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978-1-107-07245-9 - Symplectic Topology and Floer Homology: Volume 1: Symplectic
Geometry and Pseudoholomorphic Curves

Yong-Geun Oh

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