INFLATION AND STRING THEORY

The past two decades have seen transformative advances in cosmology and string theory. Observations of the cosmic microwave background have revealed strong evidence for a period of inflationary expansion in the very early universe, while new insights about compactifications of string theory have led to a deeper understanding of inflation in a framework that unifies quantum mechanics and general relativity.

Written by two of the leading researchers in the field, this complete and accessible volume provides a modern treatment of inflationary cosmology and its connections to string theory and elementary particle theory. After an up-to-date experimental summary, the authors present the foundations of effective field theory, string theory, and string compactifications, setting the stage for a detailed examination of models of inflation in string theory. Three appendices contain background material in geometry and cosmological perturbation theory, making this a self-contained resource for graduate students and researchers in string theory, cosmology, and related fields.

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To Anna, Fritz, and Julian

and

to Josephine, Aelwen, and Vala

Contents

Preface

The past two decades of advances in observational cosmology have brought about a revolution in our understanding of the universe. Observations of type Ia supernovae [1, 2], measurements of temperature fluctuations in the cosmic microwave background (CMB) – particularly by the *Wilkinson Microwave Anisotropy Probe* (WMAP) [3–7] and the *Planck* satellite [8–10] – and maps of the distribution of large-scale structure (LSS) [11] have established a standard model of cosmology, the ΛCDM model. This is a universe filled with 68% dark energy, 27% dark matter, and only 5% ordinary atoms [8]. There is now decisive evidence that large-scale structures were formed via gravitational instability of primordial density fluctuations, and that these initial perturbations originated from quantum fluctuations [12–17], stretched to cosmic scales during a period of inflationary expansion [18–20]. However, the microphysical origin of inflation remains a mystery, and it will require a synergy of theory and observations to unlock it.

In the standard cosmology without inflation, causal signals travel a finite distance between the time of the initial singularity and the time of formation of the first neutral atoms. However, the CMB anisotropies display vivid correlations on scales larger than this distance. This causality puzzle is known as the *horizon problem*. The horizon problem is resolved if the early universe went through an extended period of *inflationary expansion*, i.e. expansion at a nearly constant rate, with

$$|\dot{H}| \ll H^2 \ ,$$

where $H \equiv \dot{a}/a$ is the Hubble parameter associated with a Friedmann–Robertson–Walker spacetime,

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{d}\boldsymbol{x}^2 \ .$$

Because space expands quasi-exponentially during inflation, $a(t) \propto e^{Ht}$, homogeneous initial conditions on subhorizon scales are stretched to apparently acausal superhorizon scales. Besides explaining the overall homogeneity of the universe, inflation also creates small primordial inhomogeneities, which eventually provide the seeds for large-scale structures. These perturbations are inevitable in a quantum-mechanical treatment of inflation: viewed as a quantum field, the expansion rate H experiences local zero-point fluctuations, $\delta H(t, \boldsymbol{x})$, which lead to spatial variations in the density after inflation, $\delta\rho(t, \boldsymbol{x})$. If inflation is correct, then CMB observations are probing the quantum origin of structure in the universe. By measuring the statistical properties of the CMB anisotropies we learn about the physics of inflation and about the precise mechanism that created the primordial seed fluctuations.

In this book we will describe two intertwined approaches to the physics of inflation: from the bottom up in effective field theory (EFT), and from the top down in string theory.

We speak of an *effective theory* when we do not resolve the small-scale (or high-energy) details of a more fundamental theory. Often this coarse-graining is done so automatically that it is not explicitly mentioned: for instance, we describe fluid dynamics and thermodynamics without reference to atoms, and computations of atomic spectra are in turn insensitive to the quark substructure of nucleons. Reasoning in terms of theories valid up to a critical energy scale is also how the history of particle physics developed, long before Wilson formalized the concept of effective theories. Effective theories are particularly useful when the full theory is unknown, or is specified but is not computable. In that case, one parameterizes the unknown physics associated with degrees of freedom at a high energy scale Λ by a collection of non-renormalizable interactions in the EFT, known as irrelevant interactions. At low energies, irrelevant interactions are suppressed by powers of E/Λ. In the limit $E/\Lambda \to 0$, the high-scale degrees of freedom *decouple*. However, in some contexts a low-energy observable is strongly affected by irrelevant interactions: such an observable is termed *ultraviolet (UV) sensitive*. As we shall explain, inflation is an ultraviolet-sensitive phenomenon.

