1

Protective measurement: an introduction

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Protective measurement, in the language of standard quantum mechanics, is a method to measure the *expectation value* of an observable on a *single* quantum system (Aharonov and Vaidman, 1993; Aharonov, Anandan and Vaidman, 1993). For a conventional impulsive measurement, the state of the measured system is strongly entangled with the state of the measuring device during the measurement, and the measurement result is one of the eigenvalues of the measured observable. By contrast, during a protective measurement, the measured state is protected by an appropriate procedure so that it neither changes nor becomes entangled with the state of the measuring device appreciably. In this way, such protective measurements can measure the expectation values of observables on a single quantum system, and in particular, the wave function of the system can also be measured as expectation values of certain observables. It is expected that protective measurements can be performed in the near future with the rapid development of weak measurement technologies (e.g. Kocsis et al., 2011; Lundeen et al., 2011). In this chapter, we will give a clear introduction to protective measurement in quantum mechanics.

1.1 Standard quantum mechanics and impulsive measurement

The standard formulation of quantum mechanics, which was first developed by Dirac (1930) and von Neumann (1955), is based on the following basic principles.

1 Physical states

The state of a physical system is represented by a normalized wave function or unit vector $|\psi(t)\rangle$ in a Hilbert space.[1] The Hilbert space is complete in the sense that every possible physical state can be represented by a state vector in the space.

[1] The Hilbert space is a complete vector space with scalar product. The common notion of state includes both proper vectors normalizable to unity in the Hilbert space and so-called improper vectors normalizable only to the Dirac delta functions.

2 Physical properties

Every measurable property or observable of a physical system is represented by a Hermitian operator on the Hilbert space associated with the system. A physical system has a determinate value for an observable if and only if it is in an eigenstate of the observable (this is often called the eigenvalue–eigenstate link).

3 Composition rule

The Hilbert space associated with a composite system is the tensor product of the Hilbert spaces associated with the systems of which it is composed. Similarly, the Hilbert space associated with independent properties is the tensor product of the Hilbert spaces associated with each property.

4 Evolution law

(1) Linear evolution

The state of a physical system $|\psi(t)\rangle$ obeys the linear Schrödinger equation $i\hbar\partial|\psi(t)\rangle/\partial t = H|\psi(t)\rangle$ (when it is not measured), where $\hbar$ is Planck's constant divided by 2π, and H is the Hamiltonian operator that depends on the energy properties of the system.

(2) Non-linear collapse evolution

If a physical system is in a state $|\psi\rangle = \sum_i c_i|a_i\rangle$, where $|a_i\rangle$ is the eigenstate of an observable A with eigenvalue a_i, then an (impulsive) measurement of the observable A will instantaneously and randomly collapse the state into one of the eigenstates $|a_i\rangle$ with probability $|c_i|^2$. This is called the collapse postulate, and the non-linear stochastic process is called the reduction of the state vector or the collapse of the wave function.

The link between the mathematical formalism and experiments is provided by the Born rule. It says that the probability of the above measurement of the observable A yielding the result a_i is $|c_i|^2$. For a continuous property such as position x, the probability of obtaining a measurement result between x and $x + \mathrm{d}x$ is $|\langle x|\psi\rangle|^2\mathrm{d}x$. Note that the Born rule can be derived from the collapse postulate by resorting to the eigenvalue–eigenstate link, but it does not necessarily depend on the postulate.

The conventional impulsive measurements can be formulated as follows. According to the standard von Neumann procedure, measuring an observable A in a quantum state $|\psi\rangle$ involves an interaction Hamiltonian

$$H_{\mathrm{I}} = g(t)PA \tag{1.1}$$

coupling the measured system to an appropriate measuring device, where P is the conjugate momentum of the pointer variable X of the device. The time-dependent coupling strength $g(t)$ is a smooth function normalized to $\int \mathrm{d}t g(t) = 1$ during the measurement interval τ, and $g(0) = g(\tau) = 0$. The initial state of the pointer at

$t = 0$ is supposed to be a Gaussian wave packet of eigenstates of X with width w_0, centered around the eigenvalue x_0, which is denoted by $|\phi(x_0)\rangle$.

