

Quantum Stochastics

The classical probability theory initiated by Kolmogorov and its quantum counterpart, pioneered by von Neumann, were created at about the same time in the 1930s, but development of the quantum theory has trailed far behind. Although highly appealing, the quantum theory has a steep learning curve, requiring tools from both probability and analysis and a facility for combining the two viewpoints.

This book is a systematic, self-contained account of the core of quantum probability and quantum stochastic processes for graduate students and researchers. The only assumed background is knowledge of the basic theory of Hilbert spaces, bounded linear operators, and classical Markov processes. From there, the book introduces additional tools from analysis, and then builds the quantum probability framework needed to support applications to quantum control and quantum information and communication. These include quantum noise, quantum stochastic calculus, stochastic quantum differential equations, quantum Markov semigroups and processes, and large-time asymptotic behavior of quantum Markov semigroups.

DR. MOU-HSIUNG CHANG served as a professor of mathematical sciences at the University of Alabama in Huntsville for twenty-eight years, where he also served as department chair for eight years, prior to joining the U.S. Army Research Office (ARO) in 2002. He has published extensively on stochastic analysis and control and on quantum stochastics, with more than 70 refereed journal articles, 40 conference papers and technical reports, and more than 80 invited technical presentations at conferences and universities.

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To my wife Yuen-Man Chang and in memory of my loving mother

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Preface

It is widely known that the classical probability theory initiated by Kolmogorov and its quantum (or noncommutative) counterpart pioneered by von Neumann were both created at about the same time. However, the subsequent developments of the latter have trailed far behind the former. This is perhaps because development of a theory of quantum stochastics requires an unusually large number of tools from operator theory and perhaps also because the probabilistic and analytical tools for understanding sample path behaviors of quantum stochastic processes have yet to be developed. This monograph is intended to provide the interested readers with a systematic and yet introductory treatment of a theory of quantum Markov processes that is in parallel to its commutative counterpart, namely, the well-known classical theory of probability and Markov processes.

This monograph can be used as an introduction and/or as a research reference for researchers and advanced graduate students who have been exposed to the theory of classical (or commutative) probability and Markov processes and have a special interest in their non-commutative counterparts. This monograph is intended to be as self-contained as possible by providing necessary review material and the proofs for almost all of the lemmas, propositions, and theorems contained herein. Some knowledge in real analysis, functional analysis, and stochastic processes will be helpful. However, no background material is assumed beyond knowledge of the basic theory of Hilbert spaces, bounded linear operators, and classical Markov processes.

This monograph is largely based on a current account of relevant research results contributed by many researchers on quantum stochastic calculus, quantum dynamical or Markov semigroups, Markov dilations, quantum Markov processes, and large time asymptotic behaviors such as those of the invariant states, recurrence and transience, ergodicity, and stability of quantum Markov semigroups/processes. The bibliography certainly not exhaustive and is likely to have omitted works by other researchers. The author apologizes for any inadvertent omissions in this monograph of their works.

The author would like to thank his colleagues at large for their strong encouragement throughout the preparation of this monograph. This monograph is written under an unfunded personal research project while the author is a program manager at the U.S. Army Research Office (ARO). The author acknowledges the stimulating working environment provided by ARO that makes the completion of this research monograph possible. However, the views and conclusions contained herein are those of the author and should not be interpreted as

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