The Cambridge Dictionary of Probability and Its Applications

Probability comes of age with this, the first dictionary of probability and its applications in English, which supplies a guide to the concepts and vocabulary of this rapidly expanding field. Besides the basic theory of probability and random processes, applications covered here include financial and insurance mathematics, operations research (including queueing, reliability, and inventories), decision and game theory, optimization, time series, networks, and communication theory, as well as classic problems and paradoxes.

The dictionary is reliable, stable, concise, and cohesive. Each entry provides a rigorous definition, a sketch of the context, and a reference pointing the reader to the wider literature. As the only dictionary on the market, this will be a guiding reference for all those working in, or learning, probability together with its applications.

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This book is dedicated to the benefactors of St John's College, Oxford

Preface

This is a dictionary of technical terms that commonly arise in probability and its applications – except statistical terms, for which the reader should consult a dictionary of statistics. Besides the basic theory of probability and random processes, applications covered here include financial and insurance mathematics, operations research (including, e.g. queueing, reliability, and inventories), decision and game theory, optimization, time series, point processes, networks, branching processes, communication theory, classic problems and paradoxes, and several other stochastic models for physical and biological processes that have attracted the analytical attention of probabilists. There is, of course, some overlap with the probabilistic aspects of statistical vocabulary. However, the book is not an encyclopaedia, and the applications of probability are many and increasing at an increasing rate; so rapidly indeed, that it might be said that probability is the new calculus. Universal coverage is therefore neither intended nor possible, but it is hoped that enough terms are defined to make the book useful to a large number of those working in, or learning, the theory of probability together with its applications. There are more than 3100 headwords, which, together with subsidiary definitions and aliases, takes the total number of terms defined to somewhere near 5000. Almost all entries also include at least one reference, which may be to the original paper, a survey paper, an important and useful consequent paper, a textbook, or a monograph. It should be noted that there has been a necessary element of randomness in choosing which to cite of several equally meritorious works.

Note

In the sample space of all possible dictionaries, there is an event on which this book contains no serious errors or omissions; but that is almost surely not the way to bet. Corrections to the former and suggestions for the latter will be gratefully received by the author and the Press, who will also be pleased to see ideas for better references.

Remark

Entries are in strict lexicographical order of their headings, ignoring spaces, punctuation, and the possessive ‘s’; for this ordering numerals and Greek letters are almost always spelt out, and terms are never inverted: e.g. see ‘stochastic matrix’, not ‘matrix, stochastic’.

Abbreviations

a.e.	almost everywhere
a.s.	almost surely
arcsine	inverse sine function
arg z	argument θ of $z = \|z\|e^{i\theta}$
Ber(p)	Bernoulli distribution, parameter p
Beta(a, b)	beta distribution, parameters a, b
$\beta(a, b)$	beta function
B.M.	Brownian motion
B(n, p)	binomial distribution, parameters n, p
cdf	cumulative distribution function
cgf	cumulant generating function
chf	characteristic function
$\chi^2(n)$	chi-squared distribution
CLT	central limit theorem
cos, cosh	cosine, hyperbolic cosine
cov	covariance
cts	continuous
det	determinant
distn	distribution
edn	edition
eqn	equation
et al.	and others
Exp(λ)	exponential distribution, parameter λ
exp	exponential function
fBm	fractional Brownian motion
fdds	finite-dimensional distributions
fn	function
$\Gamma(m, \lambda)$	gamma distribution, parameters m, λ
Geo(p)	geometric distribution, parameter p
iff	if and only if
i.i.d.	independent and identically distributed
Im(z)	imaginary part of z
inequ.	inequality
inf	infimum
LHS	left-hand side
LIL	law of the iterated logarithm
lim	limit
log	logarithm
max	maximum
mgf	moment generating function
min	minimum
m.s.	mean square
$N(\mu, \sigma^2)$	normal distribution, mean μ, variance σ^2
pde	partial differential equation
pdf	probability density function
pgf	probability generating function
pmf	probability mass function
Poi(λ)	Poisson distribution, parameter λ
rcll	right continuous with left limits
Re(z)	real part of z
resp.	respectively
$\rho(X, Y)$	correlation function
RHS	right-hand side
r.v., r.vs	random variable, random variables
r.vr, r.vrs	random vector, random vectors
r.w.	random walk
SDE	stochastic differential equation
sec, sech	secant, hyperbolic secant
sin, sinh	sine, hyperbolic sine
SLLN	strong law of large numbers
sup	supremum
tan	tangent
thm	theorem
U(a, b)	uniform distribution, parameters (i.e. end points) a, b
u.i.	uniformly integrable
var	variance
v.v.	vice versa
WLLN	weak law of large numbers
wlog	without loss of generality
w.p. 1	with probability one
wrt	with respect to