For an impulsive measurement, the interaction H_I is of very short duration and so strong that it dominates the rest of the Hamiltonian (i.e. the effect of the free Hamiltonians of the measuring device and the measured system can be neglected). Then the state of the combined system at the end of the interaction can be written as

$$|t = \tau\rangle = \mathrm{e}^{-\frac{\mathrm{i}}{\hbar}PA} |\psi\rangle |\phi(x_0)\rangle . \tag{1.2}$$

By expanding $|\psi\rangle$ in the eigenstates of A, $|a_i\rangle$, we obtain

$$|t = \tau\rangle = \sum_i \mathrm{e}^{-\frac{\mathrm{i}}{\hbar}Pa_i} c_i |a_i\rangle |\phi(x_0)\rangle , \tag{1.3}$$

where c_i are the expansion coefficients. The exponential term shifts the center of the pointer by a_i:

$$|t = \tau\rangle = \sum_i c_i |a_i\rangle |\phi(x_0 + a_i)\rangle . \tag{1.4}$$

This is an entangled state, where the eigenstates of A with eigenvalues a_i get correlated to measuring device states in which the pointer is shifted by these values a_i. Then, by the collapse postulate, the state will instantaneously and randomly collapse into one of its branches $|a_i\rangle |\phi(x_0 + a_i)\rangle$ with probability $|c_i|^2$. This means that the measurement result can only be one of the eigenvalues of the measured observable A, say a_i, with a certain probability $|c_i|^2$. The expectation value of A is then obtained as the statistical average of eigenvalues for an ensemble of identically prepared systems, namely $\langle A\rangle = \sum_i |c_i|^2 a_i$.

1.2 Weak measurement

Impulsive measurements are only one kind of quantum measurement, for which the coupling between the measured system and the measuring device is very strong, and the results are only the eigenvalues of an observable. We can also perform other kinds of measurement by adjusting the coupling strength. An interesting example is weak measurement (Aharonov, Albert and Vaidman, 1988; Aharonov and Vaidman, 1990), for which the measurement result is the expectation value of the measured observable.

A weak measurement is a standard measuring procedure with weakened coupling. Like impulsive measurements, the interaction Hamiltonian is also given by (1.1) for a weak measurement. The weakness of the interaction is achieved by preparing the initial state of the measuring device in such a way that the conjugate momentum of the pointer variable is localized around zero with small uncertainty,

and thus the interaction Hamiltonian (1.1) is small. As a simple example, let the initial state of the pointer in position space be:

$$\langle x|\phi\rangle = (w_0^2\pi)^{-1/4}e^{-x^2/2w_0^2}. \tag{1.5}$$

The corresponding initial probability distribution is

$$P_i(x) = (w_0^2\pi)^{-1/2}e^{-x^2/w_0^2}. \tag{1.6}$$

Expanding the initial state of the system $|\psi\rangle$ in the eigenstates $|a_i\rangle$ of the measured observable A, $|\psi\rangle = \sum_i c_i|a_i\rangle$, then after the interaction (1.1) the state of the system and the measuring device is:

$$|t = \tau\rangle = (w_0^2\pi)^{-1/4}\sum_i c_i|a_i\rangle e^{-(x-a_i)^2/2w_0^2}. \tag{1.7}$$

The probability distribution of the pointer variable corresponding to the final state (1.7) is:

$$P_f(x) = (w_0^2\pi)^{-1/2}\sum_i |c_i|^2 e^{-(x-a_i)^2/w_0^2}. \tag{1.8}$$

In the case of an impulsive measurement, this is a weighted sum of the initial probability distribution localized around various eigenvalues a_i. Therefore, the reading of the pointer variable at the end of the measurement always yields a value close to one of the eigenvalues. By contrast, the limit of weak measurement corresponds to $w_0 \gg a_i$ for all eigenvalues a_i. Then we can perform the Taylor expansion of the sum (1.8) around $x = 0$ up to first order and rewrite the final probability distribution of the pointer variable in the following way:

$$P_f(x) \approx (w_0^2\pi)^{-1/2}\sum_i |c_i|^2[1-(x-a_i)^2/w_0^2] \approx (w_0^2\pi)^{-1/2}e^{-(x-\sum_i |c_i|^2 a_i)^2/w_0^2}. \tag{1.9}$$

This is the initial probability distribution shifted by the value $\sum_i |c_i|^2 a_i$ (Aharonov and Vaidman, 2008). It indicates that the result of the weak measurement is the expectation value of the measured observable in the measured state:

$$\langle A\rangle \equiv \langle\psi|A|\psi\rangle = \sum_i |c_i|^2 a_i. \tag{1.10}$$

Certainly, since the width of the pointer wave packet is much greater than the shift of the center of the pointer, namely $w_0 \gg \langle A\rangle$, the above weak measurement of a single system is very imprecise. However, by performing the weak measurement on an ensemble of N identically prepared systems the precision can be improved by a factor $\sqrt{N}$. This scheme of weak measurement has been realized and proved useful in quantum optical experiments (see, e.g. Kocsis et al., 2011; Lundeen et al., 2011).

