

Cambridge University Press

978-1-107-06562-8 - Commutative Algebra and Noncommutative Algebraic Geometry:

Volume I: Expository Articles

Edited by David Eisenbud, Srikanth B. Iyengar, Anurag K. Singh,

J. Toby Stafford and Michel Van Den Bergh

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In the 2012–13 academic year, the Mathematical Sciences Research Institute, Berkeley, hosted programs in Commutative Algebra (Fall 2012 and Spring 2013) and Noncommutative Algebraic Geometry and Representation Theory (Spring 2013). There have been many significant developments in these fields in recent years; what is more, the boundary between them has become increasingly blurred. This was apparent during the MSRI program, where there were a number of joint seminars on subjects of common interest: birational geometry, D-modules, invariant theory, matrix factorizations, noncommutative resolutions, singularity categories, support varieties, and tilting theory, to name a few. These volumes reflect the lively interaction between the subjects witnessed at MSRI.

The Introductory Workshops and Connections for Women Workshops for the two programs included lecture series by experts in the field. The volumes include a number of survey articles based on these lectures, along with expository articles and research papers by participants of the programs.

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Mathematical Sciences Research Institute  
Publications

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# Commutative Algebra and Noncommutative Algebraic Geometry

## Volume I: Expository Articles

*Edited by*

**David Eisenbud**

*University of California, Berkeley  
and Mathematical Sciences Research Institute*

**Srikanth B. Iyengar**

*University of Utah*

**Anurag K. Singh**

*University of Utah*

**J. Toby Stafford**

*University of Manchester*

**Michel Van den Bergh**

*Fonds Wetenschappelijk Onderzoek - Vlaanderen  
and Universiteit Hasselt*



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[More information](#)

David Eisenbud  
de@msri.org

Srikanth B. Iyengar  
iyengar@math.utah.edu

Anurag K. Singh  
singh@math.utah.edu

J. Toby Stafford  
toby.stafford@manchester.ac.uk

Michel Van den Bergh  
michel.vandenbergh@uhasselt.be

Silvio Levy (*Series Editor*)  
Mathematical Sciences Research Institute  
levy@msri.org

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## Preface

In the 2012–13 academic year, the Mathematical Sciences Research Institute, in Berkeley, hosted programs in Commutative Algebra (Fall 2012 and Spring 2013) and Noncommutative Algebraic Geometry and Representation Theory (Spring 2013). The programs had 174 participants visiting for periods ranging between one and nine months, and many others for shorter periods and for week-long workshops.

There have been many significant developments in these fields in recent years; what is more, the once rather strict boundary between them has become increasingly blurred. This was apparent during the MSRI program, where there were a number of joint seminars on subjects of common interest: birational geometry,  $\mathcal{D}$ -modules, invariant theory, matrix factorizations, non-commutative resolution of singularities, singularity categories, support varieties, and tilting theory, to name a few. This volume is intended to reflect, and stimulate, the lively interaction between the two subjects that we witnessed at MSRI.

The Introductory Workshops and Connections for Women Workshops for the two programs included lecture series by experts in the field; the volume includes a number of survey articles based on these lectures. There are also expository articles and research papers by some of the other participants of the programs.

In addition to the editors of this volume, the organizers of the programs and the Introductory and Connections for Women workshops were Mike Artin, Georgia Benkart, Victor Ginzburg, Bernard Keller, Ellen Kirkman, Ezra Miller, Claudia Polini, Idun Reiten, Sue Sierra, Karen E. Smith, Catharina Stroppel, Alexander Vainshtein, Lauren Williams, and Efim Zelmanov. We take this opportunity to express our thanks to the participants, our co-organizers, the MSRI staff, and the National Science Foundation, which supported the programs under grant DMS 0932078000, and the National Security Agency, which supported the workshops through grants H98230-12-1-0236/0256/0296/0298.

David Eisenbud  
Srikanth B. Iyengar  
Anurag K. Singh  
J. Toby Stafford  
Michel Van den Bergh