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Continuum Mechanics and Thermodynamics of Matter

Aimed at advanced undergraduate and graduate students, this book provides a clear unified view of continuum mechanics that will be a welcome addition to the literature. Samuel Paolucci provides a well-grounded mathematical structure and also gives the reader a glimpse of how this material can be extended in a variety of directions, furnishing young researchers with the necessary tools to venture into brand new territory. Particular emphasis is given to the roles that thermodynamics and symmetries play in the development of constitutive equations for different materials.

Continuum Mechanics and Thermodynamics of Matter is ideal for a one-semester course in continuum mechanics, with 250 end-of-chapter exercises designed to test and develop the reader's understanding of the concepts covered. Six appendices enhance the material further, including a comprehensive discussion of the kinematics, dynamics, and balance laws applicable in Riemann spaces.

S. Paolucci is currently Professor of Aerospace and Mechanical Engineering and Director of C-SWARM at the University of Notre Dame.

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Contents

Preface	Page xi
1 Introduction	1
1.1 Continuum mechanics	2
1.2 Continuum	3
1.3 Mechanics	4
1.3.1 Deformation and strain	5
1.3.2 Stress field	6
1.4 Thermodynamics	6
1.5 Constitutive theory	8
1.5.1 Solids	8
1.5.2 Fluids	9
1.6 Pioneers of continuum mechanics	10
Bibliography	10
2 Tensor analysis	13
2.1 Review of linear algebra	13
2.2 Tensor algebra	16
2.3 The metric tensor and its properties	22
2.4 General polyadic tensor of order m	24
2.5 Scalar product of two vectors	27
2.6 Vector product of two vectors	27
2.7 Tensor product of two vectors	29
2.8 Contraction of tensors	29
2.9 Transpose of a tensor	31
2.10 Symmetric and skew-symmetric tensors	32
2.11 Dual of a tensor	37
2.12 Exterior product	39
2.13 Tensor fields	41
2.13.1 Cartesian coordinate system	41
2.13.2 Curve in space	42
2.13.3 Derivatives	42
2.13.4 Surface in space	45
2.13.5 Curvilinear coordinate system	45
2.14 Gradient of a scalar field	49
2.15 Gradient of a vector field	50

2.16	Covariant differentiation of a vector	52
2.17	Divergence of a vector field	55
2.18	Curl of a vector field	56
2.19	Orthogonal curvilinear coordinate system	58
2.19.1	Physical components	59
2.19.2	Gradient of a scalar field	60
2.19.3	Gradient and divergence of a vector field	60
2.19.4	Curl of a vector field	61
2.19.5	Laplacian of a scalar field	61
2.19.6	Divergence of a dyadic tensor field	62
2.20	Integral theorems and generalizations	62
2.20.1	Regions with discontinuous surfaces, curves, and points	63
	Bibliography	70
3	Kinematics	73
3.1	Deformation	76
3.1.1	Deformation gradient	76
3.1.2	Transformation of linear elements	77
3.1.3	Transformation of a surface element	80
3.1.4	Transformation of a volume element	82
3.1.5	Relations between deformation and inverse deformation gradients	83
3.1.6	Identities of Euler–Piola–Jacobi	84
3.1.7	Cayley–Hamilton theorem	85
3.1.8	Real symmetric matrices	88
3.1.9	Polar decomposition theorem	91
3.1.10	Strain kinematics	95
3.1.11	Compatibility conditions	98
3.2	Motion	100
3.2.1	Velocity and acceleration	101
3.2.2	Path lines, stream lines, and streak lines	104
3.2.3	Relative deformation	106
3.2.4	Stretch and spin	110
3.2.5	Kinematical significance of D and W	112
3.2.6	Kinematics and dynamical systems	115
3.2.7	Internal angular velocity and acceleration	119
3.3	Objective tensors	120
3.3.1	Apparent velocity	122
3.3.2	Apparent acceleration	124
3.3.3	Properties of kinematic quantities	125
3.3.4	Corotational and convected derivatives	128
3.3.5	Push-forward and pull-back operations	129
3.4	Transport theorems	131
3.4.1	Material derivative of a line integral	131
3.4.2	Material derivative of a surface integral	134
3.4.3	Material derivative of a volume integral	136
	Bibliography	146