The ultraviolet behavior of gravity is a foundational question for cosmology. To understand the nature of general relativity at high energies, we recall that the interactions dictated by the Einstein–Hilbert action can be encoded in Feynman rules, just as in ordinary quantum field theory (see [21] for a modern perspective). The coupling strength is set by Newton's constant G, which has negative mass dimension, so the interaction becomes stronger at higher energies. Moreover, when divergences do arise, they cannot be absorbed by renormalization of the terms in the classical Einstein–Hilbert Lagrangian; on dimensional grounds, the factors of the gravitational coupling from graviton loops must be offset by additional derivatives compared with the classical terms. General relativity is therefore non-renormalizable,[1] and for energies above the Planck scale,

$$M_P \equiv \sqrt{\frac{\hbar c}{G}} = 1.2 \times 10^{19}\,\mathrm{GeV}/c^2 \ ,$$

the theory stops making sense as a quantum theory: it violates unitarity. The conservative interpretation of this finding is that new physics has to come into play at some energy below the Planck scale, and any quantum field theory that

[1] Pure Einstein gravity is free of one-loop divergences, but diverges at two loops. Gravitational theories including matter fields typically diverge at one loop, except in supersymmetric cases [21].

is coupled to gravity should then be interpreted as an effective theory valid at energies below the Planck scale. This is precisely what happens in string theory: strings of characteristic size ℓ_s cut off the divergences in graviton scattering at energies of order $1/\ell_s$, where the extended nature of the string becomes important. The result is a finite quantum theory of gravity, whose long-wavelength description, at energies $E \ll 1/\ell_s$, is an effective quantum field theory that includes gravity, and whose non-renormalizable interactions include terms suppressed by the Planck scale (or the string scale). String theory therefore provides an internally consistent framework for studying quantum fields coupled to general relativity.

A striking feature of effective theories that support inflation is that they are sensitive to Planck-suppressed interactions: an otherwise successful model of inflation can be ruined by altering the spectrum and interactions of Planck-scale degrees of freedom. In *every* model of inflation, the duration of the inflationary expansion is affected by at least a small number of non-renormalizable interactions suppressed by the Planck scale. In a special class of scenarios called *large-field* models, an infinite series of interactions, of arbitrarily high dimension, affect the dynamics; this corresponds to extreme sensitivity to Planck-scale physics. The universal sensitivity of inflation to Planck-scale physics implies that a treatment in a theory of quantum gravity is required in order to address critical questions about the inflationary dynamics. This is the cardinal motivation for pursuing an understanding of inflation in string theory.

A primary subject of this book is the challenge of realizing inflationary dynamics in string theory (recommended reviews on the subject include [22–33]). Let us set the stage for our discussion by outlining the range of gains that can be expected from this undertaking.

The most conservative goal of studies of inflation in string theory is to place field-theoretic models of inflation on a firmer logical footing, giving controlled computations of quantum gravity corrections to these models. In particular, ultraviolet completion can clarify and justify symmetry assumptions made in the EFT approach. For example, realizing chaotic inflation [34] through axion monodromy in string theory [35] gives a microphysical understanding of the shift symmetry, $\phi \mapsto \phi + const.$, that ensures radiative stability of the low-energy EFT. Inflationary models relying on the shift symmetries of axions in string theory – variants of "natural inflation" [36] – have provided one of the best-controlled paths to ultraviolet-complete scenarios yielding significant gravitational waves. In favorable cases, the embedding into quantum gravity can also entail small modifications of the theory that lead to additional observational signatures. For example, in axion monodromy inflation nonperturbative corrections introduce modulations of the power spectrum [37] and the bispectrum [38]. This is an example where the structure of the ultraviolet completion could potentially be inferred from correlated signatures.