Although weak measurements, like conventional impulsive measurements, also need to measure an ensemble of identically prepared quantum systems, they are

conceptually different. For impulsive measurements, every identically prepared system in the ensemble shifts the pointer of the measuring device by one of the eigenvalues of the measured observable. By contrast, for weak measurements, every identically prepared system in the ensemble shifts the pointer of the measuring device directly by the expectation value of the measured observable.

1.3 Protective measurement

Protective measurements are improved methods of weak measurements in the sense that they can measure the expectation values of observables on a single quantum system (Aharonov and Vaidman, 1993; Aharonov, Anandan and Vaidman, 1993). For an impulsive measurement, if the measured system, prior to the measurement of an observable A, is not in an eigenstate of A, then its state will be invariably entangled with the state of the device due to the interaction. A protective measurement differs from an impulsive measurement (as well as from a weak measurement) in that the measured state is protected from being entangled and changed appreciably when the measurement is being made. A universal protection scheme is via the quantum Zeno effect. Let's see how this can be done.

1.3.1 Measurements with artificial protection

Let $|\psi\rangle$ be an arbitrary known state of a single quantum system at a given instant $t = 0$. To protect this state from being changed, we make projective measurements of an observable $P(t)$, for which $|\psi\rangle$ is a non-degenerate eigenstate, a large number of times which are dense in the measurement interval $[0, \tau]$ (Aharonov, Anandan and Vaidman, 1993). For example, $P(t)$ is measured in $[0, \tau]$ at times $t_n = (n/N)\tau, n = 1, 2, ..., N$, where N is an arbitrarily large number. At the same time, we make an impulsive measurement of observable A in the interval $[0, \tau]$, which is described by the interaction Hamiltonian (1.1). The initial state of the pointer is supposed to be a Gaussian wave packet of width w_0 centered at initial position x_0, denoted by $|\phi(x_0)\rangle$.

Then the branch of the state of the combined system after τ, in which each projective measurement of $P(t_n)$ results in the state of the measured system being in $|\psi\rangle$, is given by

$$\begin{aligned} |t = \tau\rangle &= |\psi\rangle \langle\psi|e^{-\frac{i}{\hbar}\frac{\tau}{N}H(t_N)}... |\psi\rangle \langle\psi|e^{-\frac{i}{\hbar}\frac{\tau}{N}H(t_2)} |\psi\rangle \langle\psi| \\ &\quad \times e^{-\frac{i}{\hbar}\frac{\tau}{N}H(t_1)} |\psi\rangle |\phi(x_0)\rangle \\ &= |\psi\rangle \langle\psi|e^{-\frac{i}{\hbar}\frac{\tau}{N}g(t_N)PA}... |\psi\rangle \langle\psi|e^{-\frac{i}{\hbar}\frac{\tau}{N}g(t_2)PA} |\psi\rangle \langle\psi| \\ &\quad \times e^{-\frac{i}{\hbar}\frac{\tau}{N}g(t_1)PA} |\psi\rangle |\phi(x_0)\rangle. \end{aligned} \tag{1.11}$$

Thus in the limit of $N \to \infty$, we have

$$|t = \tau\rangle = |\psi\rangle\, \mathrm{e}^{-\frac{\mathrm{i}}{\hbar}\int_0^\tau g(t)\langle\psi|A|\psi\rangle P\mathrm{d}t}\, |\phi(x_0)\rangle = |\psi\rangle\, |\phi(x_0 + \langle A\rangle)\rangle\,. \tag{1.12}$$

Since the total probability of other branches is proportional to τ^2/N to first order of N, the above state will be the state of the combined system after τ when $N \to \infty$. This demonstrates that for an arbitrary but known state of a quantum system at a given instant, we can protect the state from being changed via the quantum Zeno effect by frequent projective measurements, and an independent measurement of an observable A, which is made at the same time, yields the expectation value of the observable in the measured state.

