

MECHANICS OF AERO-STRUCTURES

Mechanics of Aero-structures is a concise textbook for students of aircraft structures, which covers aircraft loads and maneuvers, as well as torsion and bending of single-cell, multi-cell, and open thin-walled structures. Static structural stability, energy methods, and aero-elastic instability are discussed. Numerous examples and exercises are included to enhance students' facility with structural analysis.

This well-illustrated textbook is meant for third- and fourth-year undergraduate students in aerospace and aeronautical engineering programs. The material included can be covered in a one-semester course.

Key features include:

- Torsion and bending of single-cell, multi-cell, and open sections are described in detail.
- Aerodynamic loads, maneuvers, and elementary aero-elastic stability are included.
- The book begins with a description of the aerodynamics loads to motivate the students.
- Includes an in-depth description of energy methods, an essential topic.

Sudhakar Nair has taught aircraft structures for more than 30 years at the Illinois Institute of Technology. He is a Fellow of ASME, an Associate Fellow of AIAA, and a Member of ASEE, Sigma Xi, and Tau Beta Pi. He has authored numerous articles on structural mechanics and applied mathematics and is the author of two previous textbooks: *Introduction to Continuum Mechanics* and *Advanced Topics in Applied Mathematics: For Engineering and the Physical Sciences*. He was Associate Dean for Academic Affairs, Department Chair, and Chair of the Faculty Council at IIT. He received the Barnett award for the best teacher and a special commendation from the AIAA student chapter at IIT.

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Preface

This book is intended as a textbook for advanced mechanics of materials for third-year undergraduate students in the area of aerospace engineering and related fields. It is assumed that these students have had a first course in strength of materials and a course in ordinary differential equations in their second year. The material included in this book can be covered in one semester. From my experience in teaching this topic, it has been abundantly clear that students are used to following textbook descriptions topic-by-topic as opposed to following the instructor's presentations, which may be at variance with the chosen text. The large number of excellent textbooks available in physics, calculus, statics, dynamics, and strength of materials has conditioned the students to depend on "one" textbook and "one" notation.

I have followed a logical sequence for introducing students to aero-structures. In Chapter 1, the typical loads expected during a preliminary design of an aircraft are described along with certain essential design considerations such as load factor, proof load, and factor of safety. Also, aerodynamic loads in level flight and under gust conditions are included.

Elements of elasticity from a three-dimensional description to two-dimensional simplification are introduced in Chapter 2. Most students find this a difficult topic. But this is the last chance for them to see the full picture before they go to work or to graduate school to continue structural analysis. Energy methods are explained in Chapter 3 and these are used in the coming chapters wherever they are needed.

Analysis of thin-walled structures under torsion and bending, which is of specific use in aero-structures, is treated in Chapters 4 and 5. Applications to open-, single-, and multiple-cell tubes are emphasized. Shear center (and center of twist) calculations are discussed. Chapter 6 is devoted to elastic stability, including a brief primer on aero-elastic stability.

Chapter 7 considers various failure and yield criteria. Metals as well as epoxy/fiber composites are included. An introduction to fracture mechanics, fatigue, and fatigue crack propagation is also included in this chapter.

I am grateful to my colleague Dr. Roberto Cammino, who provided many suggestions for improvement. My former teacher and thesis advisor, Professor S. Durvasula (Indian Institute of Science, Bangalore), and Professor M. Nambudiripad (National Institute of Technology, Calicut) were inspirational in guiding my career toward elastic structures and I am always indebted to them. I also wish to record my appreciation of my doctoral advisor, the late Eric Reissner, whose name appears when one lists the giants in this field. I also thank the hundreds

of aerospace students who took this course with me at the Illinois Institute of Technology and provided me feedback on the material included here.

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S.N., Chicago