Dynamic Economic Analysis

Focusing on deterministic models in discrete time, this concise yet rigorous textbook provides a clear and systematic introduction to the theory and application of dynamic economic models. It guides students through the most popular model structures and solution concepts, from the simplest dynamic economic models through to complex problems of optimal policy design in dynamic general equilibrium frameworks. Chapters feature theorems and practical hints, and seventy-five worked examples highlight the various methods and results that can be applied in dynamic economic analysis. Notation and formulation is uniform throughout, so students can easily discern the similarities and differences between various model classes. Chapters include more than sixty exercises for students to self-test their analytical skills, and password-protected solutions are available for instructors on the companion website. Assuming no prior knowledge of dynamic economic analysis or dynamic optimization, this textbook is ideal for advanced students in economics.

Gerhard Sorger is Professor of Economics at the University of Vienna, Austria where he teaches primarily in the areas of macroeconomics, mathematical economics, and dynamic economic analysis.

'This book offers a comprehensive vision of economic dynamics suitable for graduate students and professionals alike. Gerhard Sorger is a leading researcher with a flair for presenting mathematically challenging theories carefully and rigorously. His text emphasizes the interplay between formal theory and applications, with detailed developments of a catalogue of economic models and examples drawn from macroeconomics, growth theory and game theory.

The book's structure builds on deterministic discrete time dynamical systems following a thorough development of basic graphical and analytical solution methods for difference equations. Highlights include the detailed presentation of basic stability theories for linear and non-linear difference equations followed by a treatment of bifurcation theory. In all cases there are numerous detailed economically motivated examples constructed to teach readers techniques and appreciate many subtle elements in the constructions. There is no other single book readily accessible in the economics literature covering the same wide range of deterministic dynamics and optimization theories with detailed illustrations of those theories in action. For example, chapters on dynamic games and time consistency illustrate basic applications with careful attention to details such as the difference between Markov perfect strategies and closed-loop ones. Recursive and Lagrangean optimization methods are compared and contrasted in the context of dynamic equilibrium models, including an application to the Ramsey problem of optimal taxation.

Sorger's book is accessible to students engaged in a self-study programme for students engaging with dynamical systems for the first time. Better yet, it offers the topics and treatments for a course in dynamics. Indeed, that is a course I would love to teach our graduate students.'

Robert A. Becker, Professor of Economics, Indiana University (Bloomington)

'This is a beautifully written book, providing a completely self-contained introduction to dynamic economic methods and models for graduate students in economics. The masterly exposition strikes a perfect balance between a user-friendly approach and a completely rigorous presentation of the subject matter. The style of writing is marked by exceptional clarity, very much in keeping with the high standards set by the author in his research contributions. The book is neatly divided into two parts, the first providing a comprehensive account of the theory of dynamical systems, and the second the theory and applications of dynamic optimization in settings with single and multiple decision makers. The chapters on autonomous difference equations and optimization techniques are real gems, and should form the core material in any course on dynamic economic analysis.'

Tapan Mitra, Goldwin Smith Professor of Economics, Cornell University

'Up to now, there are very few books available at the graduate level that introduce the necessary mathematical techniques to study macroeconomics from the viewpoint of non-linear dynamics. Gerhard Sorger is one of the few theorists who have made profound contributions to the subject. His book beautifully introduces the basic results and synthesizes the latest developments in the discrete time non-linear growth models. This book is ideally suited as a textbook for graduate courses in macroeconomics and mathematical economics. Gerhard Sorger should be congratulated on his efforts to educate young researchers. I highly recommend this book.'

Kazuo Nishimura, RIEB, Kobe University

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Dynamic Economic Analysis

Deterministic Models in Discrete Time

GERHARD SORGER

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To Renate and Helene

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Preface

The present textbook grew out of lecture notes for courses on the methods and applications of dynamic economic analysis that I have been teaching to graduate students over the years. The book is not meant to cover the whole state of the art in this area but to provide a compact presentation of the most essential concepts and results and to illustrate them by selected applications from various fields of economic research. The target readership consists of students and researchers who have little or no experience with solution techniques for dynamic economic models but who have a decent background knowledge in economics, calculus, and linear algebra. I hope that the book helps its readers to get acquainted with the basic issues and the most popular modelling frameworks of dynamic economic analysis and that it raises the appetite of its audience for a more complete and detailed study of this area.

