

Discrete and Continuum Models for Complex Metamaterials

Bringing together contributions on a diverse range of topics, this text explores the relationship between discrete and continuum mechanics as a tool to model new and complex metamaterials. Providing a comprehensive bibliography and historical review of the field, it covers mechanical, acoustic, and pantographic metamaterials, discusses naive model theory and Lagrangian discrete models, and their applications, and presents methods for pantographic structures and variational methods for multidisciplinary modeling and computation. The relationship between discrete and continuous models is discussed from both mathematical and engineering viewpoints, making the text ideal for those interested in the foundation of mechanics and computational applications, and innovative viewpoints on the use of discrete systems to model metamaterials are presented for those who want to go deeper into the field.

An ideal text for graduate students and researchers interested in continuum approaches to the study of modern materials, in mechanical engineering, civil engineering, applied mathematics, physics, and materials science.

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Edited by

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