Advanced Concepts in Quantum Mechanics

Introducing a geometric view of fundamental physics, starting from quantum mechanics and its experimental foundations, this book is ideal for advanced undergraduate and graduate students in quantum mechanics and mathematical physics.

Focusing on structural issues and geometric ideas, this book guides readers from the concepts of classical mechanics to those of quantum mechanics. The book features an original presentation of classical mechanics, with the choice of topics motivated by the subsequent development of quantum mechanics, especially wave equations, Poisson brackets and harmonic oscillators. It also presents new treatments of waves and particles and the symmetries in quantum mechanics, as well as extensive coverage of the experimental foundations.

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Preface

In the course of teaching quantum mechanics at undergraduate and post-graduate level, we have come to the conclusion that there is another original book to be written on the subject. The abstract setting foreseen by Dirac and the geometric view pioneered by von Neumann are finding new realizations, leading to further progress both in physics and mathematics, while the applications to quantum computation are opening a new era in modern science. Our emphasis is mainly on structural issues and geometric ideas, moving the reader gradually from the concepts of classical mechanics to those of quantum mechanics, but we have also inserted many problems for students throughout the text, since the book is written, in the first place, for advanced undergraduate and graduate students, as well as for research workers.

The overall picture presented here is original, and also the parts in common with a previous monograph by some of us have been rewritten in most cases. The analysis of waves and particles (Chapter 3), the treatment of symmetries in quantum mechanics (in particular, the first half of Chapter 10), the assessment of modern pictures of quantum mechanics (Chapter 12) have never appeared before in any monograph, to the best of our knowledge. The material on experimental foundations is rather rich and it cannot easily be found to the same extent elsewhere. Our presentation of classical mechanics is original and the choice of topics is motivated by the subsequent development of quantum mechanics, expecially wave equations, Poisson brackets and harmonic oscillators. The examples in Chapters 6 and 7 are frequently discussed with a care not always used in many introductory presentations in the literature. We find it also useful to offer an unified view of approximation methods, as we do in Chapter 11, which is divided into three parts: perturbation theory, the JWKB method and scattering theory.

We hope that, having acquired familiarity with symbols of differential operators, geometric formulation and tomographic picture, the reader will find it easier to follow the latest developments in quantum theory, which embodies, in the broadest sense, all we know about guiding principles and fundamental interactions in physics.

Our friend Eugene Saletan, with whom some of us worked and corresponded on the subject of dynamical systems over many years, is deeply missed. Special thanks are due to our colleagues Fedele Lizzi, Francesco Nicodemi and Luigi Rosa for discussing various aspects of the manuscript, and to our students who, never being satisfied with our writing, helped us a lot in conceiving and completing the present monograph. Last, but not least, the Cambridge University Press staff, i.e. Nicholas Gibbons, Neeraj Saxena, Zoë Pruce, Lindsay Stewart, Jeethu Abraham, Sarah Payne and the copy-editor, Zoë Lewin, have provided invaluable help in the course of completing our task.