Dynamic economic analysis is a vast area and the relevant literature is extensive. In order to achieve my goal of a compact presentation, I deliberately make two important restrictions that are also reflected in the title of the book. First, I only consider models, that are formulated in discrete time and, second, I do not deal with stochastic dynamics. The main justification of the first restriction is that I want to provide an overview of the most important concepts and methods of dynamic economic analysis without getting lost in technicalities. For some dynamic economic models, the choice between a discrete-time formulation and a continuous-time formulation is simply a matter of taste, whereas for others this choice is driven by the quest for analytical tractability. Moreover, there exist dynamic economic models that generate quite different predictions depending on which of the two formulations of time is applied. In my opinion, however, the basic issues arising in dynamic economic analysis can be illustrated with less technical effort in a discrete-time setting than in a continuous-time setting, which is why I restrict the presentation to the former case. By no means do I consider the discrete-time formulation as more relevant or more realistic than its continuous-time counterpart.

The restriction to deterministic models is a much more substantial one. Many economic situations include stochastic elements in a natural and essential way. Examples include dynamic economies that are buffeted by sequences of aggregate or idiosyncratic shocks, intertemporal decision making in the presence of uncertainty or risk, or randomization devices and mixed strategies in dynamic games. By not discussing models with these features, I do not intend to indicate that I regard them as unimportant. Rather, I have chosen not to deal with them because my own expertise is mostly in deterministic modelling and because I do not see how the methods of dynamic economic analysis for both deterministic and stochastic models can be treated in a single, reasonably sized textbook. As a consequence, there are only two instances in this book where I refer to stochastic concepts such as random variables or expected values. The first one is section 3.5 where I talk about the existence of sunspot equilibria in deterministic models with extrinsic uncertainty and the second one is section 6.3 where I discuss optimization in a deterministic environment but under partial commitment. Both of these sections can be skipped on a first reading without losing the thread.

In addition to restricting the presentation to deterministic models in discrete time, I also make many other deliberate choices on the selection of topics and the depth of their discussion. These choices are mostly based on my assessment of the relative importance of various modelling frameworks and solution techniques but also on my personal taste.

Having talked about the most important restrictions of my approach, let me also emphasize two of its focal points. The first one is that the book is targeted towards students and researchers who are primarily interested in the analytical solution or the qualitative analysis of small-scale dynamic economic models. Consequently, I refrain from discussing issues regarding the numerical implementation of the solution methods for dynamic economic models and I present many examples for which solutions can be characterized (at least qualitatively) by analytical means. And even when I discuss methods that are used in computational approaches, I focus on those aspects of these methods that are also helpful for studying dynamic models using paper and pencil. In section 5.5, for example, when introducing the value iteration method of dynamic programming – a standard method to obtain numerical solutions to dynamic optimization problems – I show how this algorithm can be used to obtain closed-form solutions of the Bellman equation as well as qualitative properties of the optimal value function.

The second point that I would like to emphasize is that I believe that the concepts and methods of dynamic economic analysis can only be properly understood by the reader if he or she has successfully applied them to specific problems. This belief results in my way of presenting the material: a blend of formal theorems (some with, others without proofs), hints about practical applications of these results, and worked out examples in the main text as well as end-of-chapter exercises. For example, even if it is tedious to derive the trajectories of a system of three or four linear difference equations, the solution to a Bellman equation in a dynamic optimization problem, or the various equilibria of dynamic games or dynamic competitive economies, I take the reader through these calculations because I consider this as essential for a thorough understanding of the proposed solution concepts in a more general setting. Sometimes

I even introduce concepts by means of examples before I define them formally; on other occasions, the examples are used to illustrate the application of the various methods in economic contexts. As a result of this approach, the text contains more than 70 worked-out examples and more than 60 end-of-chapter exercises that form an integral part of the book.

