QUANTUM FIELD THEORY APPROACH TO
CONDENSED MATTER PHYSICS

A balanced combination of introductory and advanced topics provides a new and unique perspective on the quantum field theory (QFT) approach to condensed matter physics (CMP). Beginning with the basics of these subjects, such as static and vibrating lattices, independent and interacting electrons, the functional formulation for fields, different generating functionals and their roles, this book presents a unified viewpoint illustrating the connections and relationships among various physical concepts and mechanisms. Advanced and newer topics bring the book up-to-date with current developments and include sections on cuprate and pnictide superconductors, graphene, Weyl semimetals, transition metal dichalcogenides, topological insulators and quantum computation. Finally, well-known subjects such as the quantum Hall effect, superconductivity, Mott and Anderson insulators, spin-glasses, and the Anderson-Higgs mechanism are examined within a unifying QFT-CMP approach. Presenting new insights on traditional topics, this text allows graduate students and researchers to master the proper theoretical tools required in a variety of condensed matter physics systems.

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QUANTUM FIELD THEORY APPROACH TO CONDENSED MATTER PHYSICS

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Concerning matter, we have been all wrong. What we have been calling matter is actually energy, the vibration of which has been lowered so much as to be perceptible to the senses. There is no matter.

A. Einstein
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The inception of quantum field theory (QFT) occurred in 1905, when Einstein, inspired by the work of Planck, postulated the quantization of the electromagnetic radiation field in terms of photons in order to explain the photoelectric effect. Two years later, Einstein himself made the first application of this incipient QFT in the realm of condensed matter physics (CMP). By extending the idea of quantization to the field of elastic vibrations of a crystal, he used the concept of phonons in order to obtain a successful description of the specific heat of solids, which has become one of the first great achievements of the quantum theory. Since their early days, therefore, we see that CMP and QFT have been evolving together side by side.

In 1926, the quantum theory of the electromagnetic field was formulated according to the principles of quantum mechanics, thereby providing a rational description for the dynamics of photons, which were postulated by Einstein more than 20 years before. QFT soon proved to be the only framework where the two foundations of modern physics, namely, quantum mechanics and the special theory of relativity, could be combined in a sensible way.

From then on, QFTs grew up mainly in the realm of particle physics, until they eventually became some of the most successful theories in physics. Familiar examples are the Standard Model (SM) of fundamental interactions and, more specifically, Quantum Electrodynamics (QED), which exhibits some theoretical predictions that can match the experimental results up to twelve decimal figures. It is difficult to find any other model, ever proposed, possessing such accuracy.

Condensed Matter Physics (CMP), by its turn, has proved to be one of the richest areas of physics, keeping under its focus of investigation an incredible variety of systems and materials. These exhibit a plethora of unsuspected kinds of behavior, frequently associated to different responses to all types of external agents, such as electric and magnetic fields, voltage and temperature gradients, pressure, elastic stress and so on. The understanding of these phenomena is an enterprise that is frequently as interesting as it is challenging. Furthermore, like in no other area of
physics, mastering the principles and mechanisms of the phenomena under investigation has produced countless technological by-products. These sometimes have produced such impact on the society that its whole structure has been transformed, and many human habits changed. One such example was the development of the transistor, which occurred after the physics of doped semiconductors was mastered. The whole revolution of electronics, miniaturization and informatics would have been impossible without it.

For decades, CMP made a description of solids that was based on the concept of independent electrons moving on a crystalline substrate. This picture has worked extremely well due to the peculiar properties of the quantum-mechanical behavior of electrons in a periodic potential and served for understanding an enormous amount of properties of metals, insulators and semiconductors. Adding further elements to this picture has enabled the understanding of magnetic materials. Then superconductivity, one of the most beautiful, interesting and useful phenomena in physics, was understood by including the interaction of independent electrons with the crystal lattice vibrations.

