Random matrix theory is at the intersection of linear algebra, probability theory and integrable systems, and has a wide range of applications in physics, engineering, multivariate statistics and beyond. This volume is based on a Fall 2010 MSRI program which generated the solution of long standing questions on universalities of Wigner matrices and beta-ensembles, and opened new research directions especially in relation to the KPZ universality class of interacting particle systems, and low rank perturbations. The book contains review articles and research contributions on all these topics, in addition to other core aspects of random matrix theory such as integrability and free probability theory. It will give both established and new researchers insights into the most recent advances in field and the connections among many subfields.

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The Mathematical Sciences Research Institute wishes to acknowledge support by the National Science Foundation and the *Pacific Journal of Mathematics* for the publication of this series.

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UNIVERSITY PRESS

32 Avenue of the Americas, New York, NY 10013-2473, USA

Cambridge University Press is part of the University of Cambridge.

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www.cambridge.org
Information on this title: www.cambridge.org/9781107079922

First published 2014

Printed in the United States of America

A catalog record for this publication is available from the British Library.

Library of Congress Cataloging-in-Publication Data
Random matrix theory, interacting particle systems, and integrable systems / edited by Percy Deift, Courant Institute, Peter Forrester, University of Melbourne.
pages cm. – (Mathematical Sciences Research Institute publications)
"Mathematical Sciences Research Institute."
Includes bibliographical references and index.
ISBN 978-1-107-07992-2 (hardback)
1. Random matrices. I. Deift, Percy, 1945– editor. II. Forrester, Peter (Peter John) editor. III. Mathematical Sciences Research Institute (Berkeley, Calif.)
QA196.5.R37 2014
512.9′434–dc23 2014043446

ISBN 978-1-107-07992-2 Hardback

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MSRI Publications
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Preface

In the spring of 1999, MSRI hosted a very successful and influential one-semester program on random matrix theory (RMT) and its applications. At the workshops during the semester, there was a sense of excitement as brand new and very recent results were reported. The goal of the 2010 Program has been to showcase the many remarkable developments that have taken place since 1999 and to spur further developments in RMT and related areas of interacting particle systems (IPS) and integrable systems (IS) as well as to highlight various applications of RMT.

One of the outputs of the 1999 program was volume 40 in the MSRI Publications series, entitled "Random matrix models and their applications". Looking back on this publication today, it is clear that this volume gave a representative snapshot of topics that were occupying the attention of researchers in the field then. Moreover, the papers — consisting of a mix of research articles and reviews — provide a conveniently bundled resource for researchers in the field to this day.

Since 1999 random matrix theory has captured the imagine of a whole new generation of researchers, and through a collective effort some outstanding questions have been settled, and new highly promising research areas initiated. One example of the former is work on universality questions for Wigner matrices, where the task is to show that for large dimension a symmetric matrix with independent entries of mean zero and standard deviation 1 has the same statistical properties as in the case of standard Gaussian entries. Another is universality questions for β ensembles, where one wants to show that the statistical properties are independent of the one-body potential. New research areas include the KPZ equation and related growth processes, which has led to the precise experimental realization of some random matrix distributions, and also to quite spectacular theoretical advances relating to a rigorous understanding of the replica trick via so called Macdonald processes; analytic predictions of the β-generalization of the so-called Dyson constant in the asymptotic expansion of spacing distributions in β-ensembles; and stochastic differential equations and PDEs for eigenvalue distributions in the case of a low rank perturbation leading to eigenvalue separation.

A core aim of the 2010 semester was to spur further developments in RMT and the related areas of interacting particle systems and integrable systems. It is our

hope that this new MSRI Publications volume based on the 2010 semester will lend weight to this cause. Each author was a participant of the semester. Articles on all topics nominated above relating to solutions of outstanding questions and new research areas can be found: universality for Wigner matrices (Tao and Vu), universality for β ensembles (Borodin, Shcherbina); KPZ equation (Quastel, Sasamoto, Spohn, Takeuchi), Macdonald process (O'Connell), Dyson constant and asymptotics of spacing distributions (Forrester), low rank perturbations (Baik and Wang, Maida). One should also highlight the work on RMT and numerical algorithms by Pfrang, Deift and Menon, which is in the spirit of one of the very early uses of random matrices by von Neumann and co-workers at the dawn of the computer era, and the extensive review of free probability theory by Novak, the latter being a write up of a series of lectures he delivered during the semester.

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