

FLUID DYNAMICS

This book presents a focused, readable account of the principal physical and mathematical ideas at the heart of fluid dynamics. Graduate students in engineering, applied mathematics, and physics who are taking their first graduate course in fluids will find this book invaluable in providing the background in physics and mathematics necessary to pursue advanced study.

The book includes a detailed derivation of the Navier-Stokes and energy equations, followed by many examples of their use in studying the dynamics of fluid flows. Modern tensor analysis is used to simplify the mathematical derivations, thus allowing a clearer view of the physics. Peter S. Bernard also covers the motivation behind many fundamental concepts such as Bernoulli's equation and the stream function.

Many exercises are designed with a view toward using MATLAB or its equivalent to simplify and extend the analysis of fluid motion, including developing flow simulations based on techniques described in the book.

Peter S. Bernard has 35 years' experience teaching graduate-level fluid mechanics at the University of Maryland. He is a Fellow of the American Physical Society and Associate Fellow of the American Institute of Aeronautics and Astronautics. In addition to his many research articles devoted to the physics and computation of turbulent flow, he is the coauthor of the highly regarded volume *Turbulent Flow: Analysis, Measurement, and Prediction*.

Cambridge University Press
978-1-107-07157-5 - Fluid Dynamics
Peter S. Bernard
Frontmatter
[More information](#)

Cambridge University Press
978-1-107-07157-5 - Fluid Dynamics
Peter S. Bernard
Frontmatter
[More information](#)

Fluid Dynamics

Peter S. Bernard
University of Maryland



Cambridge University Press
978-1-107-07157-5 - Fluid Dynamics
Peter S. Bernard
Frontmatter
[More information](#)

CAMBRIDGE
UNIVERSITY PRESS

32 Avenue of the Americas, New York, NY 10013-2473, USA

Cambridge University Press is part of the University of Cambridge.

It furthers the University’s mission by disseminating knowledge in the pursuit of education, learning, and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781107071575

© Peter S. Bernard 2015

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2015

Printed in the United States of America

A catalog record for this publication is available from the British Library.

Library of Congress Cataloging in Publication Data

Bernard, Peter S., author.

Fluid dynamics / Peter S. Bernard, University of Maryland.

pages cm

Includes bibliographical references and index.

ISBN 978-1-107-07157-5 (hardback)

1. Fluid dynamics. I. Title.

QC151.B387 2015

532'.05--dc23 2014044742

ISBN 978-1-107-07157-5 Hardback

Additional resources for this publication at www.cambridge.org/bernard

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party Internet Web sites referred to in this publication and does not guarantee that any content on such Web sites is, or will remain, accurate or appropriate.

Cambridge University Press
978-1-107-07157-5 - Fluid Dynamics
Peter S. Bernard
Frontmatter
[More information](#)

To my wife, Susan Bradshaw Sullivan

Cambridge University Press
978-1-107-07157-5 - Fluid Dynamics
Peter S. Bernard
Frontmatter
[More information](#)

Contents

<i>Preface</i>	<i>page xi</i>
1 Introduction	1
1.1 What Is a Fluid?	1
1.2 Molecular Structure and the Continuum Hypothesis	1
1.3 Dilatation and Vorticity	4
1.4 The Big Picture	5
1.5 Vector and Tensor Analysis	7
1.5.1 Vectors	8
1.5.2 Tensors	9
1.5.3 Skew Tensors	10
1.5.4 Gradient Tensor	11
1.5.5 Basis Vectors and Change of Coordinates	12
2 Eulerian and Lagrangian Viewpoints, Paths, and Streamlines	15
2.1 Eulerian versus Lagrangian Viewpoints	15
2.2 Fluid Particle Paths	15
2.3 Curves	18
2.4 Streamlines	21
3 Stream Function	24
3.1 Two-Dimensional Planar Flow	24
3.2 Axisymmetric Flow	30
4 Helmholtz Decomposition	36
4.1 Three-Dimensional Flow	36
4.2 Bounded Domains	38
4.3 Two-Dimensional Flow	40
5 Sources, Sinks, and Vortices	44
5.1 Sources and Sinks in Two Dimensions	44
5.2 Point Vortices	46

