

## Physics of Electronic Materials

### Principles and Applications

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Adopting a uniquely pedagogical approach, this comprehensive textbook on the quantum mechanics of semiconductor materials and devices focuses on the materials, components and devices themselves whilst incorporating a substantial amount of fundamental physics related to condensed matter theory and quantum mechanics.

Written primarily for advanced undergraduate students in physics and engineering, this book can also be used as a supporting text for introductory quantum mechanics courses, and will be of interest to anyone interested in how electronic devices function at a fundamental level.

Complete with numerous exercises, and with all the necessary mathematics and physics included in appendices, this book guides the reader seamlessly through the principles of quantum mechanics and the quantum theory of metals and semiconductors, before describing in detail how devices are exploited within electric circuits and in the hardware of computers: for example as amplifiers, switches and transistors.

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## Preface

Electronic devices play a crucial role in today's societies and in the physical sciences where they originated. Contemplating that, in just a few decades, technology guiding electrons and photons has emerged that makes possible oral and visual communication between people on opposite sides of the planet is truly a triumph of science and technology. Not to mention that, equipped with a computer with access to the internet, one can instantly access a wealth of human knowledge. The physical principles providing the understanding of the functioning of present-day electronic devices should therefore be of interest not only to physicists, electrical engineers and material scientists, but also to anyone with a general interest in how the *wired* world around us functions. Present-day information technology is based on the physical properties of semiconductors, in particular the functioning of the transistor. The intention of this book is to take the reader from the principles of quantum mechanics through the quantum theory of metals and semiconductors all the way to how devices are used to perform their duties in electric circuits: for example, functioning as amplifiers, switches and in the hardware of computers. The mechanics of arithmetic and logical operations are discussed, and it is shown how electronic devices in the present-day CMOS (complementary metal–oxide–semiconductor) technology can carry out arithmetic calculations and logic operations in computers.

We presently live not only with an extraordinary information processing technology, but also in a new type of age of creation. We are not restricted to building semiconductor structures from materials provided by their immediate availability in nature and an additional purification procedure. Today, artificial human-made semiconductor structures can be built atom by atom, allowing the construction of devices with new physical properties, such as for example the blue quantum well laser. The physical principles governing these so-called heterostructures, which are revolutionizing electronics and optoelectronics, are presented. The book presents a self-contained elementary introduction to semiconductor technology. It also describes new vistas such as the role of mesoscopic physics in materials science, and presents some of the recent advances in nanotechnology. Not only is the theory of devices presented, but also their functioning in basic electric circuits is described.

In order to understand the physics of the solid states of matter, such as, for example, the behavior of electrons in semiconductors, a knowledge of quantum mechanics is essential. The book therefore starts with a discussion of the basic properties of the solutions of the Schrödinger equation. For the interested reader, it is shown in Appendix A how to arrive at the Schrödinger equation from the superposition and correspondence principles. In Chapter 2, quantum tunneling is studied using a wave packet analysis, and the tunneling probabilities for single- and double-barrier structures are obtained.

In Chapter 3, the Sommerfeld model of a metal is introduced and its characteristics determined. It is then used to discuss a tunnel junction, and the results of Chapter 2 are used to give an elementary understanding of the scanning tunneling microscope. In Chapter 4, the Gaussian wave packet description of a metal containing impurities is introduced and used to understand basic conduction properties, the standard model of a conductor. Basic electronic circuit theory is discussed in Chapter 5.

In Chapter 6, the chemical bond that lumps atoms into molecules and semiconductors is discussed, and in Chapter 7 the energy band structure of a particle in a one-dimensional periodic potential is established. Chapter 8 considers motion in a one-dimensional periodic potential, Zener tunneling and Bloch wave dynamics, and the physics of three-dimensional crystalline solids is considered in Chapter 9.

In Chapter 10, the fundamentals of semiconductor physics are considered, namely the construction of n- and p-type semiconductors by doping with donor and acceptor impurities, respectively. The quantum dynamics of holes in semiconductors is presented, leading to the discussion in Chapter 11 of basic p–n junction device functioning and its numerous applications. In Chapter 12, the production methods of heterostructures and their physical description are presented, and their new electronic and optoelectronic possibilities discussed. In Chapter 13 transport through mesoscopic systems, such as double-barrier resonant tunneling nanostructures and quantum point contacts are presented. Their physics and applications in electronics are discussed. Finally, in Chapter 14 the implementation of arithmetic and logic in mechanical electronic machines is presented, and the functioning of a computer discussed.

The book is intended to be sufficiently broad to serve as a text for a one- or two-semester undergraduate course on semiconductor physics, mesoscopic and nanophysics and solid-state transport theory. It is also hoped that the book can serve as a useful reference book for introductory courses on quantum mechanics. The book is self-contained to the extent that it should be useful for students with basic mathematical skills to read it on their own. The basic physics needed to understand the content of the chapters is relegated to appendices, where it can be consulted if needed. A number of exercises (with solutions – indeed, always the case when the result is later used in the main text) has been provided in order to aid self-instruction.

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