ALGORITHMIC AND HIGH-FREQUENCY TRADING

The design of trading algorithms requires sophisticated mathematical models, a solid analysis of financial data, and a deep understanding of how markets and exchanges function. In this textbook the authors develop models for algorithmic trading in contexts such as: executing large orders, market making, targeting VWAP and other schedules, trading pairs or collection of assets, and executing in dark pools. These models are grounded on how the exchanges work, whether the algorithm is trading with better informed traders (adverse selection), and the type of information available to market participants at both ultra-high and low frequency.

Algorithmic and High-Frequency Trading is the first book that combines sophisticated mathematical modelling, empirical facts and financial economics, taking the reader from basic ideas to the cutting edge of research and practice.

If you need to understand how modern electronic markets operate, what information provides a trading edge, and how other market participants may affect the profitability of the algorithms, then this is the book for you.

ÁLVARO CARTEA is a Reader in Financial Mathematics at University College London. Before joining UCL he was Associate Professor of Finance at Universidad Carlos III, Madrid-Spain (2009–2012) and from 2002 until 2009 he was a Lecturer (with tenure) in the School of Economics, Mathematics and Statistics at Birkbeck – University of London. He was previously JP Morgan Lecturer in Financial Mathematics at Exeter College, University of Oxford.

SEBASTIAN JAIMUNGAL is an Associate Professor and Chair, Graduate Studies in the Department of Statistical Sciences at the University of Toronto where he teaches in the PhD and Masters of Mathematical Finance programs. He consults for major banks and hedge funds focusing on implementing advance derivative valuation engines and algorithmic trading strategies. He is also an associate editor for the SIAM Journal on Financial Mathematics, the International Journal of Theoretical and Applied Finance, the journal Risks and the Argo newsletter. Jaimungal is the Vice Chair for the Financial Engineering & Mathematics activity group of SIAM and his research is widely published in academic and practitioner journals. His recent interests include High-Frequency and Algorithmic trading, applied stochastic control, mean-field games, real options, and commodity models and derivative pricing.

JOSÉ PENALVA is an Associate Professor at the Universidad Carlos III in Madrid where he teaches in the PhD and Master in Finance programmes, as well as at the undergraduate level. He is currently working on information models and market microstructure and his research has been published in Econometrica and other top academic journals.

ALGORITHMIC AND HIGH-FREQUENCY TRADING

ÁLVARO CARTEA
University College London

SEBASTIAN JAIMUNGAL
University of Toronto

JOSÉ PENALVA
Universidad Carlos III de Madrid

CAMBRIDGE UNIVERSITY PRESS

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781107091146

First published 2015

Printed in the United Kingdom by Bell and Bain Ltd

A catalogue record for this publication is available from the British Library

Library of Congress Cataloguing in Publication data
Cartea, Álvaro.
Algorithmic and high-frequency trading / Álvaro Cartea, Sebastian Jaimungal, José Penalva.
pages cm
Includes bibliographical references and index.
ISBN 978-1-107-09114-6 (Hardback : alk. paper)
1. Electronic trading of securities–Mathematical models. 2. Finance–Mathematical models.
3. Speculation–Mathematical models. I. Title.
HG4515.95.C387 2015
332.64–dc23 2015018946

ISBN 978-1-107-09114-6 Hardback

Additional resources for this publication at www.cambridge.org/9781107091146

To my girls, in order of appearance, Victoria, Amaya, Carlota, and Penélope.

— Á.C.

To my parents, Korisha and Paul, and my siblings Shelly, Cristina and especially my brother Curt for his constant injection of excitement and encouragement along the way.

— S.J.

To Nuria, Daniel, Jose María and Adelina.
For their patience and encouragement every step of the way, and for never losing faith.

— J.P.

Contents

Preface

We have written this book because we feel that existing ones do not provide a sufficiently broad view to address the rich variety of issues that arise when trying to understand and design a successful trading algorithm. This book puts together the diverse perspectives, and backgrounds, of the three authors in a manner that ties together the basic economics, the empirical foundations of high-frequency data, and the mathematical tools and models to create a balanced perspective of algorithmic and high-frequency trading.