5.3	Accommodating Boundaries in Two Dimensions	48
5.4	Sources and Sinks in Three Dimensions	51
6	Doublets and Their Applications	57
6.1	Three-Dimensional Source/Sink Doublet	57
6.2	Doublets in Two Dimensions	61
7	Complex Potential	65
7.1	Connection to Complex Analysis	65
7.2	Flows Derived from a Power Law	67
7.3	Forces in 2D Potential Flows	71
7.4	Inviscid Flow Past a Cylinder	75
8	Accelerating Reference Frames	82
8.1	Orientation	83
8.2	Position Vector	85
8.3	Velocity	86
8.4	Acceleration and Fictitious Forces	89
9	Fluids at Rest	91
9.1	Forces in a Fluid at Rest	91
9.1.1	Micromanometer	94
9.1.2	Force on a Dam	95
9.2	Buoyancy	96
9.3	Accelerating Fluids at Rest	97
9.3.1	Accelerating Fish Tank	98
9.3.2	Rotating Bucket	99
10	Incompressibility and Mass Conservation	104
10.1	Some Useful Mathematics	104
10.2	Incompressibility	106
10.3	Mass Conservation	107
11	Stress Tensor: Existence and Symmetry	110
11.1	Existence of the Stress Tensor	110
11.2	Symmetry of the Stress Tensor	112
12	Stress Tensor in Newtonian Fluids	116
12.1	Relative Fluid Motion at a Point	116
12.2	The Stress Tensor	120
13	Navier-Stokes Equation	126
13.1	Rate of Change of Momentum	126
13.2	Surface Forces	127
13.3	The Navier-Stokes Equation	128

14 Thermodynamic Considerations	131
14.1 Overview	131
14.2 First Law of Thermodynamics	132
14.3 Perfect Gases	137
15 Energy Equation	140
16 Complete Equations of Motion	147
16.1 Differential Equations of Fluid Flow	147
16.2 Bernoulli Equation	148
16.2.1 Bernoulli Equation for Steady Flow	149
16.2.2 Bernoulli Equation for Nonsteady Flow	151
16.2.3 Crocco’s Relation	152
16.3 Control Volume Equations	153
16.3.1 Mass Conservation	153
16.3.2 Momentum Conservation	154
16.3.3 Conservation of Angular Momentum	157
16.3.4 Conservation of Energy	158
17 Applications of Bernoulli’s Equation and Control Volumes	160
17.1 Fluid Impinging on a Plate	160
17.2 Draining a Tank	165
17.3 Water Sprinkler	168
18 Vorticity	173
18.1 Vorticity Equation	173
18.2 Vortex Stretching and Reorientation	175
18.3 Kelvin’s Circulation Theorem	178
18.4 2D Vortex Methods	180
18.5 Simulation of a Wing Wake	183
19 Applications to Viscous Flow	188
19.1 The Reynolds Number	188
19.2 Unidirectional Flow	190
19.3 Flow in a Narrow Gap	191
19.4 Stokes Flow Past a Sphere	194
19.4.1 Problem Formulation	195
19.4.2 An Equation for the Stream Function	195
19.4.3 Solution for Stokes Flow	196
19.4.4 Forces on the Sphere	199
19.4.5 Self-Consistency of the Solution	200
19.5 Motion of a Sphere at Higher Reynolds Numbers	201
20 Laminar Boundary Layers	211
20.1 Boundary Layer Scaling	212
20.2 Blasius Boundary Layer	214
20.3 Falkner-Skan Boundary Layers	221

21	Some Applications to Convective Heat and Mass Transfer	225
21.1	A Thermal Boundary Layer	225
21.2	Monte Carlo Schemes for Modeling Convective Diffusion	229
21.2.1	Probabilistic Interpretation of Diffusion	229
21.2.2	Monte Carlo Model of Diffusion	230
21.2.3	Monte Carlo Simulation Including Convection	232
21.2.4	Monte Carlo Solution to a Thermal Boundary Layer	233
	APPENDIX A: Equations in Curvilinear Coordinates	239
A.1	Polar Coordinates	239
A.2	Cylindrical Coordinates	240
A.3	Spherical Coordinates	240
	APPENDIX B: Tensors	242
B.1	Divergence of a Tensor	242
B.2	Vector Cross	243
B.3	Principal Directions	244
	<i>Bibliography</i>	247
	<i>Index</i>	249