String theory is a far more constrained framework than effective field theory, and some effective theories that appear consistent at low energies do not admit ultraviolet completions in quantum gravity. Enforcing the restrictions imposed by ultraviolet completion winnows the possible models, leading to improved predictivity. For example, the Dirac–Born–Infeld (DBI) scenario [39] may be viewed as a special case of k-inflation [40]. While most versions of k-inflation are radiatively unstable, string theory makes it possible to control an infinite series of higher-derivative terms. In this case, a higher-dimensional symmetry significantly restricts the form of the four-dimensional effective action. Unlike its field theory counterpart, the observational signatures of DBI inflation are correspondingly specific [41].

String theory has also been an important source of inspiration for the development of novel effective field theories. The geometric perspective afforded by compactifications, and by D-branes moving inside them, complements the more algebraic tools used to construct effective theories in particle physics. The effective theories in D-brane inflation [42, 43], DBI inflation [39], fiber inflation [44], and axion monodromy inflation [35], for example, all exist in their own right as low-energy theories, but would likely have gone undiscovered without the approach provided by string theory. Generating effective theories from the top down in string theory also leads to modified notions of what constitutes a natural inflationary model, or a minimal one. Although we are very far from a final understanding of naturalness in string theory, one broad characteristic of existing geometric constructions is the presence of many light scalar fields, the *moduli* of the compactification. Moduli play a central role in inflation, and can affect both the background evolution and the perturbations. While theories with many "unnecessary" fields might be considered non-minimal in field-theoretic model-building, they are extremely common in string theory.

The boldest hope for the use of string theory in cosmology is that string theory will open entirely new dynamical realms that cannot be described in any effective quantum field theory with a finite number of fields, and that the resulting cosmic histories will avoid or overcome the limitations of contemporary models. While this enticing prospect has inspired work in string cosmology for more than two decades, in our opinion string theory is not yet understood at the level required for such a dramatic step. Even the low-energy effective actions governing the interactions of massless string states in non-supersymmetric vacua are not adequately characterized at present, while computing dynamics driven by the full tower of massive strings is a distant dream. Fundamental advances in the understanding of time-dependent solutions of string theory with string scale curvatures will be required if we are to move outside the aegis of the effective theory for the massless modes. In this book we will restrict our attention to conservative applications of string theory to the study of inflation: we will survey the substantial literature in which string theory underpins or informs inflationary effective theories, but does not replace them outright.

The task of making predictions in string theory is overshadowed by the problem of the *landscape*, i.e. the fact that string theory has an astronomical number of vacua (see [45] for a review). Although the dynamics that populates the landscape is poorly understood, false vacuum eternal inflation seems to be an unavoidable consequence. The cosmological constant problem, the question of pre-inflationary initial conditions, and the challenge of defining a probability measure for eternal inflation are all facets of the fundamental problem of understanding the landscape and making predictions within it. The number of vacua is too large for enumeration to be a realistic possibility [46], but it does not follow that in the landscape, "everything goes." Instead, there seem to exist strong structural constraints on the properties of the vacua in the landscape. For example, axion decay constants appear to be smaller than the Planck mass in all computationally controllable vacua [47, 48]. As we will see, this has important consequences for inflationary model-building in the context of string theory. Moreover, all four-dimensional de Sitter vacua in supersymmetric string theories are metastable, essentially because ten-dimensional Minkowski space is supersymmetric and therefore has zero energy, while a de Sitter solution has positive vacuum energy. Constructing a metastable de Sitter solution is much more difficult than finding a supersymmetric vacuum, and correspondingly, determining the prevalence of de Sitter vacua is far more subtle than counting supersymmetric solutions. In fact, de Sitter solutions appear to be exponentially sparse in comparison to unstable saddle points [49]. The formidable challenges of constructing and surveying the landscape compel us to understand dynamical selection effects in the early universe, but we have yet to see the first glimmering of a solution.