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More information

Symbols and notation

(a, b), $[a, b]$	open, closed interval
$\|A\|$	cardinality of A
$A \cup B$	union
$A \cap B$	intersection
$A \backslash B = A \cap B^c$	difference
A^T or A'	transpose
A^c	complement
$\mathcal{A}, \mathcal{B}, \mathcal{F}, \mathcal{G}, \mathcal{H}, \ldots$	algebras and σ-fields
$\mathrm{B}(n, p)$	binomial distribution
$\mathbb{C}$	complex plane
cov	covariance
$c(h), c(t)$	autocovariance
$d_{\mathrm{TV}}, d_{\mathrm{KL}}$	total variation distance, Kullback–Leibler divergence
e	base of natural logarithms
$\mathbb{E}$	expectation
$\mathbb{E}(X\|Y)$	conditional expectation
$f(x), f_X(x)$	density
$f(x\|y), f_{X\|Y}(x\|y)$	conditional density
$f(x, y)$	joint density
$F(x), F_X(x)$	cumulative distribution
$F(x, y)$	joint distribution
$F(x\|y), F_{X\|Y}(x\|y)$	conditional distribution
$G(s), G_X(s)$	generating function
$\dot{g}(x) = g'(x) = \frac{\mathrm{d}g}{\mathrm{d}x}$	derivative
$H(X)$, $H(X,Y)$, $H(X\|Y)$	Shannon entropies
i	$\sqrt{-1}$
$i, j, k, l, m, n, s, t, u, v, \ldots$	indices and time parameters
$\mathrm{I}(A)$	indicator function
$\mathrm{Im}(z), \mathrm{Re}(z)$	imaginary part of z, real part of z
$\log_2$	logarithm with base 2
max ($\vee$), min ($\wedge$)	maximum, minimum
$M(t), M_X(t)$	moment generating function
$n!$	factorial
$\binom{n}{r}$	binomial coefficient
$\mathbb{N}$	positive integers
$N(t)$	Poisson, or renewal, or counting process
$N(\mu, \sigma^2)$	normal distribution
$p(x), p_X(x)$	mass function, probability
$p(x, y)$	joint mass function
$p(x\|y), p_{X\|Y}(x\|y)$	conditional mass function
$p_{jk}, p(j, k), p_{jk}(n)$	transition probabilities
$\mathbb{P}(\cdot), \mathbb{Q}(\cdot)$	probability functions, or measures
$\mathbb{R}$	reals
var	variance
$W(t)$	Wiener process
$X, Y, Z, W, X(\omega), \ldots$	random variables
$\mathbf{X}, \mathbf{Y}, \mathbf{Z}, \ldots$	random vectors
$\|X\| = \det \mathrm{X}$	determinant of the matrix X
$\|x\|$	modulus or absolute value of x
$x^+ = x_+$	max $\{0, x\}$
$x^- = x_-$	$-$ min $\{0, x\}$
$[x]$	integer part of x
$x^{\overline{n}}, x^{\underline{n}}$	rising, falling factorial
$\bar{z}$	complex conjugate
$\mathbb{Z}$	integers
$\mathbb{Z}^+$	non-negative integers
$\beta(a, b)$	beta function
γ	Euler's constant
$\Gamma(m, \lambda)$	gamma distribution
$\Gamma(t)$	gamma function
$\delta_{ij} = \delta(i - j) = I(i = j)$	Kronecker delta
$\delta(t)$	Dirac delta
μ	mean, or other parameter
$\rho(X, Y)$	correlation
$\rho(t)$	autocorrelation
$\phi(t), \phi_X(t)$	characteristic function
$\phi(x), \Phi(x)$	standard normal density, distribution function
$\boldsymbol{\pi}$	stationary distribution
$\chi^2(n)$	chi-squared distribution
ω	elementary event, or sample point
Ω	sample space
$\emptyset$	empty set, or impossible event
$\|\| \cdot \|\|$	norm
∇, ∇^2	gradient, Laplacian
$\propto$	proportional