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A GUIDED TOUR OF MATHEMATICAL METHODS FOR THE PHYSICAL SCIENCES THIRD EDITION

Mathematical methods are essential tools for all physical scientists. This book provides a comprehensive tour of the mathematical knowledge and techniques that are needed by students across the physical sciences. In contrast to more traditional textbooks, all the material is presented in the form of exercises. Within these exercises, basic mathematical theory and its applications in the physical sciences are well integrated. In this way, the mathematical insights that readers acquire are driven by their physical-science insight. This third edition has been completely revised: new material has been added to most chapters, and two completely new chapters on probability and statistics and on inverse problems have been added. This guided tour of mathematical techniques is instructive, applied, and fun. This book is targeted for all students of the physical sciences. It can serve as a stand-alone text or as a source of exercises and examples to complement other textbooks.

> We dedicate this book to our loving and beloved families: Idske, Hylke, Hidde, and Julia, and Mila, Sasha, and Niels. We also dedicate this book to our "scientific families": our teachers and mentors, in particular Guust Nolet and John Scales, our colleagues, and our students.

A GUIDED TOUR OF MATHEMATICAL METHODS FOR THE PHYSICAL SCIENCES THIRD EDITION

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Cover image: The image shows a soap bubble on a rotating table. The internal reflection of light within the soap film causes variations in the thickness of the film to show up as different colors. This laboratory experiment is used to study vortices in rotating systems; the experiment can be seen as a small-scale version of a hurricane. This figure is taken with permission from the following publication: Meuel, T., Y.L. Xiong, P. Fischer, C.H. Bruneau, M. Bessafi, and H. Kellay, *Intensity of vortices: from soap bubbles to hurricanes*, Scientific Reports, 3, p3455, 2013.

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