

978-1-107-06879-7 - Quantum Phase Transitions in Transverse Field Spin Models: From Statistical Physics to Quantum Information

Amit Dutta, Gabriel Aeppli, Bikas K. Chakrabarti, Uma Divakaran, Thomas F. Rosenbaum and Diptiman Sen Frontmatter

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From Statistical Physics to Quantum Information

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#### To our parents

Doll Dutta and Sukomal Dutta
Dorothee Aeppli and Alfred Aeppli
Pratima Chakrabarti and Bimal K. Chakrabarti
Savithry E. S. and A. P. Divakaran
Hanna L. Rosenbaum and Martin M. Rosenbaum
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### **Preface**

In recent years, there has been an upsurge of studies interconnecting the phenomena of quantum phase transitions, non-equilibrium dynamics, and quantum information and computation. These studies are important from the viewpoint of fundamental physics as well as for developing new quantum technologies. This book is the first attempt to connect these different fields, mentioning both the promises and the problems and incorporating discussions of the most recent technological developments. While there are several books on quantum phase transitions, for example, those by S. Sachdev (Cambridge University Press, 2011) and S. Suzuki et al., (Springer, 2013), the present book emphasizes several different aspects not discussed in earlier books or reviews. We build up from preliminary discussions of the basic phenomenology in the introductory chapter to full exegeses of important models, with further details presented in the appendices. We hope that this structure will enable the beginner to navigate smoothly through the more involved discussions. Concise summaries at the end of each chapter should permit the reader to easily get a sense of the scope of the book.

The book describes generic theories of the scaling of quantum information theoretic measures close to a quantum critical point (QCP) and of the residual energy in the final state reached following a passage through a QCP. This non-adiabatic passage in turn generates non-trivial quantum correlations in the final state which, in some cases, are found to satisfy some intriguing scaling relations. All these theories are illustrated employing the transverse Ising and other transverse field models and their variants. The advantage of using the transverse field Ising model is two-fold: (i) the one-dimensional version with a nearest-neighbor interaction is exactly soluble (and the QCP is conformally invariant), and (ii) the model can be mapped to a classical Ising model with one added dimension using the Suzuki-Trotter or the path integral formalism. remarkable properties of the these models have been exploited thoroughly over the last fifty years, but especially in the last two decades to understand quantum phase transitions and their connection to information processing, non-equilibrium dynamics, While these models have turned out to be useful in and quantum annealing. understanding the scaling of the defect density and related quantum information theoretic measures following a quantum quench, the success of the quantum annealing method in some multivariable optimization problems has raised the expectation of achieving a quantum annealer and hence an efficient quantum computer in the near

We also briefly discuss Tomonaga—Luttinger liquids, topological phase transitions, and related systems. The purpose here is to expose the reader to recent research on the dynamics of information processing that involve these classes of models and quantum phase transitions. Finally, we would like to emphasize that this book presents full discussions of the experimental realizations of quantum transverse field models, including the dynamics of quantum annealing and their connection to attempts to realize a quantum computer.



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Our efforts will be successful if readers, especially readers in the earlier stages of their careers, find this book useful. We hope that this book will lead to further research on the interface of quantum statistical physics, non-equilibrium dynamics, and quantum information processing and computation, leading to further development in quantum technologies.

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