in academic scholars in general. The basic point here is that although science does provide solutions to problems faced by society, solutions which vary from the technological ones to the understanding of society’s inner workings, finding these solutions and concluding that they are safe to use takes an unpredictable amount of time. Therefore, science cannot be expected to provide ready answers to society’s problems, and should not be put in the position of doing so, especially by scientists themselves.

Second, the most basic and essential goal of the scientific enterprise is to understand nature, that is, to reveal its inner workings. However, if the researcher starts the investigation also aiming at influencing policy, these two goals become mixed up. As a result, the introduction of interpretation biases into the scientist’s analysis and conclusions becomes an almost inevitability. That happens because it is all too common to have a tendency to interpret evidence as supporting the validity of the views one already holds, of reinforcing earlier convictions. Therefore, objectivity will end up compromised even when scientists firmly believe they are motivated by a noble cause. This results in the classical situation where the scientist may become too attached to a certain theory and loses perspective of his/her own biased analyses and conclusions. So, as the classical Greek playwright Euripides (480–406 BC) already knew a long time ago, “a bad end comes from a bad beginning” (Euripides, 2008, pp. 28–29: *Aeolus*).

Finally, it is important to recognize that science and politics are two different things and that most scientists lack the proper political training and advice to interfere in state politics. Thus, when scientists try to influence state police they run the risk of becoming entangled in political feuding, of which they often have little understanding and no control, and then possibly end up being used as political pawns. The chain of events that led to the Manhattan Project for the construction and use of the first nuclear bomb and its aftereffects remain in the collective memory of physicists as a reminder of these risks, since some of the physicists who played a key scientific role in that chain of events, but later raised objections
Preface

on moral and ethical grounds about the use of such device and the militaristic path taken by politics, were ostracized and publicly humiliated (Bird and Sherwin, 2006; Monk, 2012). As citizens, scientists are entitled to their own political views, but the condition of being a scientist should not be confused with that of a politician or activist.

So, the primary task of the scientist is to reveal the truth about nature. But, it is up to society to decide if, how, and when to use this acquired knowledge.
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The proposal of writing a book on income distribution from the viewpoint of econophysics was first raised during a conversation with Simon Capelin, the editor for physical sciences at Cambridge University Press. I am deeply grateful for his enthusiastic response to the early concept of this book and his continuous encouragement, such that I finally took the task of developing the initially vague plan into a feasible book project. I am also grateful to the staff at Cambridge University Press, especially to Sarah Lambert, for their help with some legal and technical aspects regarding publishing.

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