Self-Exciting Fluid Dynamos

Exploring the origins and evolution of magnetic fields in planets, stars and galaxies, this book gives a basic introduction to magnetohydrodynamics, and surveys the observational data with particular focus on geomagnetism and solar magnetism. Pioneering laboratory experiments that seek to replicate particular aspects of fluid dynamo action are also described. The authors provide a complete treatment of laminar dynamo theory and of the mean-field electrodynamics that incorporates the effects of random waves and turbulence. Both dynamo theory and its counterpart, the theory of magnetic relaxation, are covered. Topological constraints associated with conservation of magnetic helicity are thoroughly explored, and major challenges are addressed in areas such as fast-dynamo theory, accretion-disc dynamo theory and the theory of magnetostrophic turbulence. The book is aimed at graduate-level students in mathematics, physics, earth sciences and astrophysics, and will be a valuable resource for researchers at all levels.

KEITH MOFFATT FRS is Emeritus Professor of Mathematical Physics at the University of Cambridge. He has served as Head of the Department of Applied Mathematics and Theoretical Physics and as Director of the Isaac Newton Institute for Mathematical Sciences in Cambridge. A former editor of the *Journal of Fluid Mechanics*, he has published papers in fluid dynamics and magnetohydrodynamics and was a pioneer in the development of topological fluid dynamics. He is a Fellow of the Royal Society, a member of Academia Europæa, and a Foreign Member of the Academies of France, Italy, the Netherlands and the USA. He has been awarded numerous prizes, most recently the 2018 Fluid Dynamics Prize of the American Physical Society.

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Preface

In fifty years almost every book begins to require notes either to explain forgotten allusions and obsolete words; or to subjoin those discoveries which have been made by the gradual advancement of knowledge; or to correct those mistakes which time may have discovered.

Samuel Johnson, *Letters*, 1774

This book is an update and amplification of a research monograph *Magnetic Field Generation in Electrically Conducting Fluids* first published 40 years ago (Moffatt 1978a). Despite the passage of time, much of the work described in that monograph remains at the core of dynamo theory, and it has provided a useful text for graduate students approaching the subject for the first time. Nevertheless, as Samuel Johnson so aptly recorded, it is now desirable "to subjoin those discoveries which have been made by the gradual advancement of knowledge" and "to correct those mistakes which time may have discovered".

It is now exactly 100 years since Larmor (1919) enunciated his famous question "How could a rotating body such as the Sun become a magnet?" A reasonably convincing answer can now be given to this question! And "a rotating body such as the Sun" now includes planets, stars and galaxies, which nearly all exhibit internally generated magnetic fields.

Our knowledge of the self-exciting dynamo process has advanced dramatically over the last 40 years, stimulated by a wealth of satellite data, by great advances in computational power and by the extremely challenging laboratory experiments that seek to replicate either laminar or turbulent dynamo action. Mean-field electrodynamics, greatly developed since the late 1960s, remains at the heart of the subject; this theory takes account of a fluctuating velocity field, in the form of either weak random waves or strong 'Kolmogorov-type turbulence'. It is the mean helicity of this type of flow that is known to be particularly conducive to dynamo action in any conducting fluid of sufficiently large extent. Helicity, and its topological interpretation, therefore continues to play a central role in our approach.

The work is presented here in three parts: Part I treats the theoretical and observational background; Part II covers the foundations of the 'kinematic' dynamo theory that pertains to arbitrary velocity fields; and Part III incorporates dynamics governed by the Navier–Stokes equations in a rotating conducting fluid, including the 'back-reaction' of the Lorentz force distribution associated with a dynamo-generated magnetic field. Much of the presentation is completely new; we draw particular attention to the treatment of the VKS experiment (§9.12), fast dynamo action (Chapter 10), low-dimensional models of the geodynamo (Chapter 11), dynamic equilibration and quenching of the α-effect (Chapter 12), magnetorotational instability (§14.7), magnetostrophic turbulence (§15.6) and the final two chapters, 16 and 17, on magnetic relaxation, which can be viewed as 'the other side of the dynamo-theory coin', important in relation to equilibration in a statistically steady state.

Helicity, a topological invariant of the Euler equations of ideal fluid flow, still plays a central role in dynamo theory. Indeed we may confidently assert that turbulence having non-zero mean helicity will always generate a large-scale magnetic field in an electrically conducting fluid of sufficient spatial extent. The mean-field theories on which this assertion is based are presented in Chapters 7–9. These chapters provide a fundamental basis for the dynamical theories presented in subsequent chapters. Magnetic helicity, a similar topological invariant of the equations of ideal magnetohydrodynamics, plays a correspondingly central role in the theory of magnetic relaxation, providing in general a lower bound on the energy of any magnetic field of non-trivial topology.

We have taught much of the content of this book over many years in courses at graduate level in Cambridge, at the Ecole Normale Supérieure (ENS) and at the Ecole Polytechnique ('X'). However, some of the material, particularly in later chapters, records recent work, as yet presentable only at research seminar level.

We acknowledge the invaluable comments and vital input of many collaborators, who have helped to shape our ideas over many years – Philippe Cardin, Atta Chui, Stéphan Fauve, David Gérard-Varet, Andrew Gilbert, David Hughes, Dominique Jault, Yoshifumi Kimura, David Loper, Dionysis Linardatos, Krzysztof Mizerski, Gordon Ogilvie, Michael Proctor, Glyn Roberts, Renzo Ricca, Andrew Soward, Steve Tobias, Juri Toomre, Vladimir Vladimirov and Nigel Weiss all deserve special mention; above all, the late Konrad Bajer (1956–2014), who was involved in initial planning for this volume, but sadly did not live to see its completion.

Finally, we record our deep gratitude to Linty and Ludivine, without whose constant understanding, encouragement and support this work could not have been accomplished!

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