The organization of this book is as follows: in Chapter 1, we define inflation as an extended period of quasi-de Sitter evolution, and show how quantum fluctuations during this era lead to primordial density fluctuations and anisotropies in the CMB. We review the current observational evidence in favor of the inflationary hypothesis. In Chapter 2, we discuss the effective field theory approach to the physics of inflation. We explain why the effective theories supporting inflation are unusually sensitive to UV physics, and highlight the importance of symmetries for the radiative stability of inflationary models. In Chapter 3, we provide the groundwork for a discussion of inflation in string theory. We first give a brief overview of string theory, emphasizing those aspects that are particularly relevant for research in string cosmology. We examine string compactifications, discuss some leading mechanisms for moduli stabilization, and critically analyze proposals for metastable de Sitter vacua. In Chapter 4, we outline how inflation can arise in this context. In Chapter 5, we provide a more detailed discussion of several classes of inflationary models in string theory. We end, in Chapter 6, by describing some challenges and opportunities for the field.

In an effort to make this book self-contained, and accessible for a reader who is entering the field, we have included extensive background material in the appendices. In Appendix A, we collect mathematical concepts, definitions, and results that will be helpful for following the discussion in Chapters 3–5. In Appendix B, we present the effective theory of adiabatic fluctuations during inflation [50, 51]. In Appendix C, we introduce cosmological perturbation theory and derive the primordial perturbations from inflation.

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Notation and conventions

Throughout this book, we will employ natural units with $\hbar = c \equiv 1$. Moreover, the reduced Planck mass,

$$M_{\rm pl}^{-2} \equiv 8\pi G = \left(2.4 \times 10^{18}\,\text{GeV}\right)^{-2} ,$$

is often set equal to one.

Our metric signature is mostly plus, $(- + + + \cdots)$. We use t for physical time and τ for conformal time. We denote ten-dimensional spacetime coordinates by X^M, four-dimensional spacetime coordinates by x^μ, three-dimensional spatial coordinates by x^i, and three-dimensional vectors by $\boldsymbol{x}$. The coordinates of extra dimensions are y^m. Worldsheet coordinates of strings and branes are σ^a. The spacetime metric in ten dimensions is G_{MN}, while for the four-dimensional counterpart we use $g_{\mu\nu}$. The metric of the three noncompact spatial directions is g_{ij}, while the metric of the six-dimensional internal space is g_{mn}. The worldsheet metric is h_{ab}. The notation $(\partial\phi)^2$ means $g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ or $G^{MN}\partial_M\phi\partial_N\phi$, depending on the context.

The letter π stands both for $3.14159\cdots$ and for the Goldstone boson of spontaneously broken time translations. We use $\mathcal{R}$ (not ζ) for the curvature perturbation in comoving gauge. Our Fourier convention is

$$\mathcal{R}_{\boldsymbol{k}} = \int \mathrm{d}^3x\, \mathcal{R}(\boldsymbol{x})\, e^{i\boldsymbol{k}\cdot\boldsymbol{x}} .$$

The power spectrum for a statistically homogeneous field is defined by

$$\langle \mathcal{R}_{\boldsymbol{k}} \mathcal{R}_{\boldsymbol{k}'} \rangle = (2\pi)^3 P_{\mathcal{R}}(k)\delta(\boldsymbol{k} + \boldsymbol{k}') .$$

We also use the dimensionless power spectrum

$$\Delta^2_{\mathcal{R}}(k) \equiv \frac{k^3}{2\pi^2} P_{\mathcal{R}}(k) .$$

The Hubble slow-roll parameters are

$$\varepsilon \equiv -\frac{\dot{H}}{H^2} , \qquad \tilde{\eta} \equiv \frac{\dot{\varepsilon}}{H\varepsilon} ,$$

where overdots stand for derivatives with respect to physical time t. The potential slow-roll parameters are

$$\epsilon \equiv \frac{M_{\rm pl}^2}{2}\left(\frac{V'}{V}\right)^2 , \qquad \eta \equiv M_{\rm pl}^2 \frac{V''}{V} ,$$

where primes are derivatives with respect to the inflaton ϕ, and $V(\phi)$ is the potential energy density.

We define the string length and the string mass, respectively, as

$$\ell_{\rm s}^2 \equiv \alpha' , \qquad M_{\rm s}^2 \equiv \frac{1}{\alpha'} ,$$

where α' is the Regge slope. The ten-dimensional gravitational coupling is

$$2\kappa^2 = (2\pi)^7 (\alpha')^4 .$$

Beware of factors of 2π in alternative definitions of these quantities in the literature.