By a conventional impulsive measurement on a single quantum system, one obtains one of the eigenvalues of the measured observable, and the expectation value of the observable can only be obtained as the statistical average of eigenvalues for an ensemble of identically prepared systems. Thus it seems surprising that a protective measurement can yield the expectation value of the measured observable directly from a single quantum system. In fact, the appearance of expectation values as measurement results is quite natural when the measured state is not changed and the entanglement during conventional measurements does not take place as for protective measurements (Aharonov, Anandan and Vaidman, 1993). In this case, the evolution of the combining state is

$$|\psi(0)\rangle\, |\phi(0)\rangle \to |\psi(t)\rangle\, |\phi(t)\rangle\,, t > 0, \tag{1.13}$$

where $|\psi(t)\rangle$ is the same as $|\psi(0)\rangle$ up to a phase factor during the measurement interval $[0, \tau]$. The interaction Hamiltonian is still given by (1.1). Then, by Ehrenfest's theorem we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\psi(t)\phi(t)|X\,|\psi(t)\phi(t)\rangle = g(t)\langle\psi(0)|A\,|\psi(0)\rangle\,, \tag{1.14}$$

where X is the pointer variable. This further leads to

$$\langle\phi(\tau)|X\,|\phi(\tau)\rangle - \langle\phi(0)|X\,|\phi(0)\rangle = \langle\psi(0)|A\,|\psi(0)\rangle\,. \tag{1.15}$$

This means that the shift of the center of the pointer of the device gives the expectation value of the measured observable in the measured state.

1.3.2 Measurements with natural protection

In some special cases, the universal protection procedure via the quantum Zeno effect is not necessary, and the system's Hamiltonian can help protect its state from changing when the measurement interaction is weak and adiabatic. For example, for a quantum system in a discrete non-degenerate energy eigenstate, the system

itself supplies the protection of the state due to energy conservation. By the adiabatic theorem, the adiabatic interaction during the measurement ensures that the measured system cannot make a transition from one discrete energy eigenstate to another. Moreover, according to first-order perturbation theory, for any given value of P, the energy of the measured energy eigenstate shifts by an infinitesimal amount: $\delta E = \langle H_{\rm I}\rangle = g(t)P\langle A\rangle$, and the corresponding time evolution $e^{-iP\langle A\rangle/\hbar}$ then shifts the pointer by the expectation value $\langle A\rangle$. For degenerate energy eigenstates, we may not use the universal protection procedure either. The simplest way is to add a protective potential to change the energies of other states and lift the degeneracy. Then the measured state remains unchanged, but is now protected by energy conservation like non-degenerate energy eigenstates.

As a simple example, we consider a quantum system in a discrete non-degenerate energy eigenstate $|E_n\rangle$. In this case, the system itself supplies the protection of the state and no artificial protection is needed. The interaction Hamiltonian for a protective measurement of an observable A in this state is also given by (1.1) as for conventional impulsive measurements. But differently from impulsive measurements, for which the interaction is very strong and almost instantaneous, the protective measurements make use of the opposite limit where the interaction of the measuring device with the system is weak and adiabatic, and thus the free Hamiltonians cannot be neglected. Let the total Hamiltonian of the combined system be

$$H = H_{\rm S} + H_{\rm D} + H_{\rm I}, \tag{1.16}$$

where $H_{\rm S}$ and $H_{\rm D}$ are the free Hamiltonians of the measured system and the measuring device, respectively, and $H_{\rm I} = g(t)PA$ is the interaction Hamiltonian. As before, we suppose the time-dependent coupling strength $g(t)$ is a smooth function normalized to $\int {\rm d}t g(t) = 1$ in the measurement interval $[0, T]$, and $g(0) = g(T) = 0$, and the initial state of the pointer is a Gaussian wave packet of width w_0 centered at initial position x_0, denoted by $|\phi(x_0)\rangle$.

