

FUNDAMENTALS OF AEROSPACE NAVIGATION AND GUIDANCE

This text covers fundamentals used in the navigation and guidance of modern aerospace vehicles, in both atmospheric and space flight. It can be used as a textbook supporting a graduate-level course on aerospace navigation and guidance, a guide for self-study, or a resource for practicing engineers and researchers.

It begins with an introduction that discusses why navigation and guidance ought to be considered together and delineates the class of systems of interest in navigation and guidance. The book then presents the necessary fundamentals in deterministic and stochastic systems theory and applies them to navigation. Next, the book considers guidance problems under a variety of assumptions, leading to the scenarios of homing, ballistic, and midcourse guidance. Then, the book treats optimization and optimal control for application in optimal guidance. In the final chapter, the book introduces problems in which two competing controls exercise authority over a system, leading to differential games. Fundamentals of Aerospace Navigation and Guidance features examples illustrating concepts and homework problems at the end of all chapters.

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To our families, Joséphine, Monique, Orianne, and Louis, Michael, Leo, and Mia.

With a special commendation for Mia, who, at age three months, listened to the formal read-through of the whole book.





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Preface

This book arose out of lecture notes for the graduate-level course AE584 Guidance, Navigation, and Avionics, taught in the Department of Aerospace Engineering at the University of Michigan at Ann Arbor. The course was originally introduced by Professor Emeritus Robert M. Howe and was taught for a number of years by the authors.

This book can be used as a textbook supporting a graduate-level formal course on aerospace navigation and guidance. It can also be used as a guide for self-study on the subject. Finally, it can be used as a reference text. To enhance its usability, the book contains numerous examples illustrating concepts and homework problems at the end of all chapters. A partial solution manual for these homework problems is available with limited circulation.

Although this book uses many mathematical results, it is not a mathematics text. The results are stated with some rigor, but only a few proofs of special interest are given. The interested reader may consult the bibliography listed at the end of each chapter for proofs and additional information.

The prerequisite knowledge includes undergraduate instruction in elementary linear algebra, differential equations, and flight mechanics, which are standard in aerospace engineering curricula. Key definitions and mathematical results are given in the appendix.

The intended audience for this book encompasses graduate students in engineering specializing in flight dynamics, guidance, navigation, and control, together with practicing engineers and researchers in the field. In the authors' experience, this text maps to a two-semester graduate sequence, with the first semester covering roughly the first six chapters and the second semester covering roughly the last four chapters.

The authors acknowledge and are thankful for the intellectual contributions of their teachers, colleagues, and especially several generations of students who have learned this material at Michigan. The authors also acknowledge and are thankful for financial support from the Air Force Research Laboratory, the Air Force Office of Scientific Research, the Boeing Company, the Office of Naval Research, and the National Science Foundation.

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