## Continuum Mechanics and Thermodynamics of Matter

Aimed at advanced undergraduate and graduate students, this book provides a clear unified view of continuum mechanics that will be a welcome addition to the literature. Samuel Paolucci provides a well-grounded mathematical structure and also gives the reader a glimpse of how this material can be extended in a variety of directions, furnishing young researchers with the necessary tools to venture into brand new territory. Particular emphasis is given to the roles that thermodynamics and symmetries play in the development of constitutive equations for different materials.

*Continuum Mechanics and Thermodynamics of Matter* is ideal for a one-semester course in continuum mechanics, with 250 end-of-chapter exercises designed to test and develop the reader's understanding of the concepts covered. Six appendices enhance the material further, including a comprehensive discussion of the kinematics, dynamics, and balance laws applicable in Riemann spaces.

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# Preface

The goal of this text is to introduce students to the topic of continuum mechanics, with analysis of the kinematic and mechanical behavior of materials modeled under the continuum assumption. This includes the derivation of fundamental balance equations, based on the classical laws of physics, and the development of constitutive equations characterizing the behavior of idealized materials. Such background provides the starting point for the studies of thermoelasticity, fluid mechanics, and viscoelasticity that are provided in the text. Furthermore, the material covered also imparts students with sufficient background for studying more advanced topics in continuum mechanics, such as wave propagation, polar materials, mixture theory, shell theory, piezoelectricity, and electromagnetic and magnetohydrodynamic fluid mechanics.

A few years ago, I was involved in a project that required fundamental understanding of immiscible multiphase mixtures. I was not (and am still not) satisfied with the current formulations of continuum mechanics in this area, but this is a subject that will be taken up in future publications. Nevertheless, my extensive studies necessitated a deeper understanding of many aspects of single-phase continuum mechanics. Such studies provided me valuable insights and have enabled me to write the present book as an outgrowth of my efforts. At the same time, they have enabled me to become a better teacher of the subject. I hope that the results might be useful to other teachers and students as well. The book is intended for use by students in engineering, science, and applied mathematics. As pre-requisites, a student should have knowledge of multivariable calculus, linear algebra, and differential equations, which are standard in undergraduate programs of engineering and science.

I started writing this book in 2004 to fill a number of gaps that I felt limited my understanding and application of the beautiful theory of continuum mechanics – especially on the relation between continuum mechanics and thermodynamics. I became quite dissatisfied with existing textbooks. Some were delightful but superficial, others wonderful but ancient. Of course many excellent monographs existed, such as *The Classical Field Theories* by Truesdell and Toupin, *The Non-Linear Field Theories of Mechanics* by Truesdell and Noll, and *Mechanics of Continua* by Eringen. Unfortunately, such books were and have been out of print for quite a while and, in the case of the first two, they are challenging works that are not intended for use in a classroom. In the end, as I started to research the material, I fell in love with the subject. I sensed a unifying approach to teaching it that I wanted to develop and then to share. Since then, a number of good texts have appeared, but I feel that the need for the present book still exists. The present text is designed for a one-semester course in continuum mechanics. While covering the standard material, the book also provides a well-grounded mathematical structure and glimpses of how such material can be extended in a variety of directions. Thus, a major aim of the present text is not only providing a sound basis of continuum mechanics but, just as importantly, providing the tools for someone to venture into new territory. I hope that this aspect does not detract from the presentation and does not confuse the student.

Particularly in a subject such as continuum mechanics, many symbols and many fonts are used to refer to each specific quantity introduced. This is done to make the presentation clear. However, I have found this to be an absolutist approach that often burdens the reader to recall the meaning of way too many symbols. While I have retained the rigor, I have not tried to be an absolutist in this respect. Any symbol that is re-used, its meaning is made clear from the context. The text is divided into nine chapters, and each chapter includes exercise problems to test and extend the understanding of concepts presented.

Chapter 1 provides the essential understanding for the need of treating the behavior of common materials through the mathematical artifice of a continuum description. In addition, the different subject areas that make up continuum mechanics are introduced.

In Chapter 2, the essential mathematics for treating continuum problems is provided. Here, we define tensors and cover the algebra and multivariate calculus associated with these objects. In addition, we discuss integral theorems and their generalizations when discontinuous surfaces are present in a continuum region. Here, and throughout the text, we try to make the student comfortable in dealing with three different forms of representing tensors and the associated equations they enter in: by their representation of a coordinate-independent geometrical object, $\mathbf{A}$; by the matrix representing its components, $A$; and by the specific component elements, $a^i_j$.

Chapter 3 provides a comprehensive discussion of the kinematics of a continuum body. The deformation and motion of such a body are treated using Lagrangian and Eulerian descriptions. In addition, generalized balance laws are formulated and the important concept of frame-invariance is introduced and utilized.