The state of the combined system after T is then given by

$$|t = T\rangle = e^{-\frac{i}{\hbar}\int_0^T H(t){\rm d}t}\,|E_n\rangle\,|\phi(x_0)\rangle\,. \tag{1.17}$$

By ignoring the switching on and switching off processes, the full Hamiltonian (with $g(t) = 1/T$) is time-independent and no time-ordering is needed.[2] Then we obtain

$$|t = T\rangle = e^{-\frac{i}{\hbar}HT}\,|E_n\rangle\,|\phi(x_0)\rangle\,, \tag{1.18}$$

[2] The change in the total Hamiltonian during these processes is smaller than PA/T, and thus the approximate treatment given below is valid. For a more strict analysis see Dass and Qureshi (1999).

where $H = H_S + H_D + PA/T$. We further expand $|\phi(x_0)\rangle$ in the eigenstate of H_D, $\left|E_j^d\right\rangle$, and write

$$|t = T\rangle = e^{-\frac{i}{\hbar}HT} \sum_j c_j |E_n\rangle |E_j^d\rangle. \tag{1.19}$$

Let the exact eigenstates of H be $\left|\Psi_{k,m}\right\rangle$ and the corresponding eigenvalues be $E(k,m)$; we have

$$|t = T\rangle = \sum_j c_j \sum_{k,m} e^{-\frac{i}{\hbar}E(k,m)T} \langle \Psi_{k,m}|E_n, E_j^d\rangle |\Psi_{k,m}\rangle. \tag{1.20}$$

Since the interaction is very weak, the total Hamiltonian H can be thought of as $H_0 = H_S + H_D$ perturbed by PA/T. Using the fact that PA/T is a small perturbation and that the eigenstates of H_0 are of the form $|E_k\rangle |E_m^d\rangle$, the perturbation theory gives

$$\begin{aligned} \left|\Psi_{k,m}\right\rangle &= |E_k\rangle |E_m^d\rangle + O(1/T), \\ E(k,m) &= E_k + E_m^d + \frac{1}{T}\langle A\rangle_k \langle P\rangle_m + O(1/T^2). \end{aligned} \tag{1.21}$$

Note that it is a necessary condition for (1.21) to hold that $|E_k\rangle$ is a non-degenerate eigenstate of H_S. Substituting (1.21) in (1.20) and taking the limit $T \to \infty$ yields

$$|t = T\rangle_{T\to\infty} = \sum_j e^{-\frac{i}{\hbar}(E_n T + E_j^d T + \langle A\rangle_n \langle P\rangle_j)} c_j |E_n\rangle |E_j^d\rangle. \tag{1.22}$$

For the case where P commutes with the free Hamiltonian of the device,[3] i.e., $[P, H_D] = 0$, the eigenstates $|E_j^d\rangle$ of H_D are also the eigenstates of P, and thus the above equation can be rewritten as

$$|t = T\rangle_{T\to\infty} = e^{-\frac{i}{\hbar}E_n T - \frac{i}{\hbar}H_D T - \frac{i}{\hbar}\langle A\rangle_n P} |E_n\rangle |\phi(x_0)\rangle. \tag{1.23}$$

It can be seen that the third term in the exponent will shift the center of the pointer by an amount $\langle A\rangle_n$:

$$|t = T\rangle_{T\to\infty} = e^{-\frac{i}{\hbar}E_n T - \frac{i}{\hbar}H_D T} |E_n\rangle |\phi(x_0 + \langle A\rangle_n)\rangle. \tag{1.24}$$

This indicates that the result of the protective measurement is the expectation value of the measured observable in the measured state, and moreover, the measured state is not changed by the protective measurement (except for an overall phase factor).

It is worth noting that since the position variable of the pointer does not commute with its free Hamiltonian, the pointer wave packet will spread during the measurement interval. For example, the kinematic energy term $P^2/2M$ in the free Hamiltonian of the pointer will spread the wave packet without shifting the center, and the

[3] For the derivation for the case $[P, H_D] \neq 0$ see Dass and Qureshi (1999).

width of the wave packet at the end of interaction will be $w(T) = [\frac{1}{2}(w_0^2 + \frac{T^2}{M^2 w_0^2})]^{\frac{1}{2}}$ (Dass and Qureshi, 1999). However, the spreading of the pointer wave packet can be made as small as possible by increasing the mass M of the pointer, and thus it will not interfere with resolving the shift of the center of the pointer in principle.