This book has grown out of the authors' interest in the field of algorithmic and high-frequency finance and from graduate courses taught at University College London, University of Toronto, Universidad Carlos III de Madrid, IMPA, and University of Oxford. Readers are expected to have basic knowledge of continuous-time finance, but it assumes that they have no knowledge of stochastic optimal control and stopping. To keep the book self-contained, we include an appendix with the main stochastic calculus tools and results that are needed. The treatment of the material should appeal to a wide audience and it is ideal for a graduate course on Algorithmic Trading at a Master's or PhD level. It is also ideal for those already working in the finance sector who wish to combine their industry knowledge and expertise with robust mathematical models for algorithmic trading. We welcome comments! Please send them to `algo.trading.book@gmail.com`.

Brief guide to the contents

This book is organised into three parts that take the reader from the workings of electronic exchanges to the economics behind them, then to the relevant mathematics, and finally to models and problems of algorithmic trading.

Part I starts with a description of the basic elements of electronic markets and the main ways in which people participate in the market: as active traders exploiting an informational advantage to profit from possibly fleeting profit opportunities, or as market makers, simultaneously offering to buy and sell at advantageous prices.

A textbook on algorithmic trading would be incomplete if the development of strategies was not motivated by the information that market participants see in electronic markets. Thus it is necessary to devote space to a discussion of

data and empirical implications. The data allow us to present the context which determines the ultimate fate of an algorithm. By looking at prices, volumes, and the details of the limit order book, the reader will get a basic overview of some of the key issues that any algorithm needs to account for, such as the information in trades, properties of price movements, regularities in the intraday dynamics of volume, volatility, spreads, etc.

Part II develops the mathematical tools for the analysis of trading algorithms. The chapter on stochastic optimal control and stopping provides a pragmatic approach to material which is less standard in financial mathematics textbooks. It is also written so that readers without previous exposure to these techniques equip themselves with the necessary tools to understand the mathematical models behind some algorithmic trading strategies.

Part III of the book delves into the modelling of algorithmic trading strategies. The first two chapters are concerned with optimal execution strategies where the agent must liquidate or acquire a large position over a pre-specified window and trades continuously using only market orders. Chapter 6 covers the classical execution problem when the investor's trades impact the price of the asset and also adjusts the level of urgency with which she desires to execute the programme. In Chapter 7 we develop three execution models where the investor: i) carries out the execution programme as long as the price of the asset does not breach a critical boundary, ii) incorporates order flow in her strategy to take advantage of trends in the midprice which are caused by one-sided pressure in the buy or sell side of the market, and iii) trades in both a lit venue and a dark pool.

In Chapter 8 we assume that the investor's objective is to execute a large position over a trading window, but employs only limit orders, or uses both limit and market orders. Moreover, we show execution strategies where the investor also tracks a particular schedule as part of the liquidation programme.

Chapter 9 is concerned with execution algorithms that target volume-based schedules. We develop strategies for investors who wish to track the overall volume traded in the market by targeting: Percentage of Volume, Percentage of Cumulative Volume, and Volume Weighted Average Price, also known as VWAP.

The final three chapters cover various topics in algorithmic trading. Chapter 10 shows how market makers choose where to post limit orders in the book. The models that are developed look at how the strategies depend on different factors including the market maker's aversion to inventory risk, adverse selection, and short-term lived trends in the dynamics of the midprice.

Finally, Chapter 11 is devoted to statistical arbitrage and pairs trading, and Chapter 12 shows how information on the volume supplied in the limit order book is employed to improve execution algorithms.

