Quantum Stochastics

The classical probability theory initiated by Kolmogorov and its quantum counterpart, pioneered by von Neumann, were created at about the same time in the 1930s, but development of the quantum theory has trailed far behind. Although highly appealing, the quantum theory has a steep learning curve, requiring tools from both probability and analysis and a facility for combining the two viewpoints.

This book is a systematic, self-contained account of the core of quantum probability and quantum stochastic processes for graduate students and researchers. The only assumed background is knowledge of the basic theory of Hilbert spaces, bounded linear operators, and classical Markov processes. From there, the book introduces additional tools from analysis, and then builds the quantum probability framework needed to support applications to quantum control and quantum information and communication. These include quantum noise, quantum stochastic calculus, stochastic quantum differential equations, quantum Markov semigroups and processes, and large-time asymptotic behavior of quantum Markov semigroups.

Dr. Mou-Hsiung Chang served as a professor of mathematical sciences at the University of Alabama in Huntsville for twenty-eight years, where he also served as department chair for eight years, prior to joining the U.S. Army Research Office (ARO) in 2002. He has published extensively on stochastic analysis and control and on quantum stochastics, with more than 70 refereed journal articles, 40 conference papers and technical reports, and more than 80 invited technical presentations at conferences and universities.

CAMBRIDGE SERIES IN STATISTICAL AND PROBABILISTIC MATHEMATICS

This series of high-quality upper-division textbooks and expository monographs covers all aspects of stochastic applicable mathematics. The topics range from pure and applied statistics to probability theory, operations research, optimization, and mathematical programming. The books contain clear presentations of new developments in the field and also of the state of the art in classical methods. While emphasizing rigorous treatment of theoretical methods, the books also contain applications and discussions of new techniques made possible by advances in computational practice.

A complete list of books in the series can be found at www.cambridge.org/statistics.
Recent titles include the following:

11. *Statistical Models*, by A. C. Davison
12. *Semiparametric Regression*, by David Ruppert, M. P. Wand, and R. J. Carroll
13. *Exercises in Probability*, by Loïc Chaumont and Marc Yor
14. *Statistical Analysis of Stochastic Processes in Time*, by J. K. Lindsey
15. *Measure Theory and Filtering*, by Lakhdar Aggoun and Robert Elliott
16. *Essentials of Statistical Inference*, by G. A. Young and R. L. Smith
17. *Elements of Distribution Theory*, by Thomas A. Severini
18. *Statistical Mechanics of Disordered Systems,* by Anton Bovier
19. *The Coordinate-Free Approach to Linear Models*, by Michael J. Wichura
20. *Random Graph Dynamics*, by Rick Durrett
21. *Networks*, by Peter Whittle
22. *Saddlepoint Approximations with Applications*, by Ronald W. Butler
23. *Applied Asymptotics*, by A. R. Brazzale, A. C. Davison, and N. Reid
24. *Random Networks for Communication*, by Massimo Franceschetti and Ronald Meester
25. *Design of Comparative Experiments*, by R. A. Bailey
26. *Symmetry Studies*, by Marlos A. G. Viana
27. *Model Selection and Model Averaging*, by Gerda Claeskens and Nils Lid Hjort
28. *Bayesian Nonparametrics*, edited by Nils Lid Hjort et al.
29. *From Finite Sample to Asymptotic Methods in Statistics*, by Pranab K. Sen, Julio M. Singer, and Antonio C. Pedrosa de Lima
30. *Brownian Motion*, by Peter Mörters and Yuval Peres
31. *Probability (Fourth Edition)*, by Rick Durrett
32. *Analysis of Multivariate and High-Dimensional Data*, by Inge Koch
33. *Stochastic Processes*, by Richard F. Bass
34. *Regression for Categorical Data*, by Gerhard Tutz
35. *Exercises in Probability (Second Edition)*, by Loïc Chaumont and Marc Yor
36. *Statistical Principles for the Design of Experiments*, by R. Mead, S. G. Gilmour, and A. Mead

Quantum Stochastics

Mou-Hsiung Chang
Mathematical Sciences Division,
U.S. Army Research Office

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314-321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi - 110025, India

79 Anson Road, #06-04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781107069190

First published 2015

A catalogue record for this publication is available from the British Library

Library of Congress Cataloging in Publication data
Chang, Mou-Hsiung, author.
Quantum stochastics / Mou-Hsiung Chang, Mathematical Sciences Division, U.S. Army Research Office.
pages cm. – (Cambridge series in statistical and probabilistic mathematics)
Includes bibliographical references and index.
ISBN 978-1-107-06919-0 (hardback)
1. Stochastic processes. 2. Probabilities. 3. Quantum theory. I. Title.
QA274.C44 2014
519.2′3–dc23 2014023639

ISBN 978-1-107-06919-0 Hardback

To my wife Yuen-Man Chang and in memory of my loving mother

Cambridge University Press
978-1-107-06919-0 — Quantum Stochastics
Mou-Hsiung Chang
Frontmatter
More Information

Contents

Preface

It is widely known that the classical probability theory initiated by Kolmogorov and its quantum (or noncommutative) counterpart pioneered by von Neumann were both created at about the same time. However, the subsequent developments of the latter have trailed far behind the former. This is perhaps because development of a theory of quantum stochastics requires an unusually large number of tools from operator theory and perhaps also because the probabilistic and analytical tools for understanding sample path behaviors of quantum stochastic processes have yet to be developed. This monograph is intended to provide the interested readers with a systematic and yet introductory treatment of a theory of quantum Markov processes that is in parallel to its commutative counterpart, namely, the well-known classical theory of probability and Markov processes.

This monograph can be used as an introduction and/or as a research reference for researchers and advanced graduate students who have been exposed to the theory of classical (or commutative) probability and Markov processes and have a special interest in their noncommutative counterparts. This monograph is intended to be as self-contained as possible by providing necessary review material and the proofs for almost all of the lemmas, propositions, and theorems contained herein. Some knowledge in real analysis, functional analysis, and stochastic processes will be helpful. However, no background material is assumed beyond knowledge of the basic theory of Hilbert spaces, bounded linear operators, and classical Markov processes.

This monograph is largely based on a current account of relevant research results contributed by many researchers on quantum stochastic calculus, quantum dynamical or Markov semigroups, Markov dilations, quantum Markov processes, and large time asymptotic behaviors such as those of the invariant states, recurrence and transience, ergodicity, and stability of quantum Markov semigroups/processes. The bibliography certainly not exhaustive and is likely to have omitted works by other researchers. The author apologizes for any inadvertent omissions in this monograph of their works.

The author would like to thank his colleagues at large for their strong encouragement throughout the preparation of this monograph. This monograph is written under an unfunded personal research project while the author is a program manager at the U.S. Army Research Office (ARO). The author acknowledges the stimulating working environment provided by ARO that makes the completion of this research monograph possible. However, the views and conclusions contained herein are those of the author and should not be interpreted as necessarily representing the official policies or endorsements, either expressed or implied, of the U.S. Army. Last, but not least, the author would also like to thank Diana Gillooly, Dana Bricken, and Jeanie Lee of Cambridge University Press and Minaketan Dash of Aptara for their editorial assistance throughout the production of this book.