By the 1980s, however, the discovery of the quantum Hall effect and the following efforts employed to understand it brought two important features to the center of attention in the realm of CMP. The first one is the existence of material systems where the electrons, rather than being independent, are strongly correlated due to interactions. The second one is the fact that the physical properties of certain states of matter are determined by sophisticated topological constraints that fix the value of some quantities with an incredible accuracy and guarantee the conservation of others, a fact that would not be otherwise anticipated. Both features usually lead to unsuspected results.

Since that time, a large number of new materials either have been developed or are being designed that present strongly correlated electrons, topological phases or both. For understanding such a large amount of new sophisticated advanced materials, an efficient method, capable of describing the quantum-mechanical properties of a system of interacting many-particle systems and their possibly nontrivial topological aspects, was required. QFT was the natural response to this demand. By then, it had become one of the most powerful theoretical tools available in physics, with applications ranging from particle physics to quantum computation, passing through hadron physics, nuclear physics, quantum optics, cosmology, astrophysics and, most of all, condensed matter physics, which is the subject of this book.

Here I present a QFT approach to many different condensed matter systems that have attracted the interest of the scientific community, always trying to explore the beauty, depth and harmony that are provided by a unified vision of physics in such approaches. This not only fosters a deeper understanding of the subject; it opens new ways of looking at it.
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An extremely interesting example of the interplay between CMP and QFT is the Anderson–Higgs mechanism, which plays a central role in the Standard Model, and its relation to the Meissner–Ochsenfeld effect of superconductivity. Here, the Landau–Ginzburg field of the superconducting system plays the role of the Anderson–Higgs field of the SM, the only difference being the gauge group. In both cases, a mass is effectively generated to the gauge field, which causes the corresponding propagators to decay exponentially. In the former case, this exponential decay accounts for the extremely short range of the weak interaction, whereas in the later it leads to an extremely short penetration length for the magnetic field inside the bulk of a superconductor, thereby effectively expelling it from inside superconductors, a phenomenon known as the Meissner–Ochsenfeld effect. The fact that the particle excitations associated to the Landau–Ginzburg field reveal themselves as electron-bound states (Cooper pairs) strongly suggests, both on logical and esthetic grounds, that the Anderson–Higgs boson particle should also be composite. This should be a central issue in the realm of particle physics in the near future.

Another beautiful example that is explored in this book is the equivalence between the Yukawa mechanism of mass generation for leptons and quarks in the SM and the Peierls mechanism of gap generation in polyacetylene. Both involve identical trilinear interactions containing a Dirac field, its conjugate and a scalar field. In the former case, the lepton or quark Dirac fields interact with the Anderson–Higgs field, whereas in the later the electron, which can be shown to be described by a Dirac field, interacts with the elastic vibrations field of the polymer lattice. In both cases, the scalar field acquires a nonzero vacuum expectation value: the first one by a judicious choice of the Anderson–Higgs potential, while the second one by the dimerization of the polyacetylene chain. Therefore, the same mechanism that causes polyacetylene to be an insulator generates the mass of all familiar matter. This amazing unification of phenomena that are separated by more than ten orders of magnitude in energy indicates the existence of a deep, underlying unity in physics. A universal unified vision of this science is, consequently, required nowadays. This book is aimed to provide such a unified picture of CMP and QFT.

Writing a book on applications of QFT in CMP, however, is a formidable challenge, in view of the large number of excellent books that already exist on the subject, some of them listed under Further Reading at the back of this book. One can, indeed, always ask: why another book on QFT in CMP? Nevertheless, because of its peculiar characteristics, which include a balanced combination of introductory, advanced and traditionally known material, I feel that this book has its own place in the literature and will be helpful and useful to a broad group of readers.

The book has been divided into three parts. Part I provides a four-chapter introduction to CMP. Part II contains eight chapters on QFT, including an introduction.
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that starts from the very basic principles of QFT as well as a description of the main features of QFT, which may be relevant for applications in CMP. Part III is made up of eighteen chapters covering different applications of QFT in CMP, which include metals, Fermi liquids, Mott insulators, Anderson insulators, polarons, polyacetylene, materials exhibiting the Kondo effect, quantum magnetic chains, quantum magnetic planar systems, the spin-fermion system, spin glasses, superfluids, conventional superconductors, Dirac superconductors, cuprate superconductors, pnictide superconductors, systems presenting the quantum Hall effect, graphene, silicene, transition metal dichalcogenides, topological insulators, Weyl semimetals and systems that are candidates for topological quantum computation.