Preface

This book is inspired by a graduate-level course in fluid dynamics that I have taught at the University of Maryland for many years. The typical student taking this course, which is the starting point for graduate studies in fluid mechanics, has had one undergraduate course on fluids and a limited exposure to vector and tensor analysis. Consequently, the goal of this book is to provide a background in the physics and mathematics of fluid mechanics necessary for the pursuit of advanced studies and research at the graduate level. It is my experience that an effective route to these objectives is via a synthesis of the best features of two very excellent books, namely, *An Introduction to Fluid Dynamics* by George Batchelor, which presents the physics of fluid mechanics with exceptional clarity, and *An Introduction to Continuum Mechanics* by M. E. Gurtin (and now expanded and revised as *The Mechanics and Thermodynamics of Continua* by Gurtin, Fried, and Anand), which demonstrates the advantages of direct tensor notation in simplifying the expression of physical laws. Thus, to a large extent, this book combines the physics of Batchelor with the mathematics of Gurtin. The hope is that, in this way, an environment is created that helps make the subject of fluid dynamics clear, focused, and readily understandable. As a practical matter, this book should serve as an effective stepping-stone for new graduate students to enhance their accessibility to the books by Batchelor and Gurtin as well as those by many others.

Stylistically, this book follows an arc through the material that builds steadily toward the derivation and then application of the Navier-Stokes equations. The sequence of topics is also chosen so as to provide some significant exposure to examples of fluid flow and problem solving, before a relatively long and unavoidable set of chapters that deal in detail with the derivation of the flow equations. Most of what is in this book is covered in a one-semester course at Maryland, and no attempt is made to provide the depth of topics covered by Batchelor or Gurtin nor the comprehensive treatment of the subject matter typically found in other advanced textbooks. After studying this book, it is hoped that students will be well prepared to venture in any number of directions into more specialized and advanced topics in fluid dynamics.

Among the topics in the book, some represent a review of subjects normally encountered in undergraduate fluids courses (e.g., Chapter 9, on fluids at rest). This is intended to keep the book self-contained, to aid in the review of this material and

as a needed introduction to these topics for the occasional applied math or other nonengineering student who has never previously studied fluid mechanics.

The problems at the end of the chapters attempt to reflect the graduate level of the book by pursuing directions that are often somewhat challenging rather than repeating the formulaic engineering problems that are traditional to the undergraduate curriculum. For many of the problems, students are strongly encouraged to take full advantage of high-order computer languages such as MATLAB to help derive relations via symbolic manipulation, to solve algebraic and differential equations, and to calculate and plot numerical results. For example, in the case of MATLAB, facility with using commands such as *diff*, *int*, *solve*, *dsolve*, *subs*, *ode45*, and *bvp4c* greatly reduces the labor necessary to solve many problems in this book. In some cases, without the power of the symbolic solvers, the difficulty in obtaining solutions can be quite formidable if attempted with pencil and paper.

Some of the material in the book is specifically designed to be a launching point for writing computer code (e.g., with MATLAB) that solves interesting flow problems and displays results in the form of animations. Such material includes Sections 18.4 and 18.5, on the discrete vortex method; Section 19.5, on the motion of a sphere and other bodies; and Section 21.2, on the use of the Monte Carlo method for simulating scalar transport in fluid flows. In each of these cases, the numerical simulations can be carried out with a modest investment in programming yet bring to life intriguing aspects of fluid flow.

The author would like to express his great appreciation to Professor Bruce Berger for his many contributions toward improving the quality and clarity of the exposition in this book. I also appreciate the insights of Carl Biagetti of the Space Telescope Science Institute and graduate student Eric Leonard in reading some of the chapters.