4	Mechanics and thermodynamics	149
4.1	Balance law	149
4.2	Fundamental axioms of mechanics	151
4.3	Fundamental axioms of thermodynamics	154
4.4	Forces and moments	156
4.5	Rigid body dynamics	158
4.6	Stress and couple stress hypotheses	161
4.6.1	Stress and couple stress tensors	163
4.7	Local forms of axioms of mechanics	165
4.8	Properties of stress vector and tensor	169
4.8.1	Principal stresses and principal stress directions	169
4.8.2	Mean stress and deviatoric stress tensor	172
4.8.3	Lamé’s stress ellipsoid	172
4.8.4	Mohr’s circles	173
4.9	Work and heat	174
4.10	Heat flux hypothesis	175
4.11	Entropy flux hypothesis	176
4.12	Local forms of axioms of thermodynamics	178
4.13	Field equations in Euclidean frames	181
4.14	Jump conditions in Euclidean frames	183
	Bibliography	188
5	Principles of constitutive theory	191
5.1	General constitutive equation	192
5.2	Frame indifference	194
5.3	Temporal material smoothness	196
5.4	Spatial material smoothness	196
5.5	Spatial and temporal material smoothness	198
5.6	Material symmetry	199
5.7	Reduced constitutive equations	208
5.7.1	Constitutive equation for a simple isotropic solid	212
5.7.2	Constitutive equation for a simple (isotropic) fluid	212
5.8	Isotropic and hemitropic representations	213
5.9	Expansions of constitutive equations	215
5.10	Thermodynamic considerations	216
5.10.1	Thermodynamic states	216
5.10.2	Thermodynamic potentials	226
5.10.3	Thermodynamic processes	232
5.10.4	Thermodynamic equilibrium and stability	235
5.10.5	Potential energy and strain energy	239
5.11	Entropy and nonequilibrium thermodynamics	242
5.11.1	Coleman–Noll procedure	242
5.11.2	Müller–Liu procedure and Lagrange multipliers	242
5.12	Jump conditions	244
5.12.1	Characterization of jump conditions	245
5.12.2	Material singular surface	248
5.12.3	Equilibrium jump conditions	251
	Bibliography	264

6	Spatially uniform systems	271
6.1	Material with no memory	272
6.2	Material with short memory of volume	275
6.3	Material with longer memory of volume	276
6.4	Material with short memory	278
	Bibliography	281
7	Thermoelastic solids	283
7.1	Clausius–Duhem inequality	284
7.2	Material symmetries	289
7.3	Linear deformations of anisotropic materials	296
7.3.1	Propagation of elastic waves in crystals	299
7.4	Nonlinear deformations of anisotropic materials	304
7.5	Linear deformations of isotropic materials	305
7.6	Nonlinear deformations of isotropic materials	306
7.6.1	Special nonlinear deformations	309
	Bibliography	335
8	Fluids	339
8.1	Coleman–Noll procedure	340
8.2	Müller–Liu procedure	342
8.3	Representations of \mathbf{q}^d and $\boldsymbol{\sigma}^d$	347
8.4	Propagation of sound	357
8.5	Classifications of fluid motions	360
8.5.1	Restrictions on the type of motion	360
8.5.2	Specializations of the equations of motion	371
8.5.3	Specializations of the constitutive equations	372
	Bibliography	380
9	Viscoelasticity	383
9.1	Introduction	383
9.2	Kinematics	386
9.2.1	Motion with constant stretch history	390
9.3	Constitutive equations	396
9.3.1	Constitutive equations for motion with constant stretch history	399
9.3.2	Fading memory	406
9.3.3	Constitutive equations of differential type	408
9.3.4	Constitutive equations of integral type	410
9.3.5	Constitutive equations of rate type	417
	Bibliography	432
	Appendices	437
A	Summary of Cartesian tensor notation	439
	Bibliography	441
B	Isotropic tensors	442
	Bibliography	446
C	Balance laws in material coordinates	447

<i>CONTENTS</i>	ix
	Bibliography 448
D	Curves and surfaces in space 449
	D.1 Space curve 451
	D.2 Balance law for a space curve 454
	D.3 Space surface 455
	D.4 Balance law for a flux through a space surface 473
	Bibliography 482
E	Representation of isotropic tensor fields 483
	E.1 Scalar function 483
	E.2 Vector function 483
	E.3 Symmetric tensor function 484
	Bibliography 485
F	Legendre transformations 487
	Bibliography 489
Index	491

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Preface

The goal of this text is to introduce students to the topic of continuum mechanics, with analysis of the kinematic and mechanical behavior of materials modeled under the continuum assumption. This includes the derivation of fundamental balance equations, based on the classical laws of physics, and the development of constitutive equations characterizing the behavior of idealized materials. Such background provides the starting point for the studies of thermoelasticity, fluid mechanics, and viscoelasticity that are provided in the text. Furthermore, the material covered also imparts students with sufficient background for studying more advanced topics in continuum mechanics, such as wave propagation, polar materials, mixture theory, shell theory, piezoelectricity, and electromagnetic and magnetohydrodynamic fluid mechanics.