This book covers topics for which there is, so far, no complete understanding and, consequently, about which no consensus has been reached in the community. Cuprate and pnictide superconductors, for instance, are examples of such topics. Besides these, the book includes some very recent advanced topics, such as Weyl semimetals, topological insulators and materials potentially relevant for quantum computation. I am aware that the inclusion of such topics in the book is a bold venture; nevertheless, I decided to face it and take the involved risks. I feel the inclusion of these topics has made this work much more interesting and exciting. I hope the reader will understand this point and will share the constructive attitude that stands behind the inclusion of such topics.

The book can be used in many different ways. Chapters 1–7 can be used as an introductory course in CMP and QFT. After this introduction, one can follow the sequence of QFT subjects presented in Chapters 8–12, which comprise classical and quantum topological excitations, order-disorder duality, bosonization and anyons, statistical transmutation and Pseudo Quantum Electrodynamics. Then, after a bridge between QFT and CMP offered in Chapter 13, the reader will find in Chapters 14–30 the QFT approach to a variety of materials and mechanisms of CMP.

Alternatively, the book contains several avenues that will take the reader along certain sequences of QFT procedures, which play an important part in different CMP systems. The first of such avenues starts with symmetries (Chapter 7), and then order-disorder duality and quantum topological excitations (Chapter 9), bosonization and generalized statistics (Chapter 10), bosonization of polarons (Chapter 15), bosonization of quantum magnetic systems in 1d (Chapter 18) and anyons with non-Abelian statistics (Chapter 30).

A second avenue deals with electromagnetic interaction of planar systems. It starts with pseudo quantum electrodynamics (Chapter 12) and then goes to graphene (Chapter 27) and silicene and transition metal dichalcogenides (Chapter 28). A third starts with symmetries (Chapter 7), followed by classical Sine–Gordon solitons (Section 8.3), quantum Sine–Gordon solitons (Section 10.6), 2d...
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Coulomb gas (Section 18.3), application to copper benzoate (Section 18.4.2) and application to the Kosterlitz–Thouless transition (Section 18.4.3). Then, we have an avenue on superconductivity, which starts in superconductivity (Section 4.4), then goes to electron-phonon interactions (Section 3.7), from this to superconductivity of regular electrons (Sections 23.1–2) and then superconductivity of Dirac electrons (Sections 23.3–6). The reader is kindly invited to find further avenues as such.

The book is mainly meant for researchers, postdocs and graduate students in the areas of CMP, QFT, materials science, statistical mechanics and related areas. Nevertheless, being self-contained in the sense that no previous knowledge of either CMP or QFT is required, the book can also be used by undergraduate students who feel inclined toward QFT and CMP.

I want to express my gratitude to people who contributed in different ways toward the completion of this book. First of all, my editor Simon Capelin, who in the many phases of this work never hesitated to provide his unconditional support. To Roland Köberle, who followed the writing of the book for 3 years, thank you for numerous useful suggestions. Thank you to Curt Callan for taking the time to read the manuscript, to Mucio Continentino for the constructive critical reading of selected chapters, to Hans Hansson for helpful suggestions and to Cristiane de Morais Smith for invaluable comments and remarks. I would also like to thank Vladimir Gritsev, Amir Caldeira, Chico Alcaraz, Nestor Caticha, Luis Agostinho Ferreira and Carlos Aragão for (hopefully) reading the manuscript. Special thanks also go to my collaborators of the most recent years: Van Sérgio Alves, Leandro Oliveira do Nascimento and Lizardo Nunes for the fruitful exchange of points of view. I also take this opportunity to thank my home institution, the Institute of Physics of the Federal University of Rio de Janeiro, for all the support received along many years.

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