Chapter 4 is devoted to the fundamental laws of mechanics and thermodynamics. The corresponding global and local forms of the governing equations are developed, and the role that discontinuous surfaces embedded within a continuum region play is discussed. In this chapter, we also consider the effects of the microstructure that underlies the continuum body and subsequently write the balance equations for polar materials. This is done to provide the student interested in this topic a starting point from which to pursue further studies (e.g., the modeling of liquid crystals). In order to focus on major concepts, the text following this chapter only deals with non-polar materials. In this chapter, the stress and couple stress tensors as well as the heat and entropy fluxes are naturally introduced and their properties discussed. In addition, we examine the local equations resulting from Euclidean and Galilean transformations and their implications.

Chapter 5 covers the principles of constitutive theory, where thermodynamics plays an essential role. This chapter provides a unifying theory regardless of the type of matter and forms the centerpiece of the text. The constitutive equations represent macro thermo-mechanical models of real materials. Here, the principles of frame-indifference, causality, equipresence, material smoothness, memory, symmetry, and thermodynamics are systematically utilized to obtain reduced forms of constitutive equations for general materials, and for solids and fluids in particular. Tables are provided for expedient formulations of constitutive equations of isotropic materials. The development of such tables is clearly described and illustrated. Thermodynamics plays an integral part of constitutive theory and many thermodynamic tensor quantities are developed – these reduce to well-known scalar quantities encountered in classical thermodynamics. Formulations are given using the different thermodynamic potentials of internal energy, entropy, Helmholtz free energy, Gibbs free energy, and enthalpy, and the corresponding Maxwell relations provide very useful relations among thermodynamic quantities. Here we also discuss the concepts of thermodynamic equilibrium and stability. In addition, the critical role that the second law of thermodynamics plays in the reduction of constitutive equations is explored using the conventional Coleman–Noll procedure and the more general Müller–Liu procedure that makes use of Lagrange multipliers. Lastly, in this chapter we provide a comprehensive discussion of jump conditions across discontinuous surfaces, and their role in describing material and non-material singular surfaces, including boundary conditions, shocks, and phase-change interfaces.

Chapter 6 is provided to clearly illustrate many of the constitutive theory concepts to the case of spatially uniform material bodies. Here, many of the thermodynamic concepts can easily be applied within a mathematical setting that is not overly burdensome to the student.

This is followed by Chapter 7 where constitutive equations of thermoelastic solids are rigorously developed using the Coleman–Noll procedure. Material symmetries and crystal microscopic structures are fully discussed and linear and nonlinear constitutive equations for non-isotropic and isotropic thermoelastic solids are considered. In addition, we examine a number of fundamental nonlinear equilibrium deformations.

Fluids are discussed in Chapter 8. Here, we provide a rigorous development of constitutive equations using both the Coleman–Noll and the Müller–Liu procedures. General representations of the stress tensor and heat flux are provided. Their simplifications leading to Euler equations, Newtonian equations, the second-order representation, and the Reiner–Rivlin fluid are fully developed. Lastly, comprehensive classifications of fluid motions are provided. The classifications are in the general areas of kinematically restricted types of motions, specialized equations of motion, and specialized constitutive equations.

In Chapter 9, we treat the subject of viscoelasticity. The additional kinematics considerations, aspects of constitutive theory, and general classes of motions of materials having memory are provided. Lastly, the concept of fading memory and application of finite linear viscoelasticity undergoing simple deformations are considered. The treatment of phenomenological constitutive equations, while important, is intentionally left out.

The book includes six appendices that enhance the material presented in the chapters. Of particular note is an appendix that provides a comprehensive discussion of the kinematics, dynamics, and balance laws applicable in Riemannian spaces, such as arbitrary surfaces and curves embedded in the three-dimensional Euclidean space.

Lastly, bibliographies pertinent to material provided in each individual chapter is given at the end of the specific chapter.

I would like to conclude by thanking Jim Jenkins, who first introduced me to continuum mechanics while I was a graduate student in Theoretical and Applied Mechanics at Cornell University, and a number of authors who provided me guidance and inspiration throughout my journey in understanding continuum mechanics and thermodynamics of material bodies. Foremost among them, in alphabetical order, R. Aris, R.M. Bowen, H.B. Callen, D.B. Coleman, D.G.B. Edelen, J.L. Ericksen, A.C. Eringen, I-S. Liu, I. Müeller, W. Noll, R.S. Rivlin, G.F. Smith, A.J.M. Spencer, R. Toupin, and C. Truesdell. In addition, I would like to thank the many students who have taken my course in Continuum Mechanics at the University of Notre Dame; they have provided me useful feedback through multiple versions of the material. In particular, I would like to thank Dr. Gianluca Puliti who drew most of the figures in the text.

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