1.3.3 Measurements of the wave function of a single system

Since the wave function can be reconstructed from the expectation values of a sufficient number of observables, the wave function of a single quantum system can be measured by a series of protective measurements. Let the explicit form of the measured state at a given instant t be $\psi(x)$, and the measured observable A be (normalized) projection operators on small spatial regions V_n having volume v_n:

$$A = \begin{cases} \frac{1}{v_n}, & \text{if } x \in V_n, \\ 0, & \text{if } x \notin V_n. \end{cases} \tag{1.25}$$

A protective measurement of A then yields

$$\langle A \rangle = \frac{1}{v_n} \int_{V_n} |\psi(x)|^2 \mathrm{d}v, \tag{1.26}$$

which is the average of the density $\rho(x) = |\psi(x)|^2$ over the small region V_n. Similarly, we can measure another observable $B = \frac{\hbar}{2mi}(A\nabla + \nabla A)$. The measurement yields

$$\langle B \rangle = \frac{1}{v_n} \int_{V_n} \frac{\hbar}{2m\mathrm{i}} (\psi^* \nabla \psi - \psi \nabla \psi^*) \mathrm{d}v = \frac{1}{v_n} \int_{V_n} j(x) \mathrm{d}v. \tag{1.27}$$

This is the average value of the flux density $j(x)$ in the region V_n. Then when $v_n \to 0$ and after performing measurements in sufficiently many regions V_n we can measure $\rho(x)$ and $j(x)$ everywhere in space. Since the wave function $\psi(x, t)$ can be uniquely expressed by $\rho(x, t)$ and $j(x, t)$ (except for an overall phase factor), the whole wave function of the measured system at a given instant can be measured by protective measurements.

1.4 Further discussion

Protective measurement is a surprising measuring method, by which one can measure the *expectation value* of an observable on a *single* quantum system, even if the system is not in an eigenstate of the measured observable. This remarkable feature makes protective measurements quite distinct from conventional impulsive measurements. It is unsurprising that there appeared numerous objections to the validity and meaning of protective measurements (see, e.g. Unruh, 1994; Rovelli, 1994; Ghose and Home, 1995; Uffink, 1999, 2013). Although misunderstandings

have been clarified (Aharonov, Anandan and Vaidman, 1996; Dass and Qureshi, 1999; Vaidman, 2009; Gao, 2013), it is still debatable whether protective measurement has important implications for our understanding of quantum mechanics, especially for the ontological status of the wave function. In the following, we will emphasize three key points that may help us understand protective measurement, and briefly review the current state of debate on its possible implications.

First of all, a single quantum system being in an arbitrary known state can be protectively measured in principle. The state of the system being protected to be unchanged permits the state as well as the expectation values of observables in the state to be measurable. In this sense, protective measurements are not a special kind of quantum measurement, but the *very* way to measure the actual state of a quantum system at a given instant. By comparison, a non-protective measurement such as an impulsive measurement will change the measured state, and the resulting measurement outcome (i.e. one of the eigenvalues of the measured observable) does not reflect the original state of the measured system. Besides, when a quantum system interacts with another system under non-protective conditions, its state also evolves in time, and thus the expectation values of observables do not manifest themselves explicitly in the interaction either. For example, the interaction between two charged quantum systems is not directly dependent on the expectation values of their charges, but described by the potential terms in the Schrödinger equation (see Chapter 15).

Next, a realistic protective measurement can never be performed on a single quantum system with absolute certainty. For example, for a realistic protective measurement of an observable A on a non-degenerate energy eigenstate whose measurement interval T is finite, there is always a tiny probability proportional to $1/T^2$ of obtaining a different result $\langle A\rangle_{\perp}$, where $\perp$ refers to a normalized state in the subspace normal to the measured state as picked out by first-order perturbation theory. However, this effect can be made arbitrarily small when the measurement interval T is arbitrarily long. In this sense, an ideal protective measurement can measure the expectation values of observables on a single quantum system with certainty *in principle*.

Thirdly, we stress that the validity of the scheme of protective measurements does not rely on the standard von Neumann formulation of measurements. In the above formulation of protective measurement, the measuring system can be a microscopic system such as an electron, and the shift of the center of the wave packet of the measuring system is only determined by the Schrödinger equation. Since the state of the measured system is not changed during the protective measurement, a large number of identically prepared measuring systems can be used to protectively measure the original measured system, and the centers of their wave packets have the same shift. Then the shift can be read out by conventional