Let me now briefly outline the structure and the contents of the book. The material is organized according to the different modelling frameworks and the corresponding solution methods. Part I deals with those models that do not involve dynamic optimization. It is entitled 'Difference equations' and consists of four chapters. In chapter 1, I introduce the basic terminology for the study of difference equations and I explain what is understood by a solution to such an equation. I start with the simple case of one-dimensional maps and show how the solutions of these equations can conveniently be studied by graphical means. I then turn to more general explicit difference equations, distinguishing between first-order equations and higher-order equations, between autonomous ones and non-autonomous ones, and between pre-determined variables and jump variables. Some basic results about the existence of solutions as well as about their dependence on initial conditions are also presented. Finally, I point out that implicit difference equations may have no solutions at all or that they may have multiple solutions. Chapter 1 also introduces many examples of dynamic economic models, including the Solow–Swan growth model, the replicator dynamics from evolutionary game theory, the cobweb model (or hog cycle model), the basic New-Keynesian model for monetary policy analysis, and a simple version of an overlapping generations model with money.

Chapter 2 is devoted to the study of linear difference equations. After introducing some terminology for this type of equation, I show that the set of all trajectories of a linear difference equation forms a linear manifold of the same dimension as the system domain and I explain how this result suggests a general procedure for determining the trajectories of homogeneous and non-homogeneous linear difference equations starting in a given initial state. I then consider the special case of homogeneous linear difference equations with constant coefficients, and derive the form of all basis solutions by separately treating the four possible combinations of single or multiple and real or complex eigenvalues of the system matrix. Quite deliberately, I present this material not in an abstract way using the Jordan normal form of the system matrix but in a way that is hopefully more accessible to beginners. I continue with a detailed discussion of systems of two linear difference equations with constant coefficients and the extremely useful tool of the (T, D)-diagram. The application of this device is illustrated by the analysis of Samuelson's multiplier-accelerator model and the basic New-Keynesian model. Finally, I show how particular solutions of non-homogeneous linear difference equations can be found by the variation-of-constants formula or by a guess-and-verify approach.

In chapter 3, I turn to non-linear but autonomous difference equations. After introducing the important concepts of invariant sets, fixed points, and periodic points and stating a few fundamental results about these concepts, I explain the Hartman–Grobman theorem which provides the basis for all local linearization techniques. After that, various definitions of stability or instability of fixed points and periodic points are presented and the corresponding stability conditions are derived. In addition to those that can be obtained by local linearization techniques, we also encounter Lyapunov's direct method which requires neither smoothness nor hyperbolicity. Section 3.4 introduces the definition of saddle point stability by means of a simple overlapping generations model with capital, and argues that this concept of stability is the appropriate one for economic models including jump variables. The connections between the numbers of pre-determined variables and jump variables, on the one hand, and the occurrence of saddle point stability and indeterminacy, on the other hand, are clearly spelt out. Several examples illustrate these phenomena. The chapter concludes with a discussion of sunspot equilibria and their relation to the indeterminacy of fixed points.

Chapter 4, the last one in the first part of the book, discusses a number of more advanced results on one-dimensional maps. In particular, I treat monotonicity properties of trajectories, local bifurcations, and deterministic chaos. The occurrence of all these phenomena in economic models is illustrated by examples. This chapter is primarily meant to serve as an appetizer for students who want to obtain a deeper understanding of the intricacy and beauty of dynamical systems theory and its applications in economics. The presentation is deliberately restricted to one-dimensional maps, although many of the concepts and results are relevant for and can be generalized to higher-dimensional systems of difference equations.

Part II of the book is entitled 'Dynamic optimization' and also consists of four chapters. It deals with economic models in which some agents make intertemporal choices. Chapter 5 sets the stage by formulating a class of standard infinite-horizon dynamic optimization problems, and by explaining the most popular solution methods for this class of problems. I deal in turn with the Euler equation, the transversality condition, the Lagrangian approach, and the recursive approach (dynamic programming). For each of these approaches, I derive necessary and sufficient optimality conditions and apply them to the tradeoff between consumption and saving that is ubiquitous in optimal growth models and in many other economic situations as well. Special attention is devoted to the class of stationary discounted problems, which occur frequently in economic applications. For this class of problems, there exist additional results on the local dynamics around optimal steady states (turnpike theorems) as well as very strong results related to the recursive approach. In particular, I present sufficient conditions for the convergence of the value iteration algorithm and I use this algorithm to derive monotonicity, concavity, and smoothness properties of the optimal value function.

