

## ATOMS AND MOLECULES INTERACTING WITH LIGHT

### Atomic Physics for the Laser Era

This in-depth textbook with a focus on atom–light interactions prepares students for research in a fast-growing and dynamic field. Intended to accompany the laser-induced revolution in atomic physics, it is a comprehensive text for the emerging era in atomic, molecular, and optical science.

Utilizing an intuitive and physical approach, the text describes two-level atom transitions, including appendices on Ramsey spectroscopy, adiabatic rapid passage, and entanglement. With a unique focus on optical interactions, the authors present multi-level atomic transitions with dipole selection rules, along with M1/E2 and multiphoton transitions. Conventional atomic structure topics are discussed in some detail, beginning with the hydrogen atom, and these are interspersed with material rarely found in textbooks, such as an intuitive description of quantum defects. The final chapters examine modern applications and include many references to current research literature. The numerous exercises and multiple appendices throughout enable advanced undergraduate and graduate students to balance theory with experiment.

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Atomic Physics for the Laser Era

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## Preface

By any measure, atomic physics is among the fastest-growing, most dynamic, and best-recognized areas of physics. The student attendance at conferences devoted to this subject area has burgeoned, and the number of new university tenure track positions in atomic physics is disproportionately high. Recently there have been several documents from national and international science-oriented agencies extolling the growth and importance of atomic physics, with statements such as “Light influences our lives today in ways we could never have imagined a few decades ago,” and “AMO science (not only) provides the basis for new technology, it is also a source of the intellectual capital on which science and technology depend for growth and development.” The General Assembly of the United Nations and UNESCO declared 2015 as the “International Year of Light”, further underscoring its importance to the world. Applications are not restricted to further exploration of our specialized field, but have expanded to include substantial impact on other areas of physics such as condensed matter, quantum information, thermodynamics, and fluid mechanics. For these and other reasons, we have decided to provide an introductory text appropriate to the emerging laser era in atomic, molecular, and optical science.

This book is intended for multiple purposes. First and foremost, we are experimentalists, so the material is presented in an intuitive and very physical tone. Many ideas are developed from the classical physics perspective rather than from mathematical formalism. We try to connect the concepts with measurements where it makes sense to do so, and to motivate each topic by the observations that produced the information about it.

Our intent is to use it as a text for a course in atomic physics. It requires a knowledge of quantum mechanics at the level of the well-known textbooks by Griffiths or Liboff, and of elementary electricity and magnetism. Thus it is suitable for an advanced undergraduate course or a beginning graduate course (certainly for a course that serves both populations together). In addition, we have tried to write

in a sufficiently familiar style that a student can read and understand the material even without the benefit of a course. That is, the material is presented in a descriptive mode to maximize understanding. Some applications and detailed calculations relevant to a particular chapter are provided as appendices to the specific chapter.

Atomic physics underwent a renaissance with the advent of tunable lasers in the 1970s, and we have taken this revolution to heart. Modern atomic physics is intimately coupled to the interaction of atoms with laser light, and we have chosen to emphasize this aspect. Since almost all students in physics or chemistry have some minimal notions about atomic structure before they undertake a course devoted to this subject, this book begins with several chapters on transitions of two-level systems driven by a single frequency of light. We have chosen this approach because it is most appropriate for the frontiers of research in atomic physics at present. Most of the books currently available emphasize atomic structure first, and the interaction with light is treated as a secondary topic. Thus this book is distinct because its initial approach is based on the interaction of atoms with light as opposed to the elements of atomic structure.

The text begins with a discussion of classical physics as it relates to atomic physics because there are so many striking similarities. The second chapter has a discussion of two-level systems with appendices on Ramsey spectroscopy, adiabatic rapid passage, entanglement, and other topics. Then we generalize to multilevel systems with selection rules, the usual radiative approximations, and a discussion of electromagnetic fields from various sources. In the next chapter we relax these approximations to include M1/E2 transitions and multiphoton processes. The following chapters introduce spontaneous emission, the density matrix, field quantization, and several related topics.

These discussions of electromagnetic radiation at the start are not to suggest that atomic structure is neglected. Beginning with the hydrogen atom in Chapter 7, the conventional topics are discussed in some detail. After chapters on the fine and hyperfine structure of hydrogen including measurements of the Lamb shift and fine structure, there are chapters on helium and heavier atoms, followed by a treatment of external fields (Zeeman and Stark effects). What is important is that they are interspersed with material that is rarely found in textbooks, such as intuitive description of quantum defects. We have found that students rarely understand why the dominant transitions of many atoms are between two states of the same principal quantum number: the Bohr energy formula is often considered sacrosanct. This second part of the book continues with a discussion of the structure of Rydberg atoms, helium, and heavier atoms. These are followed by a chapter on molecular structure and a second one on the paradigm of molecular physics,  $H_2$ .

The third part of the book has separate chapters on various applications of the first two parts. These include but are not limited to laser cooling, trapping, BEC,

applications to fundamental physics such as parity violation, exotic atoms, and three-level systems. References are made to the earlier sections of the book where the underlying science has been introduced. Since the development of these applications are contemporary, there are many more references to the current literature than to the standard textbooks that discuss the topics of the first two parts. These are quite likely to be outdated by further progress, whereas the standard topics in Parts I and II will stand the test of time.

Some of the material in Parts I and II has a small overlap, including a few figures, with the early chapters of our previous work, “*Laser Cooling and Trapping*” [1], where the background needed to understand laser cooling and trapping is summarized. However, this book provides more depth and more complete description of the atomic physics needed for present-day research in this field. By contrast, Chapters 19 and 20 here present a summary of laser cooling and trapping, but much more detailed discussion can be found in Ref. [1]. Students are encouraged to consult that book for further studies.

The chapters are complemented by exercises of two types. Some of these exercises are simply mathematical calculations to derive something in the text that is dismissed with “it can be shown that . . .”. Others are extensions of the text where the students are asked to find something new based on a different approach than that taken in the text. Since most of Part III addresses current progress, it does not lend itself as well to exercises as do Parts I and II, so it contains only around 10% of the approximately 130 exercises in the book.

Although there are many systems of units in use, this book is restricted to SI units. Atomic units are very convenient and are summarized in an appendix, but SI units enable numerical evaluation of formulas needed for laboratory work where experimental parameters must be chosen. We think that the price of the frequent appearance of  $4\pi\epsilon_0$  and other constants is worth it.