A few years ago, I was involved in a project that required fundamental understanding of immiscible multiphase mixtures. I was not (and am still not) satisfied with the current formulations of continuum mechanics in this area, but this is a subject that will be taken up in future publications. Nevertheless, my extensive studies necessitated a deeper understanding of many aspects of single-phase continuum mechanics. Such studies provided me valuable insights and have enabled me to write the present book as an outgrowth of my efforts. At the same time, they have enabled me to become a better teacher of the subject. I hope that the results might be useful to other teachers and students as well. The book is intended for use by students in engineering, science, and applied mathematics. As pre-requisites, a student should have knowledge of multivariable calculus, linear algebra, and differential equations, which are standard in undergraduate programs of engineering and science.

I started writing this book in 2004 to fill a number of gaps that I felt limited my understanding and application of the beautiful theory of continuum mechanics – especially on the relation between continuum mechanics and thermodynamics. I became quite dissatisfied with existing textbooks. Some were delightful but superficial, others wonderful but ancient. Of course many excellent monographs existed, such as *The Classical Field Theories* by Truesdell and Toupin, *The Non-Linear Field Theories of Mechanics* by Truesdell and Noll, and *Mechanics of Continua* by Eringen. Unfortunately, such books were and have been out of print for quite a while and, in the case of the first two, they are challenging works that are not intended for use in a classroom. In the end, as I started to research the material, I fell in love with the subject. I sensed a unifying approach to teaching it that I wanted to develop and then to share. Since then, a number of good texts have appeared, but I feel that the need for the present book still exists. The present text is designed for a one-semester course in continuum mechanics. While cover-

ing the standard material, the book also provides a well-grounded mathematical structure and glimpses of how such material can be extended in a variety of directions. Thus, a major aim of the present text is not only providing a sound basis of continuum mechanics but, just as importantly, providing the tools for someone to venture into new territory. I hope that this aspect does not detract from the presentation and does not confuse the student.

Particularly in a subject such as continuum mechanics, many symbols and many fonts are used to refer to each specific quantity introduced. This is done to make the presentation clear. However, I have found this to be an absolutist approach that often burdens the reader to recall the meaning of way too many symbols. While I have retained the rigor, I have not tried to be an absolutist in this respect. Any symbol that is re-used, its meaning is made clear from the context. The text is divided into nine chapters, and each chapter includes exercise problems to test and extend the understanding of concepts presented.

Chapter 1 provides the essential understanding for the need of treating the behavior of common materials through the mathematical artifice of a continuum description. In addition, the different subject areas that make up continuum mechanics are introduced.

In Chapter 2, the essential mathematics for treating continuum problems is provided. Here, we define tensors and cover the algebra and multivariate calculus associated with these objects. In addition, we discuss integral theorems and their generalizations when discontinuous surfaces are present in a continuum region. Here, and throughout the text, we try to make the student comfortable in dealing with three different forms of representing tensors and the associated equations they enter in: by their representation of a coordinate-independent geometrical object, \mathbf{A} ; by the matrix representing its components, A ; and by the specific component elements, a_j^i .

Chapter 3 provides a comprehensive discussion of the kinematics of a continuum body. The deformation and motion of such a body are treated using Lagrangian and Eulerian descriptions. In addition, generalized balance laws are formulated and the important concept of frame-invariance is introduced and utilized.

Chapter 4 is devoted to the fundamental laws of mechanics and thermodynamics. The corresponding global and local forms of the governing equations are developed, and the role that discontinuous surfaces embedded within a continuum region play is discussed. In this chapter, we also consider the effects of the microstructure that underlies the continuum body and subsequently write the balance equations for polar materials. This is done to provide the student interested in this topic a starting point from which to pursue further studies (e.g., the modeling of liquid crystals). In order to focus on major concepts, the text following this chapter only deals with non-polar materials. In this chapter, the stress and couple stress tensors as well as the heat and entropy fluxes are naturally introduced and their properties discussed. In addition, we examine the local equations resulting from Euclidean and Galilean transformations and their implications.