In all the models and examples from chapter 5, the principle of optimality applies, which means that it is irrelevant whether the decision maker solves the model once and for all at the start of the planning horizon or whether he or she solves the model sequentially in each period of time. Chapter 6, on the other hand, deals with dynamic optimization models in which changes in the constraints or the preferences of the optimization problem occur over the course of time, which implies that the optimal solution from a certain point in time onwards may differ from what the decision maker considered optimal at an earlier time. In such a situation, it makes a difference whether the decision maker optimizes once and for all and commits to that solution or whether he or she optimizes sequentially. In the literature, this difference is discussed under the heading of 'commitment versus discretion'. I start by describing various mechanisms that lead to the dynamic inconsistency of optimal solutions, that is, to situations in which the decision maker wants to deviate from previously made plans. Optimal solutions in these situations can only be implemented if commitment is possible. I continue by explaining solution concepts for models in which the decision maker lacks any commitment power. Finally, I consider an intermediate case and discuss a model in which it is assumed that the decision maker has limited or partial commitment power.

The last two chapters of the book deal with two classes of models that are frequently used in economic research: dynamic games and dynamic competitive equilibrium models. In both of these frameworks, there exist multiple interacting decision makers. Whereas the decision makers in a dynamic game are aware of these interactions and exploit them strategically, the interactions in a competitive model are indirect ones and the decision makers are not aware of them. Various solution concepts for these types of models are presented and discussed. In chapter 7 on dynamic games, I emphasize the generic multiplicity of Nash equilibria, a phenomenon that is rooted in the different possible presentations of optimal solutions (informational non-uniqueness). I also show how the equilibrium refinement of Markov perfection can be applied to obtain unique predictions. Finally, I point out a number of technical and conceptual difficulties that arise in dynamic games with a hierarchical decision making process (for example, dynamic Stackelberg games).

Chapter 8 presents two different definitions of a dynamic competitive equilibrium: the sequence formulation and the recursive formulation. I illustrate by examples from industrial organization and macroeconomics how these definitions can be applied. After doing that, I turn to policy problems in which, in addition to the competitively acting agents, there also exists a policy maker. The policy maker is able to affect the preferences and constraints of all competitive agents by his or her choice of the policy variables and acts like a Stackelberg leader in a game. The structure of equilibria in these models is already quite complex and only very limited results can be obtained by analytical means. I present two such applications from the area of fiscal and monetary policy. The situation becomes especially complicated if we assume that the policy maker lacks commitment power and seeks a dynamically consistent solution. The discussion in this chapter clearly illustrates the limits of a pencil-and-paper approach to the study of realistic dynamic economic models with multiple interacting agents, and it underlines the need for computational methods. The presentation of such methods, however, should be left to those who have sufficient expertise in developing and applying them and is therefore beyond the scope of the present book.

I have taught various selections of the material covered in this book to students of economics both at Master level and at Ph.D. level. I do not think that the book is suitable for Bachelor students of economics. In the Master program in economics, I usually teach selected material from chapters 1, 2, 3, and 5, leaving out some details and proofs but illustrating all concepts and results by economic examples. Sometimes I add material from chapter 4 at the end in order to encourage the students to delve deeper into dynamical systems theory. In the Ph.D. program in economics, I usually start with chapter 5 (going through all the proofs) and continue by presenting material from chapters 6–8 along with original journal articles. In all my courses on dynamic economic analysis, I require the students to solve problem sets on a regular basis. You can find many of these problems in the exercises collected at the end of each chapter.

I would like to thank the institutions at which I was encouraged to teach courses on dynamic economic analysis, and the students who challenged me to convey my knowledge in a comprehensible way. In addition, I would like to acknowledge the encouragement, support, and guidance that I have received from the following three colleagues: Gustav Feichtinger introduced me to dynamic optimization and its applications in economics more than 30 years ago. He thereby sowed the seeds for my interest in dynamic economic analysis and prepared the stage for my academic career. Ngo Van Long supported me at various phases throughout my career in a number of ways: as a host, a co-author, and a friend. Last but not least, Tapan Mitra's sharp insights into dynamic models and his incredible clear way of thinking and writing have been constant sources of inspiration for me. Of course, none of these persons is responsible for any shortcomings or mistakes in the present textbook. Finally, I would like to thank my family for letting me spend hours and hours in front of my laptop preparing the manuscript. I hope that they do not resent me for having focused so much on this project.