Style of the book

In choosing the content and presentation of the book we have tried to provide a rigorous yet accessible overview of the main foundational issues in market

microstructure, and of some of the empirical themes of electronic trading, using the US equities market as the one most familiar to readers. These provide the basis for a thorough mathematical analysis of models of trade execution, volume-based algorithms, market making, statistical arbitrage, pairs trading, and strategies based on order flow information. Most chapters in Part III end with exercises of varying levels of difficulty. Some exercises closely follow the material covered in the chapter and require the reader to: solve some of the problems by looking at them from a different perspective; fill in the gaps of some of the derivations; see it as an invitation to experiment further. We have set up a website, `http://www.algorithmic-trading.org`, from which readers can download datasets and MATLAB code to assist in such experimentation.

This book does not cover any of the information technology aspects of algorithmic trading. Nor does it cover in detail certain aspects of market quality or discuss regulation issues.

Acknowledgements

We are thankful to those who took the time to read parts of the manuscript and gave us very useful feedback: Ali Al-Aradi, Gene Amromin, Robert Almgren, Ryan Francis Donnelly, Luhui Gan, John Goodacre, Hui Gong, Tianyi Jia, Hoi Kong, Tim Leung, Siyuan Li, Eddie Ng, Zhen Qin, Jason Ricci, Anton Rubisov, Mark Stevenson, Mikel Tapia and Jamie Walton. We also thank the students who have taken our courses at University College London, University of Toronto, University of Oxford, IMPA and Universidad Carlos III de Madrid.

Álvaro is grateful for the hospitality and generosity of the Finance Group at Saïd Business School, University of Oxford, with special thanks to Tim Jenkinson and Colin Mayer, and the Department of Statistical Sciences, University of Toronto, where a great deal of this book was written.

Sebastian is grateful for the hospitality of the Mathematical Institute, University of Oxford and the Department of Mathematics, University College London, where parts of this book were written.

José is grateful for the hospitality of the Department of Mathematics, University College London and the Department of Finance at Cass Business School where parts of this book were written, as well as his home institution, the Business Department of the Universidad Carlos III for allowing him to make these visits. He also wishes to thank Artem Novikov of TradingPhysics for his availability and help in accessing the data and clarifying specific issues faced by traders and technicians in high-frequency trading environments.

May 2015, Oxford, London, Toronto, Madrid, Mallorca

How to Read this Book

This book is aimed at those who want to learn how to develop the mathematical aspects of Algorithmic Trading. It is ideal for a graduate course on Algorithmic Trading at a Master's or PhD level, and is also ideal for those already working in the finance sector who wish to combine their industry knowledge and expertise with robust mathematical models for algorithmic trading.

Much of this book can be covered in an intensive one semester/term course as part of a Graduate course in Financial Mathematics/Engineering, Computational Finance, and Applied Mathematics. A typical student at this stage will be learning stochastic calculus as part of other courses, but will not be taught stochastic optimal control, or be proficient in the way modern electronic markets operate. Thus, they are strongly encouraged to read Part I of the book to: gain a good understanding of how electronic markets operate; understand basic concepts of microstructure theory that underpin how the market reaches equilibrium prices in the presence of different types of risks; and, study stylised statistical issues of the dynamics of the prices of stocks in modern electronic markets. And to read Part II to learn the stochastic optimal control tools which are essential to Part III where we develop sophisticated mathematical models for Algorithmic and High-Frequency trading.

Those with a solid understanding of stochastic calculus and optimal control, may skip Part II of the book and cover in detail Part III. However, we still encourage them to read Part I to gain an understanding of the stylised statistical features of the market, and to develop a better intuition of why algorithmic models are designed in particular ways or with specific objectives in mind.

For a shorter and more compact course on algorithmic trading, students should focus on learning about the limit order book, Chapter 1, then optimal control in Part II, and then concentrate on selected Chapters in Part III, for instance Chapters 6, 8 and 10.

Readers in the financial industry who have some knowledge of how electronic markets are organised may want to skip Chapter 1 but are encouraged to read the other chapters which cover microstructure theory and the empirical and statistical evidence of stock prices before delving into the details of the mathematical models in Part III.