Chapter 5 covers the principles of constitutive theory, where thermodynamics plays an essential role. This chapter provides a unifying theory regardless of the type of matter and forms the centerpiece of the text. The constitutive equations represent macro thermo-mechanical models of real materials. Here, the principles of frame-indifference, causality, equipresence, material smoothness, memory, sym-

PREFACE

xiii

metry, and thermodynamics are systematically utilized to obtain reduced forms of constitutive equations for general materials, and for solids and fluids in particular. Tables are provided for expedient formulations of constitutive equations of isotropic materials. The development of such tables is clearly described and illustrated. Thermodynamics plays an integral part of constitutive theory and many thermodynamic tensor quantities are developed – these reduce to well-known scalar quantities encountered in classical thermodynamics. Formulations are given using the different thermodynamic potentials of internal energy, entropy, Helmholtz free energy, Gibbs free energy, and enthalpy, and the corresponding Maxwell relations provide very useful relations among thermodynamic quantities. Here we also discuss the concepts of thermodynamic equilibrium and stability. In addition, the critical role that the second law of thermodynamics plays in the reduction of constitutive equations is explored using the conventional Coleman–Noll procedure and the more general Müller–Liu procedure that makes use of Lagrange multipliers. Lastly, in this chapter we provide a comprehensive discussion of jump conditions across discontinuous surfaces, and their role in describing material and non-material singular surfaces, including boundary conditions, shocks, and phase-change interfaces.

Chapter 6 is provided to clearly illustrate many of the constitutive theory concepts to the case of spatially uniform material bodies. Here, many of the thermodynamic concepts can easily be applied within a mathematical setting that is not overly burdensome to the student.

This is followed by Chapter 7 where constitutive equations of thermoelastic solids are rigorously developed using the Coleman–Noll procedure. Material symmetries and crystal microscopic structures are fully discussed and linear and nonlinear constitutive equations for non-isotropic and isotropic thermoelastic solids are considered. In addition, we examine a number of fundamental nonlinear equilibrium deformations.

Fluids are discussed in Chapter 8. Here, we provide a rigorous development of constitutive equations using both the Coleman–Noll and the Müller–Liu procedures. General representations of the stress tensor and heat flux are provided. Their simplifications leading to Euler equations, Newtonian equations, the second-order representation, and the Reiner–Rivlin fluid are fully developed. Lastly, comprehensive classifications of fluid motions are provided. The classifications are in the general areas of kinematically restricted types of motions, specialized equations of motion, and specialized constitutive equations.

In Chapter 9, we treat the subject of viscoelasticity. The additional kinematics considerations, aspects of constitutive theory, and general classes of motions of materials having memory are provided. Lastly, the concept of fading memory and application of finite linear viscoelasticity undergoing simple deformations are considered. The treatment of phenomenological constitutive equations, while important, is intentionally left out.

The book includes six appendices that enhance the material presented in the chapters. Of particular note is an appendix that provides a comprehensive discussion of the kinematics, dynamics, and balance laws applicable in Riemannian spaces, such as arbitrary surfaces and curves embedded in the three-dimensional Euclidean space.

Lastly, bibliographies pertinent to material provided in each individual chapter

is given at the end of the specific chapter.

I would like to conclude by thanking Jim Jenkins, who first introduced me to continuum mechanics while I was a graduate student in Theoretical and Applied Mechanics at Cornell University, and a number of authors who provided me guidance and inspiration throughout my journey in understanding continuum mechanics and thermodynamics of material bodies. Foremost among them, in alphabetical order, R. Aris, R.M. Bowen, H.B. Callen, D.B. Coleman, D.G.B. Edelen, J.L. Ericksen, A.C. Eringen, I-S. Liu, I. Müller, W. Noll, R.S. Rivlin, G.F. Smith, A.J.M. Spencer, R. Toupin, and C. Truesdell. In addition, I would like to thank the many students who have taken my course in Continuum Mechanics at the University of Notre Dame; they have provided me useful feedback through multiple versions of the material. In particular, I would like to thank Dr. Gianluca Puliti who drew most of the figures in the text.

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