

### **Applying Graph Theory in Ecological Research**

Graph theory can be applied to ecological questions in many ways, and more insights can be gained by expanding the range of graph theoretical concepts applied to a specific system. But how do you know which methods might be used? And what do you do with the graph once it has been obtained?

This book provides a broad introduction to the application of graph theory in different ecological systems, providing practical guidance for researchers in ecology and related fields. Readers are guided through the creation of an appropriate graph for the system being studied, including the application of spatial, spatio-temporal, and more abstract structural process graphs. Simple figures accompany the explanations to add clarity, and a broad range of ecological phenomena from many ecological systems are covered. This is the ideal book for graduate students and researchers looking to apply graph theoretical methods in their work.

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> To my grandchildren: Monroe and Elliot in Sydney; Laura and Thomas in Edmonton



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## **Preface**

Applications of graph theory have been proliferating throughout ecology over the past several decades, whether explicitly realized or implicit in the approaches used, and not only in the cases which fall clearly into the popular category of networks. The reasons for this increased interest are as diverse as the areas of research. A basic impetus is that graphs and graph theory are about structure and provide the methods to analyze structure as abstracted from almost any ecological (or other) system. The second reason is the great popularity of network studies and network theory, originally applied to social relationships, communications (including the Internet as a prime example), transportation and the spread of disease. It is an obvious step to take network concepts and models from these sources and see how well they apply to ecological systems. Such network studies are obvious sources of inspiration for investigations of ecological interactions of all kinds (such as predation, competition, mutualism and facilitation) using the methods developed for those other systems. A third reinforcement for graph theory applications arises from the growing sophistication of ecologists in analyzing spatial data or time-ordered data or the complexities of spatio-temporal data; and, once again, methods based on graph theory provide the right mix of simplicity of concept but flexibility in application to provide valuable insights that would otherwise be impossible. Putting together interaction networks and spatio-temporal data brings a researcher to the challenges and rewards of studying the interplay of form and function (or "pattern and process" or "structure and dynamics") in ecological systems in which both form and function change through time by reciprocal influences and effects.

The book is organized in an order that reflects this range of sources. First is an introduction to thinking with graphs based on the theme of graphs and structure (Chapters 1 and 2). There are then several chapters on ecological interaction networks, first in general (Chapter 3), followed by more specific topics: predation (Chapter 4), social structure (Chapter 5), competition (Chapter 6) and mutualism (Chapter 7). The next three chapters are about locational graphs, in which the nodes have positions in one or more dimensions: time only (Chapter 8), space only (Chapter 9) and spatio-temporal (Chapter 10). Chapter 11 describes approaches to studying the dynamics of networks in the context of the reciprocal effects of form and function, focusing on the fascinating and promising methods based on graphlets. The last chapter (Chapter 12) attempts to draw together a number of the themes that emerged throughout the book and provide a synthesis of the common threads; it also takes on the risky task of making some predictions about future directions and developments to be expected in this field.



Preface

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The working title started out as "Smart Things Ecologists Can Do with Graph Theory"; and that is a good description of the intention. The book is not primarily an introduction to graph theory developed for ecologists; it is intended to make researchers aware of the wide range of possibilities for their own research projects, even when (or especially when) they have yet to be fully tried out in ecological systems. A prime example is the many forms of analysis based on graphlets that are recently developed and applied in other biological systems (e.g. protein-protein interactions) but not yet in ecology. The goal is to provide enough background that the researcher knows how and where to start and where to find some examples that will provide inspiration and support. The treatments of the various topics are very heterogeneous; some have a good range of examples to be cited (e.g. food webs or trophic networks; mutualism), but others have virtually none.

My own interest in graph theory as a useful approach to answering ecological questions related to structure started with my MSc research many years ago, and I owe a large debt to my then-supervisor, Tony Yarranton, who suggested the area and encouraged my exploration of the field. I owe thanks to John Moon, who helped me understand some of the more formal aspects of graph theory and its application (look at his *Topics on Tournaments*, if you have not already: a great example). In acknowledging people who have helped with this book, I thank the following for reading chapters, sometimes as they developed: Alex Aravind, Tan Bao, Conan Vietch, JC Cahill and Brendan Wilson. I thank Marie-Josée Fortin, especially; she read all the chapters, and some more than once! For data used in examples, there are many to be acknowledged, including Tan Bao and JC Cahill for the *Arabadopsis* competition tournament material and Gord Thomas for the rich data set on Saskatchewan weed communities. I thank NSERC Canada and UNBC for their support over many years.

I greatly enjoyed writing this book, and discovering all the exciting material I had not known was very rewarding. It is my hope that the readers will find the work equally rewarding and that it will help create pathways to more that is useful, more that is new and more that is surprising.